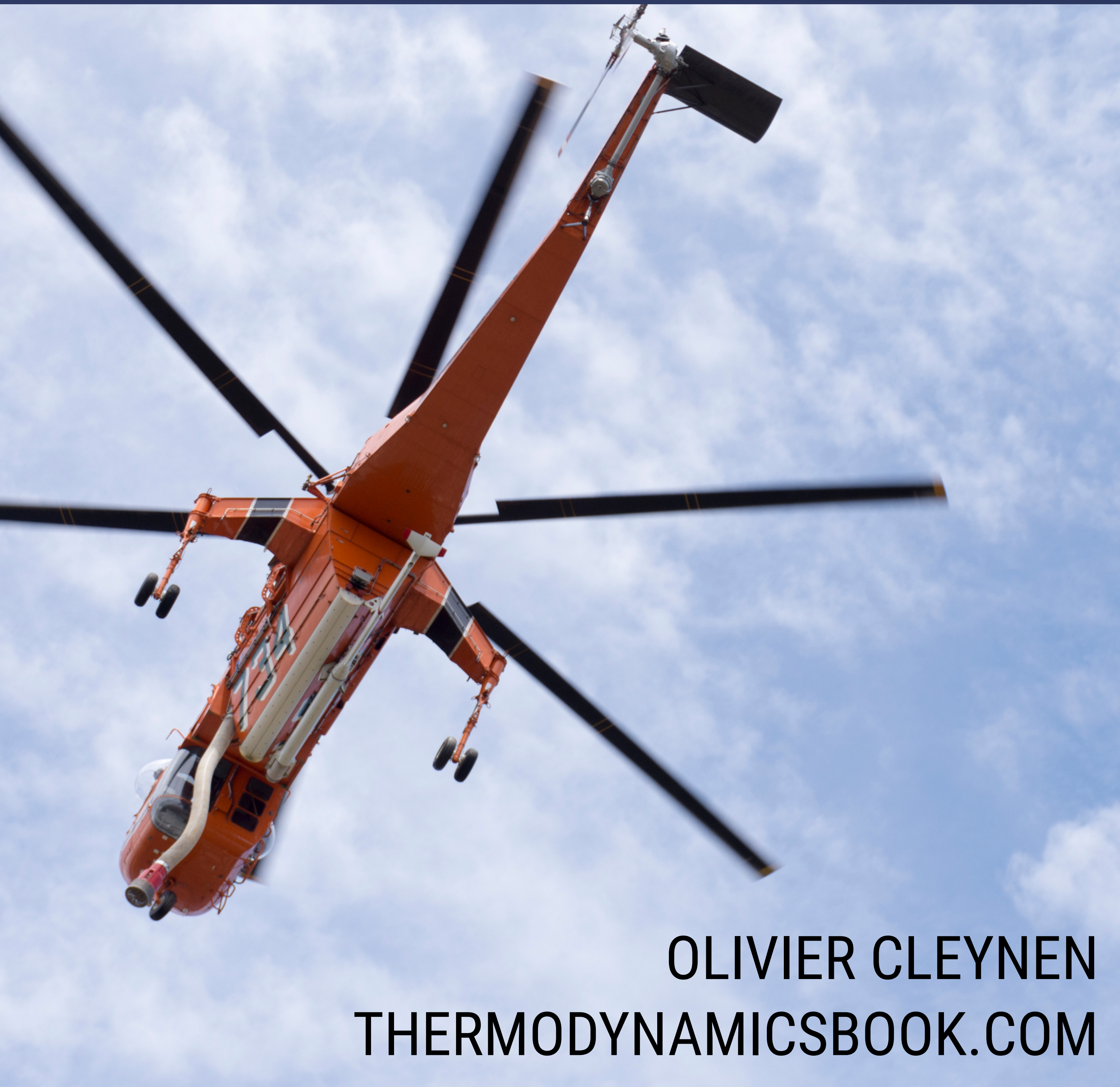
ENGINEERING THERMODYNAMICS

international edition with SI units



OLIVIER CLEYNEN

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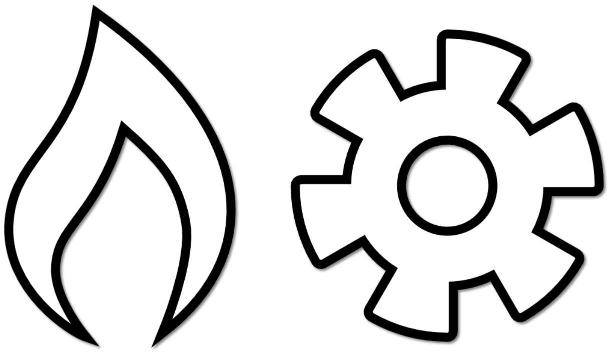
Engineering Thermodynamics



Olivier Cleynen

Engineering Thermodynamics
International edition with SI units, 2025
by Olivier Cleynen
ISBN: 9781446710067

First edition in English (June 2025), Olivier Cleynen / thermodynamicsbook.com
Translated from the third edition of the French original [51]

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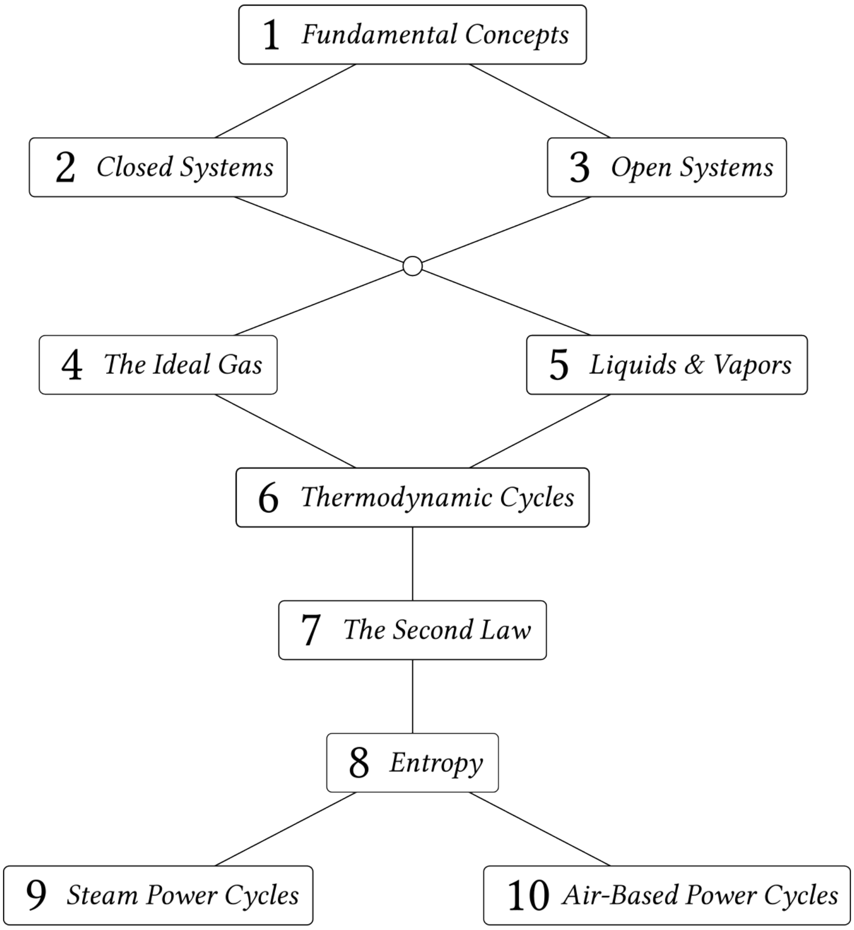
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Introduction

*

Thermodynamics is the study of the conversion of energy between two forms, heat and work. However, its beginnings date back long before these three concepts were established: for a long time, it was only a matter of focusing on the *nature of heat*. In other words, what exactly does "hot" mean? Can it be measured?

The first reflections on the nature of matter and fire date back to ancient Greece and already gave rise to atomic theory. But these were mainly philosophical constructions, based more on an organized spiritual view of the world than on actual observations.

It was not until the 17th century that serious research on this subject began. The concept of temperature, which is easier to grasp than heat, was initially the focus of attention. Designing a thermometer indeed raises many engineering and physics problems: how to link this idea of "temperature" to an observable phenomenon directly, in a predictable and reproducible way?

During these years and until 1850, thermodynamics remained at the macroscopic scale – there was no mention of atoms or molecules yet. It generated a lot of interest because it directly addressed phenomena of friction and heat transfer, which only occur in one direction, and which a Newtonian mechanistic view of the universe cannot explain.

The great development of thermal machines, at the beginning of the 19th century, caught science off guard. Early engines pumped water out of mines, but thermodynamics –which did not even bear its name at the time– could not explain how. It would take about thirty years before theory caught up with practice and a coherent vision of thermodynamics was established, allowing, for example, to predict an engine's efficiency.

In 1865, German physicist Rudolf Clausius concluded nearly a century of trial and error by making explicit the foundational principles of what would come to be known as "thermodynamics": what we now know as the two laws. In doing so, he generalized his observations on a gas balloon to the entire universe. On their side, the Scottish James Clerk Maxwell and the Austrian Ludwig Boltzmann reconciled thermodynamics with particle physics by working at the microscopic level. As the 20th century progressed, the concept of uncertainty became accepted and thermodynamics became a matter of probabilities and quantifying disorder; it even laid the physical foundations of information theory.

In the meantime, the industrial revolution had taken place. Moving away from water pumps, the heat engine transitioned to powering locomotives, then ships, automobiles, electrical generators, and aircraft. Our way of life, in which human physical strength is no longer of any importance, shows how dependent we have become on the power and precision that this engine provides. In essence, it is the reason why our environment differs so much from that of our ancestors, and from what our descendants will experience. Thermodynamics helps us understand the bewildering operation of this machine, which is both ordinary and frightening.

During this series of ten chapters on *Engineering Thermodynamics*, we will progress from the elementary behavior of fluids up to the theory of engines – with the objective of providing students with a good understanding of the operation of heat engines and a solid foundation to later study engine design and fluid mechanics.

CHAPTER 1
Fundamental Concepts

or

The Indispensable Playset of the Little Thermodynamicist

Chapter 1 – Fundamental Concepts

Executive summary for chapter 1

The first law states that energy is constant and indestructible: we *transform* it from one form to another. Work is a transfer involving a force and a displacement. Heat is a chaotic transfer associated with a potential, temperature.

Introduction

We are presenting here the essential concepts that we will use in the following chapters, attempting to answer two questions:

- What does energy represent?
- Which forms of energy are manipulated in a machine?

1.1 Concept of Energy

1.1.1 Energy

We immediately tackle one of the most difficult concepts in all of physics: *energy*.

We observe that in all phenomena, during all the transformations that we can observe in the universe, there exists a quantity that does not vary. This quantity is a measure for an abstract property (energy comes from the Greek ἐνέργεια or *energeia*, meaning “activity”, “operation”) that can take on multiple forms.

We have learned to quantify the amount of energy present in any arbitrary volume, and we strive to control its transformation from one form to another. For example, the electrical energy stored in a battery can be transformed into work in an electric motor, which can be used to operate an elevator, which can lift a load. During all these processes, the total amount of energy remains the same (figure 1.1), a fact that allows us, for example, to quantify the minimum size of battery required to lift a given load.

Figure 1.1: The chemical energy stored in the fuel that has been burned is exactly equal to the energy rejected by the exhaust pipe, plus the energy dissipated by friction, plus the kinetic energy of the moving car. All this energy is transformed into heat, *but never destroyed*, once the car is stopped (whichever the means!).

Photo CC-by-sa *by Tommi Nummelin (cropped)*

Thus, energy is primarily a concept that we use to describe the transformations we observe in the world: we could say that it is “what does not change when things change.” For the engineer, it represents above all the ability of one body to set another in motion, in a unified way (for example, with a displacement) or in a disordered way (for example, with chaotic excitation).

We measure energy in joules (J).

1.1.2 The first law

The *first law of thermodynamics* simply states:

Energy is indestructible.

> « It is important to realize that in physics today, we have no knowledge of what energy *is*. We do not have a picture that energy comes in little blobs of a definite amount. It is not that way. However, there are formulas for calculating some numerical quantity, and when we add it all together it gives "28" —always the same number. It is an abstract thing in that it does not tell us the mechanism or the *reasons* for the various formulas. »
>
> Richard Feynman, 1963 [30, 35]
> *The Feynman Lectures on Physics*

One can also state that "the energy of the universe is constant", or "energy is always conserved": it can neither be created nor destroyed. In other words, when an object receives a joule of energy, it can either store it or deliver it back to the surroundings; but in no case can it destroy it.

There are only two important laws in thermodynamics; the second one (to which we devote chapters 7 and 8) also deals with the nature of energy. Their implications are enormous and they are the result of deep and laborious intellectual work, spanning several centuries. There is no proof or demonstration of their truth, but all our observations and experiments corroborate them, so they are now universally accepted.

We will express the first law quantitatively in two different ways, one for a closed system (in chapter 2, equation 2/2) and the other for an open system (in chapter 3, equation 3/15).

1.1.3 Forms of energy

The different forms of energy that we usually identify have been revealed one by one throughout the history of physics.

Kinetic energy is possessed by a body due to its velocity (see §1.2 below). It is the easiest form of energy to identify. It has long been called *vis viva* ("living force").

Potential energy is stored due to the interaction between two objects linked by a conservative force.[1] On a macroscopic scale, its most palpable form is the potential energy of altitude, resulting from the work done on a mass against its weight (it is this work that makes climbing stairs more tiring than going down, for example). By compressing a spring, potential energy of compression is stored, which can be recovered by letting it expand again.

Chemical energy is a combination of potential energy and kinetic energy *between atoms*. Human metabolism, as well as the combustion of hydrocarbons with atmospheric oxygen used in almost all our vehicles, are both based on chemical energy transfers.

In the 20th century, it was discovered that mass, at the sub-atomic level, was also a form of energy (hence the famous $E = mc^2$ relates mass and energy). Radiant (electromagnetic) energy is also identifiable at the sub-atomic level. These forms of energy are not relevant in this book.

In thermodynamics, we will focus on three forms of energy, identifiable at the macroscopic scale:

---

[1] A force is called *conservative* when it remains the same in one direction as in the other. For example, gravity is conservative (it is the same whether going up or down) but friction is not (it always opposes the motion).

Internal energy denoted U, a concept that we use to group together all of the kinetic and potential energy of all of the molecules of a body. It represents the total amount of mechanical energy stored at the microscopic level within an object;

Heat denoted Q, which is a transfer of the kinetic energy of molecules in a chaotic manner from one body to another;

Work denoted W, which is a transfer of energy in a coherent manner from one body to another.

In general, the thermodynamic engineer wishes to capture heat from bodies that s/he wants to cool, or supply work to bodies that s/he wants to move. We will therefore study these transfers in detail.

1.1.4 Power

Power represents a flow of energy in time. Its SI unit is the joule per second, which we name the watt (W):

$$1\,\mathrm{W} \equiv 1\,\mathrm{J\,s^{-1}} \tag{1/1}$$

Other units are often used, such as the horsepower. One horsepower is approximately the power that a strong horse can deliver as work in full effort (the story in §5.5 p. 143 reveals the name of the creator of this unit). Note that several definitions for horsepower exist (see Appendix A5 p. 319); here we refer to the DIN 66036 standard used in the automotive industry:

$$1\,\mathrm{hp} = 735.5\,\mathrm{W} \tag{1/2}$$

We will denote power by placing a dot above the symbol for energy; thus we write $\dot{E}$ as a power (for example, mechanical) delivering an amount of energy E every second.

In everyday language, the term “power” is used to quantify *the maximum useful power* of a system. For example, when we say that a car has “100 horsepower”, it means has an engine capable of providing it with a power of $\dot{W}_{\text{mech.}} = 100\,\mathrm{hp}$ for a few moments—but for this, the engine receives about $\dot{Q}_{\text{combustion}} = 300\,\mathrm{hp}$ in the form of heat. Furthermore, on the road, the average mechanical power supplied by the engine probably does not exceed $20\,\mathrm{hp}$.

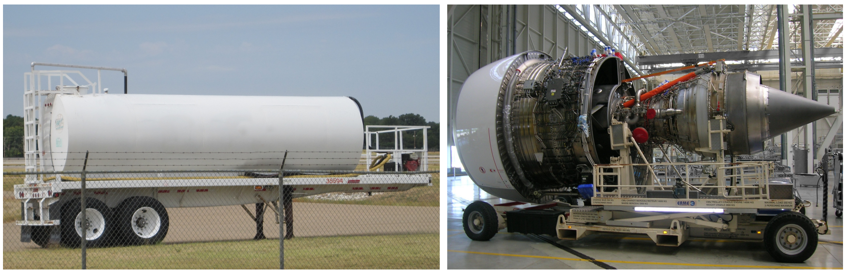


Figure 1.2: A trailer, with zero power ($\dot{Q} = 0\,\mathrm{W}$) but capable of delivering a lot of energy. The combustion of 20 t of kerosene releases about $Q = 800\,\mathrm{GJ}$ as heat;
A *Trent 900* turbofan engine, with very high power (able to supply $\dot{W} = 14\,\mathrm{MW}$ to a commercial aircraft) but devoid of energy (0 J).

Turbofan engine photo derived from a photo CC-0 by Commons User:Dr Brains
Trailer photo derived from a photo CC-BY by Thomas R Machnitzki

1.1.5 Specific energy and specific power

In many thermodynamic applications, it is interesting to quantify energy transfers independently of the amount of mass inside the machine. For example, if we want to compare the *operation* of the engines of a motorcycle and of a truck, it will be wise to divide each of the energy transfers (during compression, combustion, expansion) by the quantity of air in the cylinders, to eliminate the scale effects. To this end, we use quantities called *specific* (sometimes called *mass-specific*); and we denote them in lowercase.

Specific energy (sometimes called mass-specific energy), is measured in joules per kilogram ($\mathrm{J\,kg^{-1}}$):

$$e \equiv \frac{E}{m} \tag{1/3}$$

where e is the specific energy ($\mathrm{J\,kg^{-1}}$),
E is the energy (J),
and m is the mass of the system under consideration (kg).

Example 1.1

A fuel injector in a car engine must supply a specific heat $q_{\text{comb.}} = 300\,\mathrm{kJ\,kg^{-1}}$ regardless of the quantity of air in the cylinder. What will be the energy supplied when $m_{\text{air}} = 0.5\,\mathrm{kg}$ and when $m_{\text{air}} = 1\,\mathrm{kg}$?

The heat required will be $Q_{\text{comb.1}} = m_1 q_{\text{comb.}} = 0.5 \times 300 \times 10^3 = 150 \times 10^3\,\mathrm{J} = 150\,\mathrm{kJ}$ in the first case, and $Q_{\text{comb.2}} = m_2 q_{\text{comb.}} = 300\,\mathrm{kJ}$ in the second case.

Specific power (sometimes also called mass-specific power), has the same units as specific energy: we divide watts (joules per second) by a mass flow rate (kilos per second).

$$e \equiv \frac{\dot{E}}{\dot{m}} \tag{1/4}$$

where e is the specific power ($\mathrm{J\,kg^{-1}}$),
$\dot{E}$ is the power (W),
and $\dot{m}$ is the mass flow rate through the system ($\mathrm{kg\,s^{-1}}$).

Example 1.2

A combustion chamber in a jet engine must supply a specific power $q_{\text{comb.}} = 300\,\mathrm{kJ\,kg^{-1}}$ regardless of the air flow rate through the engine. What will be the power supplied when $\dot{m}_{\text{air}} = 0.5\,\mathrm{kg\,s^{-1}}$ and when $\dot{m}_{\text{air}} = 1\,\mathrm{kg\,s^{-1}}$?

A power $\dot{Q}_{\text{comb.1}} = \dot{m}_1 q_{\text{comb.}} = 0.5 \times 300 \times 10^3 = 150 \times 10^3\,\mathrm{W} = 150\,\mathrm{kW}$ will be required in the first case, and $\dot{Q}_{\text{comb.2}} = \dot{m}_2 q_{\text{comb.}} = 300\,\mathrm{kW}$ in the second case.

☞ Power $\dot{Q}$ and mass flow $\dot{m}$ are written with a dot (flow in time) but not the specific power as heat q, which is measured in $\mathrm{J\,kg^{-1}}$ just like specific heat.

☞ Thanks to the concepts of specific energy and specific power, we can compare the same physical process (air heated up by combustion) in this example with the previous one, in two very different machines.

It should be noted that in practice, the adjectives "specific" or "mass-specific" are often simply omitted, and that the lowercase notation is not systematically used in scientific literature.

1.2 Mechanical Energy

The student will have no difficulty quantifying *kinetic energy*:

$$E_k = \frac{1}{2}\, m\, C^2 \tag{1/5}$$

where E_k is the kinetic energy (J),
m is the mass of the body (kg),
and C is the velocity ($\mathrm{m\,s^{-1}}$, see Appendix A6 p. 321 for other units).

Of course, we also define *specific kinetic energy* correspondingly:

$$e_k \equiv \frac{E_k}{m} = \frac{1}{2}\, C^2 \tag{1/6}$$

In thermodynamics, we are mainly interested in the changes in the energy of fluids within machines. The kinetic energy of gases varies negligibly in piston/cylinder engines, but it plays a major role in jet engines, as we will see in chapter 10 (*air-based power cycles*).

The expression for *potential energy due to altitude* should also not cause any concern for the student:

$$E_p = m\, g\, z \tag{1/7}$$

$$e_p \equiv \frac{E_p}{m} = g\, z \tag{1/8}$$

where g is the gravitational acceleration (usually $9.81\,\mathrm{m\,s^{-2}}$),
and z is the altitude relative to the reference point (m, see Appendix A5 p. 319 for other units).

We will show that in machines, the change of the potential energy of the air due to altitude is always negligible, and that this is often also true for water.

Kinetic energy and potential energy due to altitude are often combined into a single term, called *mechanical energy*:

$$e_m \equiv e_k + e_p = \frac{1}{2} C^2 + g\, z \tag{1/9}$$

Example 1.3

A student is coasting down a mountain road with a bicycle. At a point with an altitude of 540 m (1772 ft), his/her speed is 10 km/h (6.214 mph). A few moments later, passing a point at an altitude of 490 m (1608 ft), his/her speed is 45 km/h (27.96 mph). The mass of the cyclist together with his/her equipment is 70 kg (154.3 lb).

How much energy has s/he dissipated in the form of friction?

The student's mechanical energy changed by

$$\Delta E_m = E_{m2} - E_{m1} = m(e_{m2} - e_{m1})$$
$$= m(gz_2 - gz_1 + \tfrac{1}{2}C_2^2 - \tfrac{1}{2}C_1^2) = m\left[g(z_2 - z_1) + \tfrac{1}{2}(C_2^2 - C_1^2)\right]$$
$$= 70\left[9.81(490 - 540) + \tfrac{1}{2}\left(\left(\tfrac{45\times10^3}{3600}\right)^2 - \left(\tfrac{10\times10^3}{3600}\right)^2\right)\right]$$
$$= 70\left[-490.5 + 74.3\right] = -2.91 \times 10^4\,\text{J} = -29.1\,\text{kJ}.$$

The student has lost 29.1 kJ of mechanical energy. This quantity was transferred to the atmosphere in the form of turbulence and heat, and to the bicycle's bearings and tires in the form of heat.

☞ Energy changes may well be negative. Kinetic energy is however always positive.

☞ Refer to Appendix A5 p. 319 for converting to and from SI units.

☞ The passage of the bicycle through the air causes observable disturbances on a macroscopic scale that we call *turbulence*. After a short time, this kinetic energy has dissipated down to a microscopic scale, warming up the atmosphere.

1.3 Work

Work is a transfer of energy. An object does work (and thus loses energy) when it exerts a force over a displacement. In mechanics, this work is quantified using vectors:

$$W \equiv \vec{F} \cdot \vec{l} \qquad (1/10)$$

where W is the work (J),
$\vec{F}$ is the vector representing the force (of magnitude F in N),
and $\vec{l}$ is the vector representing the displacement (of magnitude l in m).

In thermodynamics, we will use this equation 1/10 to quantify the work done by fluids. In order to do this, we will rewrite it by adding three particularities:

- We will measure the displacement *with the length of the object that does the work*;
- We will only consider cases where the vectors $\vec{F}$ and $\vec{l}$ are collinear;
- We will take into account the fact that $\vec{F}$ can vary as a function of $\vec{l}$.

With these three constraints, equation 1/10 becomes:

$$W_{\mathrm{A}\to\mathrm{B}} = \int_{\mathrm{A}}^{\mathrm{B}} \vec{F} \cdot \mathrm{d}\vec{l}$$

Since $\mathrm{d}\vec{l}$ is measured from the length of the object performing the work, $\mathrm{d}l$ will be negative when W is positive (work is then done *to* the object, causing its length to decrease). Finally, since $\vec{F}$ is always collinear with $\mathrm{d}\vec{l}$ in our case, we can write:

$$W_{\mathrm{A}\to\mathrm{B}} = -\int_{\mathrm{A}}^{\mathrm{B}} F \, \mathrm{d}l \tag{1/11}$$

where $W_{\mathrm{A}\to\mathrm{B}}$ is the work done between two points A and B (J),
F is the force (N),
and $\mathrm{d}l$ is the infinitesimal change in the length of the considered object (m).

On a diagram representing the force as a function of distance, this work $W_{\mathrm{A}\to\mathrm{B}}$ is represented by the area under the curve from A to B (figure 1.3). The shape of the curve, in other words, the relationship $F_{(l)}$ between F and l during the process, will determine the quantity $W_{\mathrm{A}\to\mathrm{B}}$.

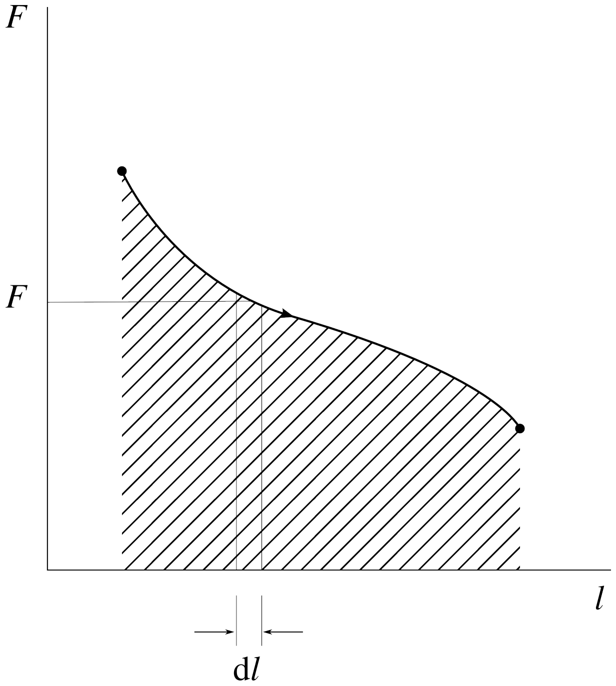


Figure 1.3: On a force-distance diagram, the work done by an object can be visualized by the area under the curve. In the case shown here, the object's length l increases, and the work will be negative (done by the object).

Diagram CC-0 *Olivier Cleynen*

Example 1.4

A spring is compressed from a length of 30 cm (11.81 in) down to a length of 5 cm (1.968 in). The spring is such that it exerts a force (in newtons) independent of its length and equal to:

$$F_{(l)} = 6 \times 10^3 \mathrm{~N}$$

What is the energy supplied to the spring in the form of work during compression?

The work done is obtained using equation 1/11, making sure to set the boundaries in SI units: $W_{\mathrm{A}\to\mathrm{B}} = -\int_{\mathrm{A}}^{\mathrm{B}} F_{(l)}\,\mathrm{d}l = -\int_{\mathrm{A}}^{\mathrm{B}} 6\times10^{3}\,\mathrm{d}l = -6\times10^{3}\int_{\mathrm{A}}^{\mathrm{B}}\mathrm{d}l = -6\times10^{3}\,[l]_{l_{\mathrm{A}}}^{l_{\mathrm{B}}} = -6\times10^{3}(0.05-0.3) = +1.5\times10^{3}\,\mathrm{J} = +1.5\,\mathrm{kJ}$.

☞ The sign of the transferred work is positive: the spring has *received* energy. This does not suprise us: its length has decreased as it was compressed.

☞ Springs with such a characteristic (independent of their length) are often ribbon springs, like those used in mechanical watches.

Example 1.5

Another spring is also compressed from a length of 30 cm down to a length of 5 cm. It is such that it exerts a force (in N) related to its length l (in m) by the relation:

$$F_{(l)} = 9\times10^{3} - 14\times10^{3}\,l$$

What is the energy supplied to the spring in the form of work during compression?

The work done is still obtained using equation 1/11, and the integral is only slightly more complex: $W_{\mathrm{A}\to\mathrm{B}} = -\int_{\mathrm{A}}^{\mathrm{B}}(9\times10^{3} - 14\times10^{3}l)\,\mathrm{d}l = -\left[9\times10^{3}\,l - \frac{1}{2}14\times10^{3}\,l^{2}\right]_{l_{\mathrm{A}}}^{l_{\mathrm{B}}} = -10^{3}\left[9l - 7l^{2}\right]_{0.3}^{0.05} = -10^{3}(0.4325 - 2.07) = +1.6375\times10^{3}\,\mathrm{J} = +1.638\,\mathrm{kJ}$.

☞ Springs with such a characteristic (force proportional to length) have regular coils.

Example 1.6

One final spring is compressed from a length of 30 cm down to a length of 5 cm. It is such that it exerts a force (in N) related to its length l (in m) by the relation:

$$F_{(l)} = 14\times10^{3} - 12\times10^{3}\,l^{0.3}$$

What is the energy supplied to the spring in the form of work during compression?

The work done is still obtained using equation 1/11, $W_{\mathrm{A}\to\mathrm{B}} = -\int_{\mathrm{A}}^{\mathrm{B}}(14\times10^{3} - 12\times10^{3}\,l^{0.3})\,\mathrm{d}l = -10^{3}\left[14\,l - \frac{1}{0.3+1}12\,l^{0.3+1}\right]_{0.3}^{0.05} = -10^{3}\,(0.5121 - 2.2703) = +1.7582\times10^{3}\,\mathrm{J} = +1.758\,\mathrm{kJ}$.

☞ Springs with such a characteristic are called progressive springs: very soft at first, but increasing rapidly in hardness. They are often used in automobile suspensions. We will see in chapter 2 (*closed systems*) that gases have a similar characteristic.

1.4 Heat

1.4.1 Temperature

For now, we define *temperature* as a body's potential for supplying or receiving heat.

The temperature of a body is a quantity that indicates its level of internal excitation. The higher the kinetic energy of its molecules, with different speeds and directions, the higher its temperature will be.

When the molecules making up a body are perfectly stationary relative to each other, the body has no internal vibration: this state defines zero temperature. In contrast, the temperature scale is open towards infinity. There is no defined maximum temperature point.

We cannot simply measure the "mean kinetic energy of the molecules" of a body, and as a result, it is very difficult to rigorously define a temperature scale (for example, what it means for a temperature to be "twice as large"). We will revisit the concept of temperature itself in chapter 4 (*the ideal gas*) and define it fully in chapter 7 (*the second law*). In the meantime, we will accept the definition given above.

Temperature is measured in kelvins (K), on a scale created for the needs of thermodynamics and rather immodestly referred to as *absolute*.

Students will likely be familiar with at least one of two temperature scales:

- the Celsius scale (°C). Simply subtracting 273.15 units from an absolute temperature (in kelvins) gives a temperature in degrees Celsius:

$$T(^\circ\mathrm{C}) \equiv T(\mathrm{K}) - 273.15 \qquad (1/12)$$

$$T(\mathrm{K}) = T(^\circ\mathrm{C}) + 273.15 \qquad (1/13)$$

- the Fahrenheit scale (°F). The translation to kelvins is a little more complex:

$$T(^\circ\mathrm{F}) = 1.8 \times [T(\mathrm{K}) - 273.15] + 32 \qquad (1/14)$$

$$T(\mathrm{K}) = \frac{T(^\circ\mathrm{F}) - 32}{1.8} + 273.15 \qquad (1/15)$$

Both of those scales precede the Kelvin scale, and they were cleverly redefined and synchronized with it in 1848 (wee will have the opportunity to study this clever manipulation in chapter 7, see §7.4 p. 193). Purists will note that the absolute unit is named kelvin and not "degree Kelvin". Some indicative temperatures are listed in table 1.1.

kelvins	degrees Celsius	
0	−273.15	Absolute zero (by definition)
10^{-10}	−273.1499999999	Lowest temperature ever reached (only a few particles)
4.22	−268.93	Helium boiling at atmospheric pressure
44	−229	Average temperature of the surface of Pluto*
184	−89.4	Lowest recorded atmospheric temperature on Earth*
273.15	0	Water melting at atmospheric pressure
327	54	Highest recorded atmospheric temperature on Earth*
373.15	100	Water boiling at atmospheric pressure
400	127	Nose of a Concorde in cruise flight*
483	200	Ordinary household oven*
485	210	Autoignition of diesel fuel*
753	480	Leading edges of a Lockheed SR-71 Blackbird in cruise*
1 100	830	Wood fire*
1 900	1 600	Space Shuttle heat shield on atmospheric re-entry*
2 500		Incandescent lamp filament
5 000		Melting point of diamond (at 12 GPa)
5 800		Surface of the Sun
16×10^{6}		Center of the Sun
3×10^{9}		Within a nuclear weapon detonation
3×10^{9}		Core of a massive star on its last day
1×10^{12}		Particles colliding within the RHIC
1.417×10^{32}		The Universe 5.391×10^{-44} s after the Big Bang

Table 1.1: Examples of temperatures. Values with an asterisk are converted approximately.

1.4.2 Heat

> « These results are inexplicable if heat be a substance. »
>
> James Joule, 1845
> *On the Changes of Temperature Produced by the Rarefaction and Condensation of Air* [8]

> « These circumstances … pressingly demand a comparison between heat and work, to be undertaken with reference to the divergent assumption that the production of work is not only due to an alteration in the *distribution* of heat, but to an actual *consumption* thereof; and inversely, that by the consumption of work heat may be *produced*. »
>
> Rudolf Clausius, 1850 [10, 11, 21]

When two bodies of different temperatures are brought into contact, their temperatures tend to equalize during a spontaneous transfer of energy. We call this form of energy *heat*.

Heat, written Q, is **a form of energy** (measured in joules). On a macroscopic scale, it is a transfer of energy in chaotic form. It can be caused in several ways, the most relevant for the engineer being:

- loss of internal energy of a body, by coming into contact with a lower temperature body;
- friction;
- disappearance of mass in a nuclear reaction;
- transformation of potential energy between atoms, by chemical reaction (especially the combustion of hydrocarbons with atmospheric oxygen).

Just like we denote heat as Q (J), we denote *specific heat* as q ($\mathrm{J\,kg^{-1}}$).

The concept of heat is very difficult to understand. It was long believed to be a fluid (the *caloric*) of very low density, capable of permeating all materials. This theory was abandoned in the mid-19th century, when it was shown that *heat is not conserved*, that is, it has the ability to disappear or appear. For example, an engine in operation receives heat (through combustion) but releases less than it received. It transforms part of it into work, which we can then use to propel a vehicle.

On a microscopic scale, in other words, when we consider the movement of individual particles, the concepts of temperature and heat are even more difficult to define (Richard Feynman [30, 35] explores this beautifully). However, this is beyond the scope of this book.

1.4.3 Thermal capacity

When the same amount of heat is supplied to two different bodies, their temperature can increase in different ways – for example, it takes less heat to raise the temperature of a kilogram of steel than a kilogram of aluminum. This tendency of a body's temperature to increase is called its *thermal capacity* (or *heat capacity*).

The specific thermal capacity of a body is defined as the amount of heat required to raise the temperature of one kilogram of the substance by one kelvin:

$$c \equiv \frac{\delta q}{\mathrm{d}T} = \frac{1}{m}\frac{\delta Q}{\mathrm{d}T} \tag{1/16}$$

where c is the *specific thermal capacity* of the substance ($\mathrm{J\,kg^{-1}\,K^{-1}}$),
δq is a (specific) infinitesimal quantity of heat ($\mathrm{J\,kg^{-1}}$),
δQ is an infinitesimal quantity of heat (J),
m is the mass (kg),
and $\mathrm{d}T$ is an infinitesimal change in temperature (K or °C).

In this equation 1/16, the infinitesimal transfer of heat is denoted by the symbol δ, while the infinitesimal change in temperature is denoted by the symbol d. This distinction is harmless and is detailed in Appendix A4 p. 318.

The specific thermal capacity of solids is generally invariant. However, for fluids, which we use extensively in machines, it is not so simple:

- By working a gas (namely, by allowing it to push on a movable wall), we significantly increase its specific thermal capacity. We will quantify this phenomenon in chapter 4 (*the ideal gas*).
- The specific thermal capacity of liquids and vapors becomes infinite (!) during boiling, which takes place over a particular range of properties. Outside of this range, the capacity becomes finite again, but it varies with temperature. We will describe these behaviors in chapter 5 (*liquids and vapors*).

Example 1.7

The specific thermal capacity of solid steel is constant (independent of temperature) and has a value of $c_{\text{steel}} = 475\,\mathrm{J\,kg^{-1}\,K^{-1}}$. How much heat is required to change the temperature of a 50 kg (110.2 lb) block of steel from a temperature of $T_\mathrm{A} = 5\,^\circ\mathrm{C}$ to a temperature of $T_\mathrm{B} = 18\,^\circ\mathrm{C}$?

We use the definition 1/16 to write, in the general case:

$$\begin{aligned} c_{\text{steel}} &= \frac{1}{m_{\text{steel}}}\frac{\delta Q}{\mathrm{d}T} \\ \delta Q &= c_{\text{steel}}\, m_{\text{steel}}\, \mathrm{d}T \\ Q_{\mathrm{A}\to\mathrm{B}} &= \int_\mathrm{A}^\mathrm{B} m_{\text{steel}}\, c_{\text{steel}}\, \mathrm{d}T \end{aligned}$$

Since the capacity c_{steel} is independent of T, this integral becomes simply: $Q_{\text{A}\to\text{B}} = m_{\text{steel}}\, c_{\text{steel}} \int_{\text{A}}^{\text{B}} \mathrm{d}T = m_{\text{steel}}\, c_{\text{steel}}(T_{\text{B}} - T_{\text{A}}) = 50 \times 475 \times (18 - 5) = +3.0875 \times 10^5\ \text{J} = +308.8\ \text{kJ}$.

☞ During integration, $\int_{\text{A}}^{\text{B}} \mathrm{d}T$ becomes ΔT (a temperature difference), while $\int_{\text{A}}^{\text{B}} \delta Q$ becomes simply $Q_{\text{A}\to\text{B}}$ (a transfer between two states). Heat, a *path quantity*, is transferred, while temperature, a *state quantity*, is increased (see Appendix A4 on this topic).

☞ In this book, when we quantify energy transfers, we convene to make their sign explicit (so we add a "+" in positive transfers).

☞ A conversion of the temperatures to kelvins would not have changed the value of ΔT. The result would then have been the same.

☞ With an electrical resistance of the power of a standard domestic heater (2 kW), it would take $\Delta t = \frac{Q_{\text{A}\to\text{B}}}{\dot{Q}} = \frac{308.8\times10^3}{2\times10^3} = 154\ \text{s}$ to warm up the steel, just over two minutes. We will see in chapter 4 (*the ideal gas*) that air at constant pressure has a specific thermal capacity three times greater than that of steel.

1.5 Hot and Cold

We conclude this chapter by revisiting some common language terms, as they are understood in thermodynamics.

Hot — For us, "hot" is not a property of objects: instead of "this object is hot", we say that its temperature is high. Instead of "this object is heating up/cooling down" we say that its temperature is increasing or decreasing. In everyday language, phrases like "it is hot" or "heat wave" also refer to temperature.

To heat — For us, "to heat up" means to supply heat. We can "heat up" an object while its temperature drops. We can also raise the temperature of an object without supplying heat (figure 1.4).

Cold — For us, the sensation of "cold" denotes a low temperature. We do not consider "cold" to be something that can be manufactured or measured. Instead, we would say that we are transfering heat away from an object (for example, a refrigerator extracts heat from a warm food item).

Fire — Fire is the term given to the emission of light (electromagnetic radiation) from a gas at high temperatures. In thermodynamics, "fire" does not have any special properties. For us, it is the same heat whether it is generated by the combustion of wood or kerosene, by friction in a brake, or by a nuclear reaction. Ultimately, the only thing that matters is the temperature at which it is transmitted!

Figure 1.4: Left: when air is compressed in a compressor, air gives heat away through the sides and the fins of the cylinders; and yet, its temperature increases. Right: by contrast, when liquid oxygen is expanded in a vane, liquid oxygen receives heat from the atmosphere (as evidenced by the condensation and frost from atmospheric air on the piping); in spite of this, its temperature drops.
Compressor photo CC-BY-SA *Fábio Teixeira*
Liquid oxygen photo public domain Jensen Stidham / USAF

Thermometer — We leave it to the student to explore how thermometers work: how can we *know* in absolute terms that a temperature is high or low?

We simply note that we humans are ourselves very poor thermometers: since the human body attempts to maintain a constant temperature, our sensations of "hot" or "cold" are intrinsically linked to heat transfer.

Even though this vocabulary probably puts us among the unsociable scientists relegated to the end of the table, it equips us better to face what's next, because in the next chapter, we will be dealing with *closed systems*.

« The principle to be followed in constructing a thermometric scale might at first sight seem to be obvious, as it might appear that a perfect thermometer would indicate equal additions of heat, as corresponding to equal elevations of temperature, estimated by the numbered divisions of its scale. It is however now recognized (from the variations in the specific heats of bodies) as an experimentally demonstrated fact that thermometry under this condition is impossible, and we are left without any principle on which to found an absolute thermometric scale. »

William Thomson
(not yet crowned *Baron Kelvin*…)
1848 [9]

A Bit of History: Measuring the Degree of Heat

*

By Philippe Depondt
Pierre and Marie Curie University, Paris

For Aristotle, in the 4th century BCE in Greece, fire was one of the four elements of matter along with water, air, and earth. The idea of measuring something, be it fire or anything else, that is, assigning a numerical value to a quantity, was completely foreign to him because his physics was essentially non-mathematical [31]: his theories were based on *qualitative* observations. The synthesis of Aristotle's ideas with Christianity was made in the 12th century by Thomas Aquinas, and these ideas were widely dominant in the scholarly world in Europe until the early 17th century (for example, Galileo had to argue largely *against* these ideas).

Until the 17th century, descriptions of the world would unfortunately remain largely qualitative. The exception provided by astronomers is telling: in establishing his heliocentric model in the early 16th century, Copernicus could rely on measurements dating back to antiquity, and then on those of Arab astronomers from the Middle Ages. Similarly, it was the remarkably rigorous and precise measurements (less than one minute of angle) carried out in Tycho Brahe's modern "laboratory" that led to Johannes Kepler's discovery of his three laws, which formed one of the foundations of Newton's dynamics.

In the case of thermodynamics, the English philosopher Francis Bacon, laying the foundations of inductive reasoning at the beginning of the 17th century, used heat as an example to illustrate his point. In order to study its nature, he proposed in the *Novum Organum* to compile all observations of phenomena in which heat appears, of phenomena where it does not appear, and finally of those where it appears "by degrees." This method remained qualitative, but at approximately the same time, there was an explosion of attempts to make truly quantitative measurements of this "degree of heat."

It seems that the first thermometer was invented around 1605 by a Dutchman named Cornelis Drebbel [39]: based on ideas dating back to Hero of Alexandria (1st century CE), it consisted of a hollow glass sphere extended by a tube pointing downwards and immersed in a colored liquid. If the sphere was heated, the liquid was pushed downwards by the expansion of the air, and conversely, if it was cooled, the liquid rose in the tube. It was thus an air thermometer (figure 1.5). This thermometer was later used to monitor fever in patients (figure 1.6), but it had the drawback of being as sensitive to changes in atmospheric pressure as to temperature.

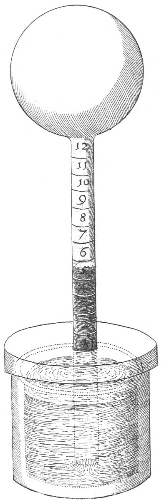


Figure 1.5: Air thermometer from the early 17th century. The ball was filled with air whose volume varies with its temperature, pushing the water from the reservoir below whose surface is at atmospheric pressure. The liquid could be colored, and its changes height were measured using a scale. Since atmospheric pressure varies with meteorological conditions, it affected the measurements: it was, in a way, a baro-thermometer.

Engraving by Robert Fludd (1626, public domain), selected by Lamouline 2005 [45]

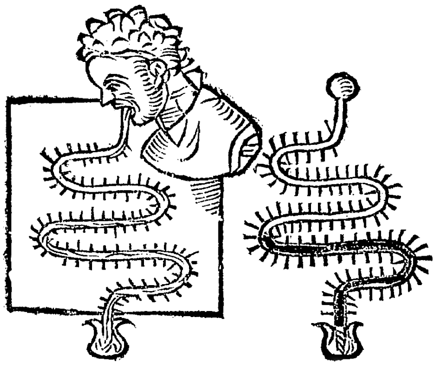


Figure 1.6: Medical thermometer from the early 17th century. The gas bulb was placed in the patient's mouth. One can well imagine that the thermometer's sensitivity to atmospheric pressure was not the biggest obstacle to its adoption…

Drawing by Santori & Avicenne (Commentaria in primam Fen primi libr Avicennae, 1625, public domain), selected by Lamouline 2005 [45]

In the middle of the century, liquid thermometers would prove to be much more reliable and easier to use. The glass bulb was now placed at the bottom of the device and filled with colored liquid that rose in a graduated tube; this tube was initially open, but it was found that by closing it, evaporation of the liquid could be prevented (figure 1.7). These improvements had been strongly supported by the Italian grand duke Ferdinando II de' Medici, and these devices were thus called "Florence thermometers."

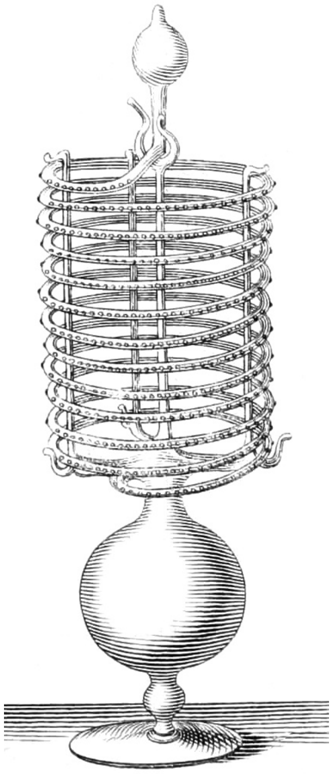


Figure 1.7: Florence thermometer from the mid-17th century. This time, it was the liquid, contained in the lower bulb, that contracted and expanded with temperature. Its changes in volume were such that a long spiral-blown glass tube was needed to measure them.

Engraving by the Accademia del cimento (Staggi di naturali esperientze, 1667, public domain), selected by Lamouline 2005 [45]

The issue of graduations remained. The number of graduations varied widely, with artisans merely attempting to reproduce what they had already created themselves; at best, thermometers built by the same person gave roughly similar results. Due to the lack of a universally accepted scale, it was impossible to make measurements in different locations with different instruments for comparison.

In the early 18th century, the Frenchman Guillaume Amontons built an air thermometer based on the measurement of a pressure difference rather than volume. Having observed that if boiling water continued to be heated, its degree of heat did not increase, he used this as a fixed point of reference. Of course, the measurements needed to be corrected by simultaneously measuring atmospheric pressure. This system allowed Amontons to make a major discovery: if the gas pressure increases as the degree of heat increases, conversely, it decreases as the degree of heat decreases. At a minimum, this pressure must become zero, as well as the degree of heat. This extrapolated minimum corresponds to, in modern units, $-239.5\,^{\circ}\mathrm{C}$... A first measure of absolute zero!

However, all these thermometers remained difficult to use, significantly limiting their dissemination. René-Antoine Ferchault de Réaumur, around the middle of the 18th century, developed a water-alcohol mixture thermometer in which the alcohol level is precisely fixed to ensure the reproducibility of the instrument. He calibrated it by choosing two references (melting ice and boiling water) and dividing this interval into 80 degrees. This scale is called the "Réaumur scale."

In 1724, in Danzig, the German Daniel Gabriel Fahrenheit described a thermometer that used the expansion of mercury and introduced a scale in which melting ice is at 32 degrees and the temperature of blood at 96 degrees; a mixture of ice, water, and sal ammoniac gave him the zero of his scale (see also section §7.4.3 p. 193).

In 1741, the Swedish Anders Celsius adopted the Réaumur scale but divided it into 100 intervals instead of 80. This convention was widely spread in France, and in 1794, at the time of the adoption of the metric system by the Convention, the Celsius scale was chosen as the official temperature scale.

The transition from the subjective sensation of hot and cold to the objective measurement of temperature with reliable instruments and a universal scale led to a large number of observations that were not self-evident until then: the temperature of a cellar is not higher in winter than in summer, iron is not "colder" than wood, etc., and all in all, this is quite recent!

Problems

1.1 Free-Wheeling Bicycle

A cyclist starts a free wheel descent. With their equipment and bike, their mass is 75 kg (165.3 lb). As they pass a point at an altitude of 1200 m (3937 ft), their speed is 50 km/h (31.1 mph). Exactly 7 min later, as they pass a point at an altitude of 950 m, their speed is 62 km/h.

1. How much energy has the cyclist dissipated in the form of friction between these two points?

Further down the descent, still freewheeling, the cyclist sees their speed stabilize at 45 km/h on a slope of 4 %.

2. What is the power with which the cyclist dissipates energy in the form of friction?

1.2 Power of a Steam Power Plant

The path followed by the water in a steam power plant can be represented in a simplified way as follows (figure 1.8):

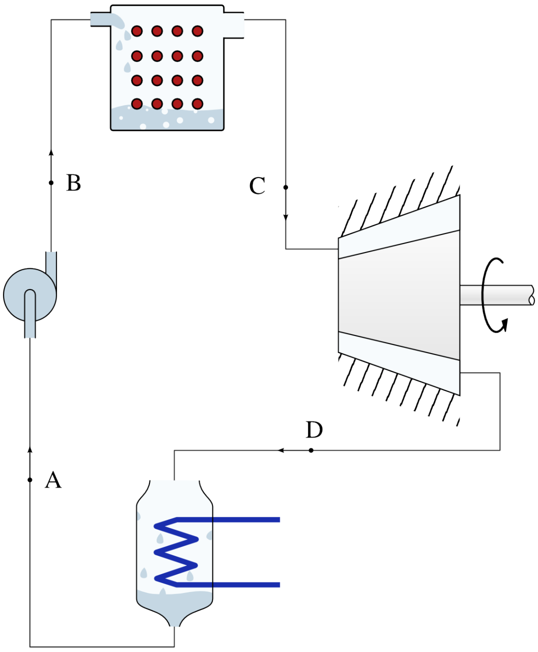


Figure 1.8: Simplified diagram of the water circuit inside a steam power plant. The water follows a complete cycle through four processes.
This circuit, called the Rankine cycle, is studied in more detail in chapter 9 (*steam power cycles*) (section 9.4.2 p. 254).

Diagram CC-BY-SA *Olivier Cleynen*

From A to B the liquid water is compressed in the pump. It receives a specific work $w_{A\to B} = +50\,\mathrm{kJ\,kg^{-1}}$, with no heat transfer.

From B to C the water is heated in the boiler, where it exits as steam. It receives a specific heat $q_{B\to C} = +450\,\mathrm{kJ\,kg^{-1}}$, without receiving any work.

From C to D the water expands in the turbine, where it delivers a specific work $w_{C\to D} = -194\,\mathrm{kJ\,kg^{-1}}$, without receiving or giving heat.

From D to A the water is cooled in a condenser, with no work transfer. It returns to its original state and properties before returning to the pump to be compressed again.

The water flow rate circulating in the plant is $15\,\mathrm{kg\,s^{-1}}$.

1. What is the specific power rejected in the form of heat in the condenser?
2. What is the power (in watts) rejected by the condenser?
3. What is the power (in watts) generated by the turbine in the form of work?
4. What is the efficiency η_{plant} of the power plant, that is, the ratio between its net power and the power it receives as heat?

1.3 Compression of Springs

In the laboratory of a company manufacturing automotive suspension systems, an engineer compares the characteristics of three springs of different geometry. To do this, s/he measures the force F (in N) exerted by each spring as a function of its length l (in m), and models these behaviors as follows:

- $F_{A\ (l)} = 8 \times 10^3 - 2 \times 10^3\ l$
- $F_{B\ (l)} = 8 \times 10^3 - 3 \times 10^3\ l^{1.6}$
- $F_{C\ (l)} = 0.1 \times 10^3\ l^{-3}$

What is the amount of work required to compress each of these springs from a length of 40 cm (15.74 in) down to a length of 12 cm (4.724 in)?

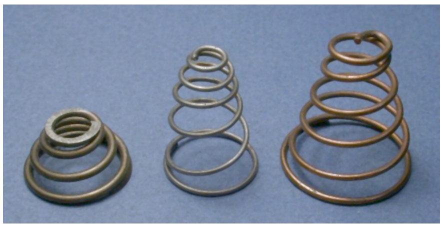


Figure 1.9: Conical springs, whose stiffness increases exponentially when compressed. We will see in chapter 2 (*closed systems*) that when fluids are compressed and expanded slowly, they behave similarly to spring C, which has a conical geometry like those shown here.

Photo CC-BY-SA *Jean-Jacques Milan*

1.4 Spring-Powered Engine

We model the operation of a gasoline engine by replacing the air in a cylinder with a spring. We want to quantify the energy received and then rejected by a powerful spring during a back-and-forth motion (similar to the air during the compression and expansion phases of a piston engine cycle).

The experiment proceeds cyclically with the following four steps (figure 1.10):

From 1 to 2: The experimenter compresses a spring from a length of 25 cm to a length of 8 cm. The spring exerts a force related to its length (in meters) by the relation:

$$F = 25.4 \times 10^3 - 40 \times 10^3 \, l \qquad (1/17)$$

where F is the force (in N);

and l is the length of the spring (in m).

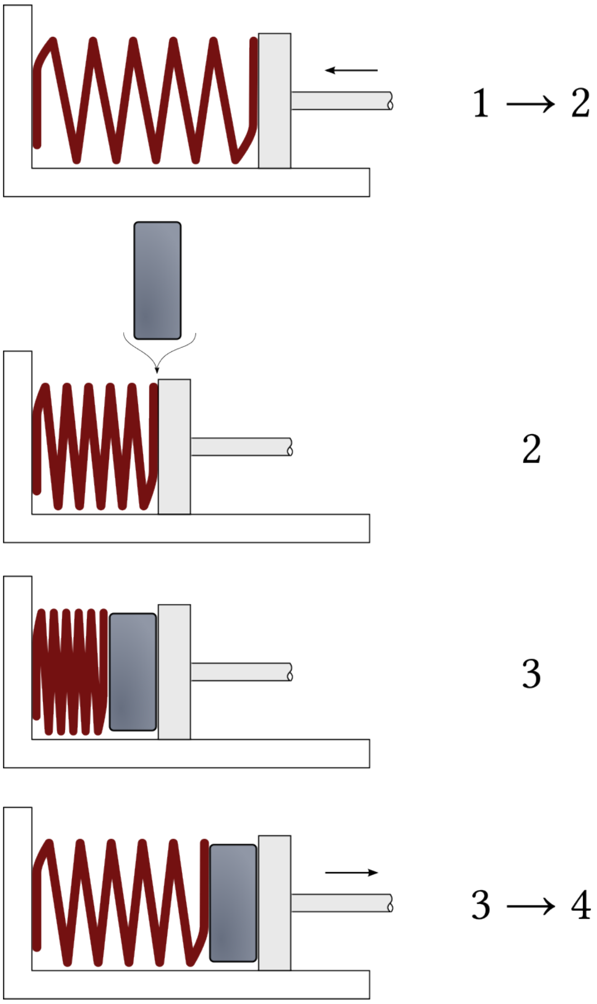


Figure 1.10: Experiment conducted with a powerful spring. The piston compresses the spring from 1 to 2, then the spring pushes back the piston from 3 to 4. On the return path, the force exerted by the spring is greater.

Diagram CC-BY-SA *Olivier Cleynen*

From 2 to 3: When the length of the spring reaches 8 cm, the experimenter blocks the piston's movement. A solid block is then inserted between the piston wall and the spring.
The force on the piston (which has not moved) increases until reaching 32 kN.

From 3 to 4: Once the block has been inserted, the experimenter reverses the motion with the piston until the final length reaches again 25 cm.

From 4 to 1: The block is removed without moving the piston, and the force on the piston returns to the value it had at the beginning of the experiment.

We want to quantify the energy received and then rejected by the {spring + block} assembly during one back and forth motion.

1. Draw the process on a diagram showing the force as a function of the inner length, qualitatively (that is, without showing numerical values).
2. How much energy did the spring receive from the experimenter during the outbound journey (from 1 to 2)?
3. What is the characteristic $F_{(l)}$ of the {spring + block} assembly during the return journey (from 3 to 4)?
4. How much energy did the spring receive from the piston during the return journey (from 3 to 4)?
5. In the end, how much energy did the experimenter receive or spend during the experiment?
6. At what frequency should the experiment be repeated for the power to reach 25 hp, that is, 18.4 kW?

1.5 Preparing a Bath

A student exhausted from doing integral calculus with springs wishes to take a bath.

Tap water arrives at a temperature of 10 °C (50 °F) in the electric water heater; it has a constant thermal capacity of $c_{\text{liquid water}} = 4.2\,\text{kJ}\,\text{kg}^{-1}\,\text{K}^{-1}$ and a constant density $\rho_{\text{liquid water}} = 10^3\,\text{kg}\,\text{m}^{-3}$.

1. How much energy is needed to heat the water to 40 °C (104 °F) in order to fill a bathtub of 270 L (10.57 US gal)?
2. How long will it take the heater to increase the temperature of the water if its heating power is $\dot{Q} = +2\,\text{kW}$?

1.6 Hydraulic Jack

We want to lift a vehicle with a mass of 1200 kg (2645.5 lb) using the hydraulic jack schematized in figure 1.11. The surface area of the left piston is 5 cm^2 (0.775 sq in).

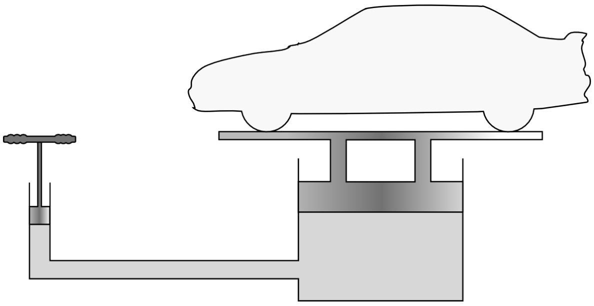


Figure 1.11: Schematic diagram of a hydraulic jack.

Diagram CC-0 *Olivier Cleynen*

The oil within the jack is assumed to be incompressible, meaning its volume is considered constant regardless of pressure.

The purpose of the setup is to allow an average-sized person to lift and hold the vehicle in place with the left piston (whose end is equipped with handles).

1. Design the right piston (under the vehicle) so that the force in the left piston does not exceed 100 N (22.48 lbf).
2. What is the power required to hold the vehicle in place?

We want to lift the vehicle by 25 cm (9.843 in), in less than 30 seconds.

3. How far would the left piston need to be pushed for this?
4. What would be the work and power to be supplied in this case?

1.7 Water Turbine

A constant flow rate of 1200 $\text{kg}\,\text{s}^{-1}$ of water passes through a small hydraulic power plant represented in figure 1.12.

- At point 1, the water arrives at a speed of 3 $\text{m}\,\text{s}^{-1}$ with a temperature of $T_1 = 5\,^\circ\text{C}$ and an altitude $z_1 = 75\,\text{m}$ (246.1 ft).
- At point 2, it exits at a speed of 2.5 $\text{m}\,\text{s}^{-1}$ with a temperature $T_2 = 5.04\,^\circ\text{C}$ and an altitude $z_2 = 4\,\text{m}$ (13.12 ft).

The water pressure is the same at 1 and 2, and the velocity profile of the water at each point is approximately uniform. The water has a specific thermal capacity of $c_{\text{liquid water}} = 4.2\,\text{kJ}\,\text{kg}^{-1}\,\text{K}^{-1}$.

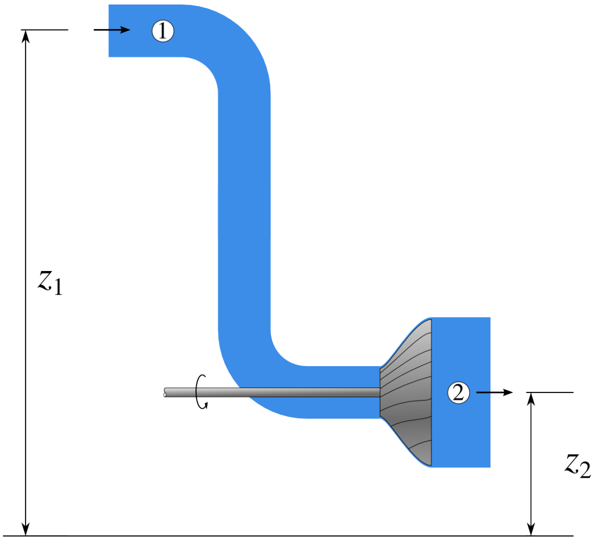


Figure 1.12: Schematic diagram of a water turbine. Water enters at the top left, rotates the turbine blades, is heated by internal friction, and exits at the bottom right of the machine.

Diagram CC-BY-SA *Olivier Cleynen*

1. What is the specific mechanical power received or supplied by the water as it passes through the power plant?
2. What is the specific power provided as heat by internal friction?
3. What is the power (in watts) released in the form of work by the turbine?

1.8 Central Heating Boiler

The boiler of the central heating system of a building, represented in figure 1.13, operates with the combustion of kerosene.

The water enters the boiler at a temperature $T_C = 20\,^\circ\text{C}$ (68 °F) and exits at $T_D = 70\,^\circ\text{C}$ (158 °F), with a flow rate $\dot{V}_{\text{water}} = 0.25\,\text{L}\,\text{s}^{-1}$ (0.054 99 imp gal).

The combustion chamber admits air at $T_A = 8\,^\circ\text{C}$ (46.4 °F) and it exits through the chimney at a temperature $T_B = 120\,^\circ\text{C}$ (248 °F); the air flow rate is $\dot{m}_{\text{air}} = 0.5\,\text{kg}\,\text{s}^{-1}$.

Specific heat of combustion of kerosene: 46.4 $\text{MJ}\,\text{kg}^{-1}$

Specific thermal capacity of liquid water: 4.18 $\text{kJ}\,\text{kg}^{-1}\,\text{K}^{-1}$

Specific thermal capacity of air at constant pressure: 1.15 $\text{kJ}\,\text{kg}^{-1}\,\text{K}^{-1}$

1. What is the hourly consumption of kerosene by the boiler?
2. What is the efficiency of the boiler, that is, the ratio between its useful heat transfer and its energy consumption?

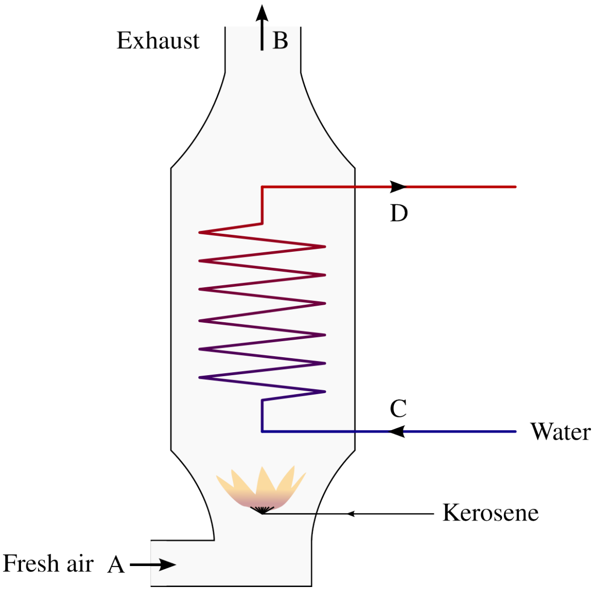


Figure 1.13: Schematic diagram of a boiler used for heating a building. The water (C → D) enters from the right and is heated by the air (A → B) mixed with kerosene.

Diagram CC-BY-SA *Olivier Cleynen*

1.9 Helicopter Turboshaft Engine

A helicopter is equipped with two turboshaft engines, namely, gas turbines whose purpose is to rotate a shaft emerging from the engine (figure 1.14). We can evaluate several characteristics of these engines without knowing the details of their internal operation.

Each of the two engines admits atmospheric air at a temperature of 15 °C. The air is compressed, heated, and then expanded, which allows work to be generated to rotate the rotors. At the engine outlet, the air is discharged at atmospheric pressure and a temperature of 360 °C (680 °F).

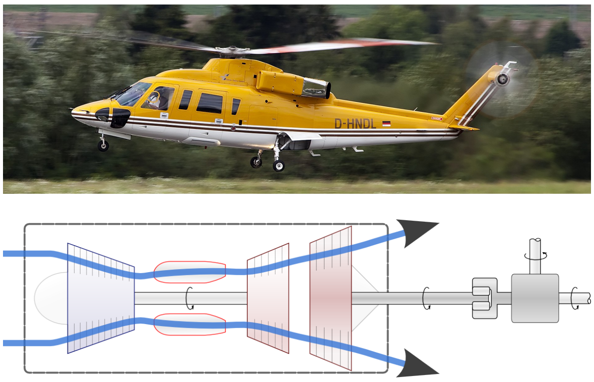


Figure 1.14: A Sikorsky S-76B helicopter, equipped with two *P&WC* PT-6B turboshaft engines, each with 980 hp. The airflow through the engines is shown in a schematic diagram. We will study these engines in more detail in chapter 10 (*air-based power cycles*).

Helicopter photo CC-BY-SA *Maarten Visser (cropped)*
Diagram CC-BY-SA *Olivier Cleynen*

At constant pressure, the specific thermal capacity of air is approximately $c_{p \text{ air}} = 1050\,\mathrm{J\,kg^{-1}\,K^{-1}}$. The combustion of kerosene releases $q_{\text{kerosene}} = 46\,\mathrm{MJ\,kg^{-1}}$.

1. What is the specific power rejected by the engines in the form of heat to the atmosphere?
   Hint: this is the specific heat that the rejected air must lose to return to its initial temperature.

The flight manual indicates that in the combustion chamber (the part of the engine where the fuel is burned), the air is admitted at a temperature of 250 °C and is heated by the combustion, at constant pressure, up to 776 °C (1428.8 °F).

2. What is the specific power generated by the engines in the form of work?
   Hint: in the end, the net energy supplied by the air in the form of work and heat had been supplied to it in the combustion chamber.

In order to maintain the helicopter in stationary flight at full load, the rotors require a total power from the two engines in the form of work of 1.32 MW (approximately 1800 hp).

3. What is the total air flow rate that needs to be admitted to the two turboshaft engines?
4. What is the total power (in W) to be supplied in the two combustion chambers?
5. What is the hourly consumption of kerosene for the helicopter in stationary flight?

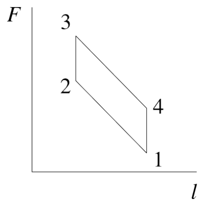
Answers

1.1 1) $E_{m2} - E_{m1} = -180\,\mathrm{kJ}$ (the $7\,\mathrm{min}$ of time, of course, does not have any importance)
2) $\dot{Q}_{\text{friction}} = m\ g\ |\dot{z}| = 368\,\mathrm{W}$.

1.2 1) In the end, the water has rejected as much energy as it has received, so $q_{\text{D}\to\text{A}} = -w_{\text{A}\to\text{B}} - q_{\text{B}\to\text{C}} - w_{\text{C}\to\text{D}} = -306\,\mathrm{kJ\,kg^{-1}}$ We ran engines for forty years before we understood this!
2) $\dot{Q}_{\text{D}\to\text{A}} = \dot{m}q_{\text{D}\to\text{A}} = -4.59\,\mathrm{MW}$
3) $\dot{W}_{\text{C}\to\text{D}} = \dot{m}w_{\text{C}\to\text{D}} = -2.91\,\mathrm{MW}$
4) $\eta_{\text{plant}} = \left|\frac{w_{\text{turbine}}+w_{\text{pump}}}{q_{\text{boiler}}}\right| = 32\,\%$ (realistic value).

1.3 1) $W_{\text{A}} = \int_{l_1}^{l_2} F_{(l)}\,\mathrm{d}l = -10^3\left[8l - \frac{1}{2}2l^2\right]_{0,4}^{0,12} = +2.094\,\mathrm{kJ}$.
2) $W_{\text{B}} = +2.138\,\mathrm{kJ}$
3) $W_{\text{C}} = +3.157\,\mathrm{kJ}$.

1.4



2) $W_{1\to2} = +3.196\,\mathrm{kJ}$
3) $F_{(l)3\to4} = 35.2\times10^3 - 40\times10^3 l$
4) $W_{3\to4} = -4.862\,\mathrm{kJ}$
5) $W_{\text{cycle}} = 1.666\,\mathrm{kJ}$
6) $f = 11.04\,\mathrm{s^{-1}}$ (11 times per second)

1.5 1) $Q_{\text{water}} = \rho_{\text{water}}V_{\text{water}}c_{\text{water}}\Delta T = +34.02\,\mathrm{MJ}$
2) $\Delta t = \frac{Q_{\text{water}}}{\dot{Q}} = 4.7\,\mathrm{h}$

1.6 1) $A_2 \leq \frac{F_2}{p_2} = \frac{F_2}{p_1} = 5.89\times10^{-2}\,\mathrm{m^2} = 589\,\mathrm{cm^2}$ (91.3 sq in)
2) $\dot{W} = 0\,\mathrm{W}$ of course, since there is no movement…
3) By calculating the swept oil volume V, $d_1 = \frac{V_1}{A_1} = \frac{V_2}{A_1} = 29.43\,\mathrm{m}$, an impracticable length, which can be avoided by adding a pumping mechanism.
4) $\dot{W}_{\text{mean}} \leq \frac{W_{\text{A}\to\text{B}}}{\Delta t} = 98.1\,\mathrm{W}$.

1.7 1) $\Delta e_m = -697.9\,\mathrm{J\,kg^{-1}}$ (so, a loss by water)
2) $q_{1\to2} = +168\,\mathrm{J\,kg^{-1}}$
3) $\dot{W}_{\text{turbine}} = \dot{m}(\Delta e_m + q_{1\to2}) = -635.9\,\mathrm{kW}$.

1.8 1) $\dot{m}_{\text{kerosene}} = \frac{\dot{Q}_{\text{kerosene}}}{q_{\text{kerosene}}} = \frac{-\dot{Q}_{\text{water}}-\dot{Q}_{\text{air}}}{q_{\text{kerosene}}} = 2.51\times10^{-3}\,\mathrm{kg\,s^{-1}} = 9.1\,\mathrm{kg\,h^{-1}}$
2) $\eta_{\text{boiler}} = \left|\frac{\dot{Q}_{\text{water}}}{\dot{Q}_{\text{kerosene}}}\right| = 44.8\,\%$

1.9 1) $q_{\text{rejected}} = +362.25\,\mathrm{kJ\,kg^{-1}}$
2) $q_{\text{chamber}} + w_{\text{shaft}} + q_{\text{atmospheric cooling}} = 0$ or $q_{\text{chamber}} + w_{\text{shaft}} - q_{\text{rejected}} = 0$; thus we have $w_{\text{shaft}} = -q_{\text{chamber}} + q_{\text{rejected}} = -190.1\,\mathrm{kJ\,kg^{-1}}$ (these powers do not depend on the mass flow, and are independent of the number of engines taken into account).
3) $\dot{m}_{\text{air}} = 6.95\,\mathrm{kg\,s^{-1}}$
4) $\dot{Q}_{\text{chambers}} = 3.836\,\mathrm{MW}$
5) $\dot{m}_{\text{kerosene}} = \frac{\dot{Q}_{\text{chambers}}}{q_{\text{kerosene}}} = 300.2\,\mathrm{kg\,h^{-1}}$ (realistic value).

CHAPTER 2
Closed Systems

or

A Short Treatise for Energy Accounting

Chapter 2 – Closed Systems

Executive summary for chapter 2

A closed system contains a fixed amount of mass. Heat and work transfers cause variations in the *internal energy* of the fluid. For work to be reversible, the movement must be infinitely slow.

Introduction

We wish to develop here a method of energy accounting applied to a fixed quantity of mass. This chapter 2 (*closed systems*) aims to answer two questions:

- How to quantify the work that can be received and provided by a body of fixed mass?
- What is reversibility, and why are we pursuing it?

2.1 Why Use a Closed System?

From now on, we want to describe and quantify energy transfers in fluids. We can adopt two different points of view to observe the fluid:

- Either we "cut out" a small piece of mass, which we closely follow as it moves along, and then quantify the energy transferred to it: this is what we call a *closed system*;
- Or we choose a fixed volume, which is constantly being *crossed* by a mass flow, and then quantify the energy transfers towards the volume: this is what we call an *open system*.

Of course, these two methods are equivalent: they will produce the same results. The choice of one or the other will simply make the analysis and quantification of transfers easier.

The use of a closed system is appropriate for analyzing machines with reciprocating motion (automotive engines, pumps and compressors, and generally all piston/cylinder machines). These machines divide the fluid into small quantities that are trapped in a chamber, where they are heated, cooled, compressed, or expanded (figure 2.1). It is then easy to identify a given mass quantity and quantify the transfers it undergoes.

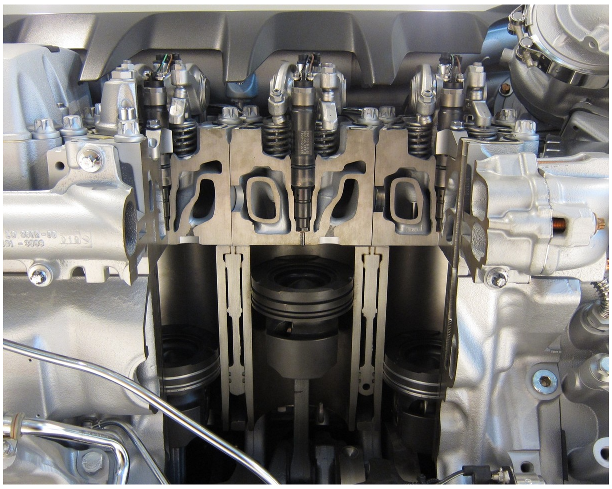


Figure 2.1: A cutaway in a truck engine reveals three pistons in their cylinders. A closed system is a useful tool for studying the air trapped in a cylinder. The photographed engine is a diesel V8 from MAN.

Photo CC-BY-SA *Olivier Cleynen*

On the contrary, to study what happens in a jet engine nozzle, for example, we would have difficulties identifying a given group of particles and quantifying the change in their properties. It would then be more convenient to use an open system, as we will study in chapter 3 (*open systems*).

Concretely, in this chapter, we want to quantify the work that can be done by a fluid in a cylinder. A car engine provides work because the air in the cylinders provides more work by expanding on the return stroke than it received by being compressed on the intake stroke (figure 2.2). How can we generate this? In order to answer this question, we need a robust method to quantify energy transfers.

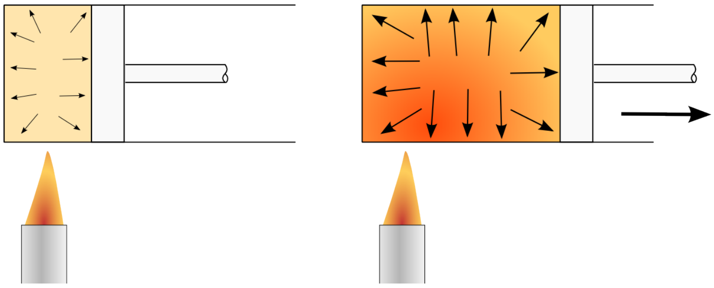


Figure 2.2: Operating principle of an engine. When heat is supplied to a fluid in a closed reservoir, it increases the forces it exerts on the walls of the reservoir. By allowing the reservoir to deform, we let the fluid perform work.

Diagram CC-0 *Olivier Cleynen*

2.2 Accounting Conventions

2.2.1 The closed system

We refer to a *closed system* as an arbitrary study subject with boundaries impermeable to mass: a given set of particles, with fixed mass. All properties of this set (pressure, temperature, volume, etc.) can change, but it always involves the same molecules, not mixed with others. For example, a gas trapped in a cylinder and compressed by a piston (figure 2.3) is perfectly described with a closed system.

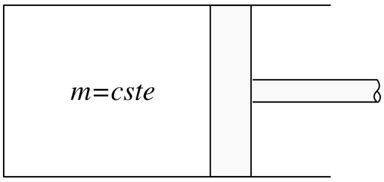


Figure 2.3: A typical closed system: a fixed mass quantity in a closed reservoir. A movable wall allows to compress it; we will also allow it to receive and lose heat.

Diagram CC-0 *Olivier Cleynen*

2.2.2 Sign conventions

In order to quantify transfers, we will use the following sign convention, illustrated in figure 2.4:

- When they are positive, transfers Q and W indicate a *receipt* by the system.
- Conversely, when they are negative, transfers Q and W indicate a *loss* from the system. The system then supplies work W and rejects heat Q.

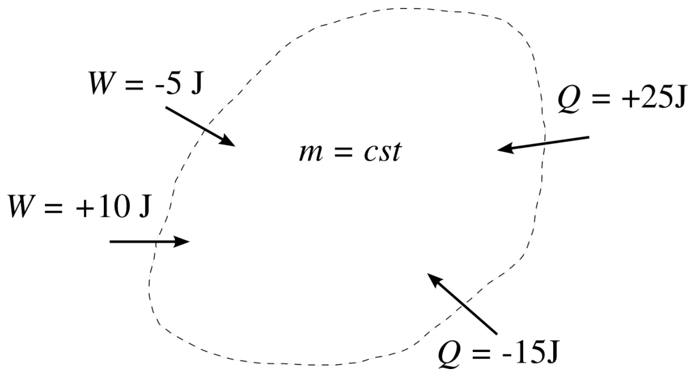


Figure 2.4: Sign conventions for a closed system. Incoming flows are positive, outgoing flows are negative; they are all represented with inward-pointing arrows. The mass quantity is constant.

Diagram CC-0 *Olivier Cleynen*

Thus, in the equations, we can systematically add the terms without needing to know the direction of the changes. Transfers are accounted for like in a bank account: expenses are negative and revenues are positive.

2.3 The First Law in a Closed System

The first law states that energy is indestructible (§1.1.2). If we supply 100 J of work to a closed system and it rejects 80 J in the form of heat, then “its” energy has increased by 20 J. We call this increase the *change of internal energy*, ΔU.

In the form of an equation, the first law in a closed system is expressed by the equation:

$$Q_{1\to2} + W_{1\to2} = \Delta U \tag{2/1}$$

for a stationary closed system;
where $\Delta U = U_2 - U_1$ is the change of internal energy (J),
$W_{1\to2}$ is the work received by the system (J),
and $Q_{1\to2}$ is the heat received by the system (J).

Unfortunately, internal energy U is sometimes very difficult to measure. We shall see in chapters 4 and 5 that bodies store this internal energy in different ways, and that it is intimately related to temperature. By definition, internal energy U is always positive, but its variation ΔU can also be negative.

Equation 2/1 can be expressed with specific quantities:

$$m\,(q_{1\to2} + w_{1\to2}) = m\,\Delta u$$

$$q_{1\to2} + w_{1\to2} = \Delta u \tag{2/2}$$

for a stationary closed system;
where $\Delta u = u_2 - u_1$ is the change of specific internal energy ($\mathrm{J\,kg^{-1}}$),
$w_{1\to2}$ is the specific work received by the system ($\mathrm{J\,kg^{-1}}$),
and $q_{1\to2}$ is the specific heat received by the system ($\mathrm{J\,kg^{-1}}$).

We can rewrite this equation 2/2 to express it in its *differential form*:

$$\delta q + \delta w = \mathrm{d}u \tag{2/3}$$

for a stationary closed system;
where $\mathrm{d}u$ is the infinitesimal change of specific internal energy ($\mathrm{J\,kg^{-1}}$),

« Let therefore be Q the whole quantity of heat which one must impart to a body while it transitions on a certain path from one state into another (whereby a withdrawn quantity of heat is counted as a negatively imparted heat quantity), so we divide this into three parts, of which the first comprehends the increase of the heat actually present in the body, the second the heat consumed for internal work and the third the heat consumed for external work. Of the first part, the same holds true as has already been said of the second, that it is independent of the manner in which the change has occurred, and therefore we can represent both parts together by a function U, of which, even if we do not yet know it more closely, we know at least this much beforehand, that it is fully determined by the initial and final state of the body. »

Rudolf Clausius, 1854
Über eine veränderte Form des zweiten Hauptsatzes der mechanischen Wärmetheorie [13]

δw is the (specific) infinitesimal work transfer ($\mathrm{J\,kg^{-1}}$),
and δq is the (specific) infinitesimal heat transfer ($\mathrm{J\,kg^{-1}}$).

In this equation 2/3, the mathematical operators d and δ have slightly different meanings: $\mathrm{d}u$, an *exact differential*, represents an infinitesimal *change* that will integrate to $\Delta u = u_2 - u_1$; on the other hand, an δw, *inexact differential*, represents an infinitesimal *transfer* that will integrate to $w_{1\to2}$. This distinction is further elaborated in Appendix A4 p. 318.

When a fluid is brought back to its initial state (same pressure, same volume, same temperature), then it contains exactly the same amount of internal energy as before. The total energy it has received (in the form of heat or work) has therefore necessarily been returned to the surroundings in one form or another. We express this statement as follows:

$$Q_\text{cycle} + W_\text{cycle} = 0 \tag{2/4}$$

for a complete thermodynamic cycle,
where W_cycle is the work received by the system (J),
and Q_cycle is the heat received by the system (J).

This equation 2/4 is the reason why the first law is often stated—without adding much to our simple statement in chapter 1 — in the following way:

"When a system has completed a full thermodynamic cycle, the algebraic sum of the heat it has supplied and the work it has done is zero."

2.4 Quantifying Work With a Closed System

Calculating work with fluids is delicate. We will proceed in three steps of increasing complexity:

- By replacing the fluid with a spring;
- By compressing the fluid infinitely slowly;
- By compressing the fluid rapidly.

2.4.1 Work as a function of volume, with a spring

Let's start by imagining that the fluid within a closed system behaves like a metal spring (figure 2.5). This is an interesting modeling approach to begin our study. We had seen in §1.3 that the work supplied or received by a spring is expressed as:

$$W_{\mathrm{A}\to\mathrm{B}} = -\int_\mathrm{A}^\mathrm{B} F\,\mathrm{d}l \tag{1/11}$$

Today, since we are using a fluid, we want to express work in terms of *pressure* and *volume* rather than force and length.

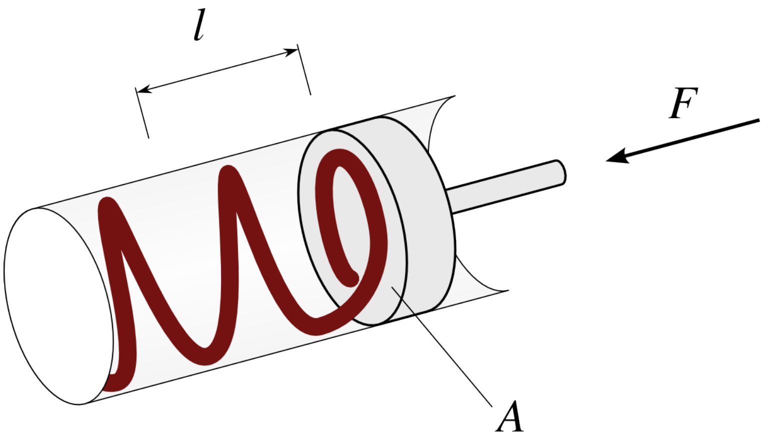


Figure 2.5: Initially, we model the fluid inside the system with a metal spring.

Diagram CC-BY-SA *Olivier Cleynen*

Pressure is defined as a force divided by an area:

$$p \equiv \frac{F}{A} \tag{2/5}$$

where p is the pressure (Pa),
F is the force (N),
and A is the area of the surface on which the force acts (m^2).

The SI unit of pressure is the Pascal,

$$1\,\text{Pa} \equiv 1\,\text{N}\,\text{m}^{-2} \tag{2/6}$$

but three other units are commonly used instead:

- the bar:
$$1\,\text{bar} \equiv 1 \times 10^5\,\text{Pa} \tag{2/7}$$
- the kilogram – force per square centimeter, which is almost equal to the bar:
$$1\,\text{kg}_\text{f}/\text{cm}^2 = 9.806\,65 \times 10^4\,\text{Pa} \tag{2/8}$$
- and the pound per square inch:
$$1\,\text{psi} = 6.894\,757 \times 10^3\,\text{Pa} \tag{2/9}$$

Note that atmospheric pressure at low altitude is of the order of one bar or one kilogram-force per square centimeter ($p_{\text{atm.std.}} \equiv 1\,\text{atm} \equiv 1.013\,25\,\text{bar} \approx 14.7\,\text{psi}$). Care is needed when reading pressure on measurement devices, which often indicate *gauge pressure* and not real pressure. This difference is described in Appendix A2 p. 316.

Volume can also be easily expressed. If the system is deformed by a piston with area A, such that its length varies by $\text{d}l$, we have:

$$\text{d}V = A\,\text{d}l \tag{2/10}$$

where $\text{d}V$ is the infinitesimal change in volume (m^3),
A is the area of the piston surface being displaced (m^2),
and $\text{d}l$ is the infinitesimal change in length of the system corresponding to the piston displacement (m).

In the SI system of units, volume is measured in m^3, but the student is likely accustomed to using at least one of three common units:

- The liter:

$$1\,\mathrm{L} \equiv 10^{-3}\,\mathrm{m}^3 \tag{2/11}$$

- The US gallon:

$$1\,\mathrm{US\ gal} \equiv 3.785\,411\,784 \times 10^{-3}\,\mathrm{m}^3 \tag{2/12}$$

- The imperial gallon:

$$1\,\mathrm{imp\ gal} \equiv 4.546\,09 \times 10^{-3}\,\mathrm{m}^3 \tag{2/13}$$

Let's now express the work of a closed system in terms of volume and pressure. By inserting equations 2/5 and 2/10 into equation 1/11, we obtain:

$$W_{\mathrm{A}\to\mathrm{B}} = -\int_{\mathrm{A}}^{\mathrm{B}} F\,\mathrm{d}l = -\int_{\mathrm{A}}^{\mathrm{B}} \frac{F}{A} A\,\mathrm{d}l$$

$$W_{\mathrm{A}\to\mathrm{B}} = -\int_{\mathrm{A}}^{\mathrm{B}} p\,\mathrm{d}V \tag{2/14}$$

for a closed system modeled by a spring,
where $W_{\mathrm{A}\to\mathrm{B}}$ is the work received by the system (J),
p is the (uniform) internal pressure (Pa),
and $\mathrm{d}V$ is the change in volume (m^3).

In order to quantify the work done on or by the system, we only need to know the relationship between p and V. In this case, this function $p_{(V)}$ is directly related to the characteristic $F_{(l)}$ of the spring. The spring stiffness and its geometry (regular or progressive coils) will ultimately determine the amount of work received or supplied by the system.

A powerful tool for understanding and analyzing work transfers is the *pressure-volume diagram*. In the case where the fluid is modeled by a spring, work can be visualized by the area under the curve (figure 2.6).

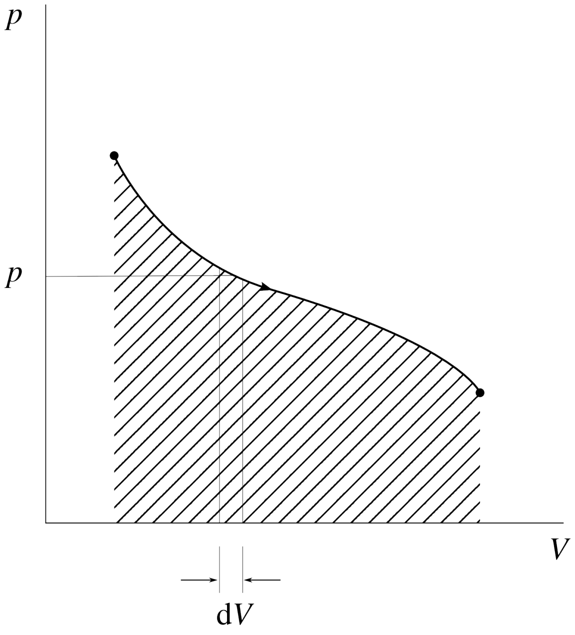
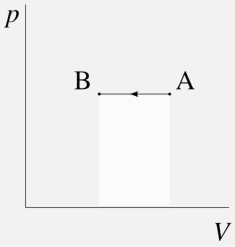


Figure 2.6: Pressure-volume diagram of a closed system modeled by a spring. In the case shown here, the volume is increasing (the piston moves away). The quantity dV remains positive, and the work is negative: the system loses energy by doing work on the piston. This figure represents the same phenomenon as in fig. 1.3 p. 17, using different physical quantities. *Diagram* CC-0 *Olivier Cleynen*

Example 2.1

A closed system consists of an empty box in which there is a spring. The pressure exerted by the spring on the walls of the box is constant at $p = 10^5\ \mathrm{Pa}$ regardless of its volume. The box is compressed from a volume $V_\mathrm{A} = 2\ \mathrm{L}$ to $V_\mathrm{B} = 1\ \mathrm{L}$. What is the work transfer?

The process can be drawn qualitatively (that is, without showing numerical values) on a pressure-volume diagram as follows:



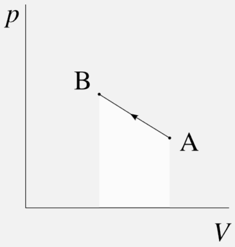
Starting from equation 2/14: $W_{\mathrm{A}\to\mathrm{B}} = -\int_\mathrm{A}^\mathrm{B} p\, \mathrm{d}V = -p_\mathrm{cst.} \int_\mathrm{A}^\mathrm{B} \mathrm{d}V = p_\mathrm{cst.} \left[V\right]_{V_\mathrm{A}}^{V_\mathrm{B}} = -10^5(1 \times 10^{-3} - 2 \times 10^{-3}) = +100\ \mathrm{J}$.

☞ The sign is positive: the box (“the system”) receives work. We always explicitly specify the sign when quantifying transfers.

Example 2.2

A closed system has an internal pressure related to its volume by the relation $p = 7 \times 10^5 - 2 \times 10^8\ V$ (in SI units). The box is compressed from a volume $V_\mathrm{A} = 2\ \mathrm{L}$ to $V_\mathrm{B} = 1\ \mathrm{L}$. How much energy has been received or supplied as work?

The process can be drawn qualitatively on a pressure-volume diagram as follows:



Once again, starting from equation 2/14: $W_{\mathrm{A}\to\mathrm{B}} = -\int_\mathrm{A}^\mathrm{B} p\, \mathrm{d}V = -\int_\mathrm{A}^\mathrm{B} (7 \times 10^5 - 2 \times 10^8 V)\, \mathrm{d}V = -\left[7 \times 10^5 V - \frac{1}{2} 2 \times 10^8 V^2\right]_{V_\mathrm{A}}^{V_\mathrm{B}} = -(700 - 100 - 1400 + 400) = +400\ \mathrm{J}$ (positive: work received by the system).

2.4.2 Work of a fluid in a slow process

When a fluid is compressed, the molecules it is made of are brought closer to each other (figure 2.7) and the collisions between them and against the walls become more frequent. On a macroscopic scale, this increase results in an increase in pressure.

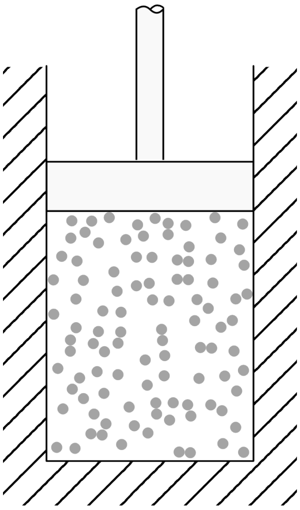


Figure 2.7: A simplistic representation of a fluid being infinitely slowly compressed without heating it. The fluid sees its temperature and pressure increase.

Diagram CC-0 *Olivier Cleynen*

We experimentally observe that when the motion is infinitely slow, a compressed fluid behaves exactly like a spring (figure 2.8). The condition "when the motion is infinitely slow" is of paramount importance, as we will see below.

> « That being stated, let us take some arbitrary gas at the temperature T …; let us represent its volume v_o by the abscissa AB, and its pressure by the ordinate CB.… The gas, during its expansion, will have developed a quantity of mechanical action whose value will be the integral of the product of the pressure by the differential of the volume, and which will be represented geometrically by the surface enclosed between the axis of the abscissas, the two coordinates CB, DE, and the portion of the hyperbola CE. »
>
> Benoît Paul Émile Clapeyron, 1834
> (the first $p - v$ diagram...)
> *Mémoire sur la puissance motrice de la chaleur* [5]

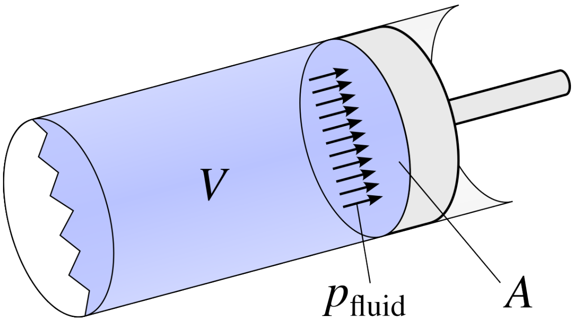


Figure 2.8: When the piston motion is infinitely slow, the fluid behaves like a spring being compressed. *Diagram* CC-BY-SA *Olivier Cleynen*

If this condition is met, we can express the work received or supplied by the system in the same way as with the spring in the previous section:

$$W_{\text{A}\rightarrow\text{B}} = -\int_{\text{A}}^{\text{B}} p \, \mathrm{d}V$$

$$w_{\text{A}\rightarrow\text{B}} = -\int_{\text{A}}^{\text{B}} p \, \mathrm{d}v \qquad (2/15)$$

for a closed system when volume changes are infinitely slow;
where $w_{\text{A}\rightarrow\text{B}}$ is the specific work received by the system ($\text{J}\,\text{kg}^{-1}$),
p is the (uniform) internal pressure (Pa),
and $\mathrm{d}v$ is the change in specific volume ($\text{m}^3\,\text{kg}^{-1}$).

On a diagram representing pressure as a function of specific volume, this work $w_{A\to B}$ is represented by the area under the curve from A to B, just like in figure 2.6. The shape of the curve, that is, the relationship between p and v as the fluid undergoes the process, will ultimately determine the quantity $w_{A\to B}$.

How exactly do fluids behave when they are compressed – in other words, by what type of "spring" can they be modeled? Experimentally, it is observed that when compressed, most gases have their pressure and volume related by a relation of the form $p\,v^k = \text{cst.}$ with k being a constant (figure 2.9). (An exception to this trend is found with liquid/vapors when they change phase, as we will see in chapter 5.)

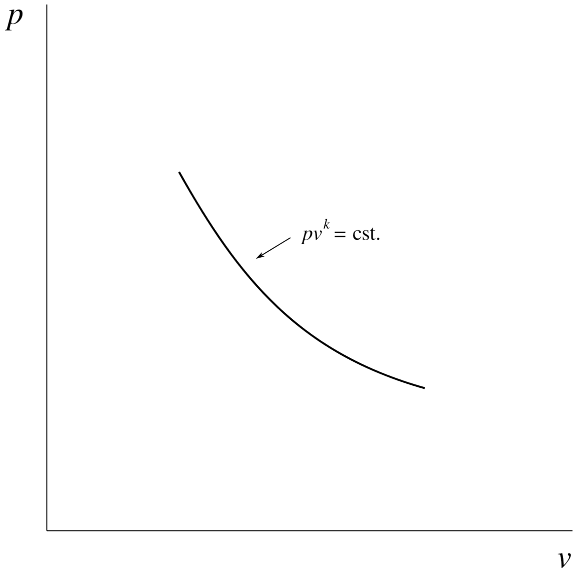


Figure 2.9: Properties of a gas when compressed, represented on a pressure-volume diagram. The relationship is similar to what would be obtained with a spring with progressive coils. *Diagram* CC-0 *Olivier Cleynen*

When heat is provided to the fluid while it is being compressed, its behavior becomes "stiffer", and the pressure increases more rapidly (figure 2.10). Conversely, when heat is taken away from it during compression, the pressure increases less rapidly. These heat transfers therefore vary the amount of work required to compress the fluid between two given volumes.

The case where no heat is added is called *adiabatic*: $Q = 0$. Care is needed here: adiabatic does not mean "at constant temperature". When a fluid is compressed without heat input, its temperature increases. In a diesel engine, for example, the air in the cylinders can reach 900 °C before combustion – which is desirable, as we will see in chapter 7 (*the second law*).

In the three processes of figure 2.10, the relation of the form $pv^k = \text{cst.}$ remains an appropriate model. The more heat is supplied during compression, the more rapidly the pressure increases – the exponent k is then larger.

Conversely, if heat is taken away during compression, the pressure increases less rapidly and a curve closer to the horizontal (with a lower exponent k) is obtained. By removing enough heat, one can even maintain a constant pressure, as we will see in chapters 4 and 5. The exponent k is then zero and we have $p = p_{\text{cst.}}$.

> « If it were true that the steam expended itself through the cylinder at a pressure equal to that of the boiler, or which stood in a fixed ratio to it as indicated by some coefficient, since it always takes the same locomotive the same number of wheel revolutions, or the same number of piston strokes to cover the same distance, it would follow that as long as these machines work at the same pressure, they should, in all cases, consume the same quantity of water for the same distance. »
>
> François-Marie Guyonneau de Pambour, 1839
> *Théorie de la machine à vapeur* [7]

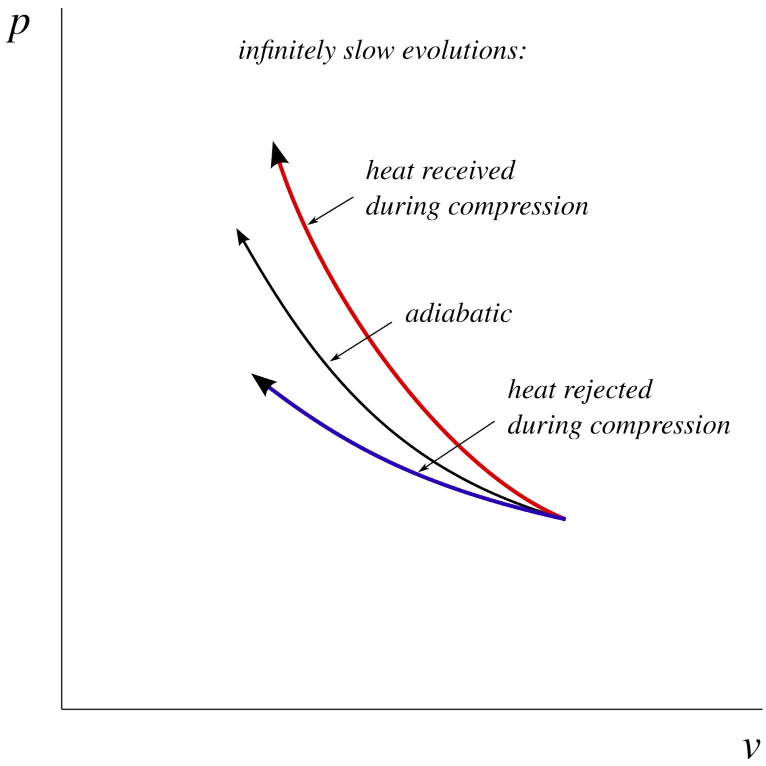


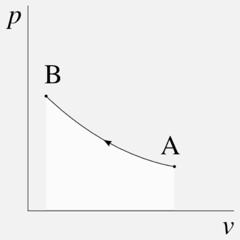
Figure 2.10: Behavior of a fluid when infinitely slowly compressed. The more heat is supplied during compression, the more rapidly the pressure increases. The adiabatic curve represents the case where no heat transfer occurs ($Q = 0$).

Diagram CC-0 *Olivier Cleynen*

Example 2.3

A gas in a cylinder is slowly compressed by a piston. It is observed that its pressure is related to its volume by the relation $pv^{1.2} = k$ (in SI units, with k being a constant). At the beginning of compression, its properties are $p_A = 1\,\text{bar}$ and $v_A = 1\,\text{m}^3\,\text{kg}^{-1}$. It is compressed until its volume reaches $v_B = 0.167\,\text{m}^3\,\text{kg}^{-1}$.
What amount of specific work has the gas received or supplied?

The process can be drawn qualitatively on a pressure-volume diagram as follows:



First, we need to calculate the value of k to determine quantitatively the relation between p and v. We obtain it with the initial conditions: $k = p_A v_A^{1.2} = 10^5 \times 1^{1.2} = 10^5$ u.SI.

☞ The physical quantity represented by k is confusing: it is measured in $\mathrm{Pa\,m^{3.6}\,kg^{-1.2}}$. This is not important for us, and it is sufficient (after properly converting the input units to SI!) to indicate "in SI units" or u.SI.

Now, we can describe p as a function of v: $p = 10^5 \times v^{-1.2}$. We just have to integrate starting from equation 2/15: $w_{\mathrm{A}\to\mathrm{B}} = -\int_{\mathrm{A}}^{\mathrm{B}} p\,\mathrm{d}v = -\int_{\mathrm{A}}^{\mathrm{B}} kv^{-1.2}\,\mathrm{d}v = -k\left[\frac{1}{-1.2+1}v^{-1.2+1}\right]_{v_\mathrm{A}}^{v_\mathrm{B}} = \frac{10^5}{0{,}2}\left[v^{-0{,}2}\right]_1^{0.167} = +2.152 \times 10^5\,\mathrm{J\,kg^{-1}} = +215.2\,\mathrm{kJ\,kg^{-1}}$.

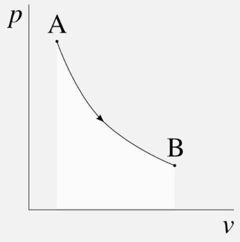
☞ The sign of $w_{\mathrm{A}\to\mathrm{B}}$ is positive: the gas has work done to it.

☞ The result may seem large, but it is important to remember that it is a mass-specific amount of work (§1.1.5) that needs to be multiplied by the mass of the gas to obtain a quantity in joules. At the initial conditions ($1\,\mathrm{kg\,m^{-3}}$), a volume of air of $1\,\mathrm{L}$ weighs just over one gram.

Example 2.4

A mass of 0.3 gram of pressurized gas in a cylinder is slowly expanded as a piston moves. It is known that its pressure and volume are related by a relation of the form $pv^{k_1} = k_2$ (where k_1 and k_2 are two constants). At the beginning of the expansion, the pressure is at 12 bar (174 psi) and the volume is 0.25 L (0.22 gal imp). Once expanded, the gas reaches ambient pressure of 1 bar (14.5 psi) with a volume of 1.76 L (0.387 gal imp). What is the work done by the gas during the expansion?

The process can be drawn qualitatively on a pressure-volume diagram as follows:



First, we need to fully know the law relating p to v; then we will proceed with the integration $-\int p\,\mathrm{d}v$ during the process to calculate the work. Let us start by calculating the specific volumes at the start and end: $v_\mathrm{A} = \frac{V_\mathrm{A}}{m} = \frac{0.25\times10^{-3}}{0.3\times10^{-3}} = 0.833\,\mathrm{m^3\,kg^{-1}}$. Similarly, $v_\mathrm{B} = \frac{V_\mathrm{B}}{m} = 5.867\,\mathrm{m^3\,kg^{-1}}$.

Now, we can calculate k_1:

$$p_\mathrm{A} v_\mathrm{A}^{k_1} = p_\mathrm{B} v_\mathrm{B}^{k_1}$$
$$\left(\frac{v_\mathrm{A}}{v_\mathrm{B}}\right)^{k_1} = \frac{p_\mathrm{B}}{p_\mathrm{A}}$$

$$k_1 \ln\left(\frac{v_\mathrm{A}}{v_\mathrm{B}}\right) = \ln\left(\frac{p_\mathrm{B}}{p_\mathrm{A}}\right)$$

$$k_1 = \frac{\ln\left(\frac{p_\mathrm{B}}{p_\mathrm{A}}\right)}{\ln\left(\frac{v_\mathrm{A}}{v_\mathrm{B}}\right)} = \frac{\ln\left(\frac{1}{12}\right)}{\ln\left(\frac{0.833}{5.867}\right)} = 1.2733$$

And with k_1, we can calculate $k_2 = p_\mathrm{A} v_\mathrm{A}^{k_1} = 12 \times 10^5 \times 0.833^{1.2733} = 9.514 \times 10^5$ u.SI.

☞ k_1 is an exponent and has no units. The units of k_2 are not interesting to us.

☞ Although it may seem laborious, this approach of "we have a general model for the trend, what are the parameters for this specific case?" is very common in physics, and extremely useful for engineers.

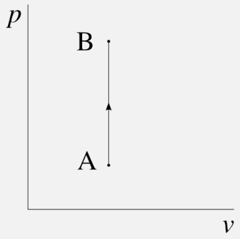
We can now quantitatively describe the properties during the process: $pv^{1.2733} = 9.514 \times 10^5$. We just have to carry out our usual integration: $w_{\mathrm{A}\to\mathrm{B}} = -\int_\mathrm{A}^\mathrm{B} p \,\mathrm{d}v = -k_2 \int_\mathrm{A}^\mathrm{B} v^{-k_1} \,\mathrm{d}v = \frac{-9.514\times10^5}{-0.2733}\left[v^{-0.2733}\right]_{0.833}^{5.867} = -1.513 \times 10^6 \,\mathrm{J\,kg^{-1}} = -1513 \,\mathrm{kJ\,kg^{-1}}$. We multiply by the mass of the gas to obtain the work: $W_{\mathrm{A}\to\mathrm{B}} = m \, w_{\mathrm{A}\to\mathrm{B}} = -453.8$ J.

☞ This calculation can be done more quickly without calculating the values of v_A, v_B, and k_2. However, to ensure reaching the correct result, it is safer and easier to quantify p and v (in SI) at all stages of the process before starting an integration.

Example 2.5

A gas confined in a sealed container is slowly heated. Its volume remains at 12 L, and its pressure changes from 1 bar to 40 bar. What is the work done?

The process can be drawn qualitatively on a pressure-volume diagram as follows:



The work is zero, of course. Since the volume does not change, dV is zero throughout the process. We can heat or cool as we wish, but as long as no wall is moved, there will be no work done.

2.4.3 Work of a fluid in a fast process

Things get more complicated when we compress and expand our fluid rapidly (figure 2.11). A complex and critically important phenomenon in thermodynamics occurs: **the pressure on the wall differs from the "average pressure" inside the fluid**.

> « We have said that at the start of the movement, the pressure equilibrium is established between the boiler and the cylinder, but as the speed of the piston increases, the latter, so to speak, escapes ahead of the steam without giving it time to establish this equilibrium, and the pressure in the cylinder necessarily drops. »
>
> François-Marie Guyonneau de Pambour, 1835
> *Traité théorique et pratique des machines locomotives* [6]

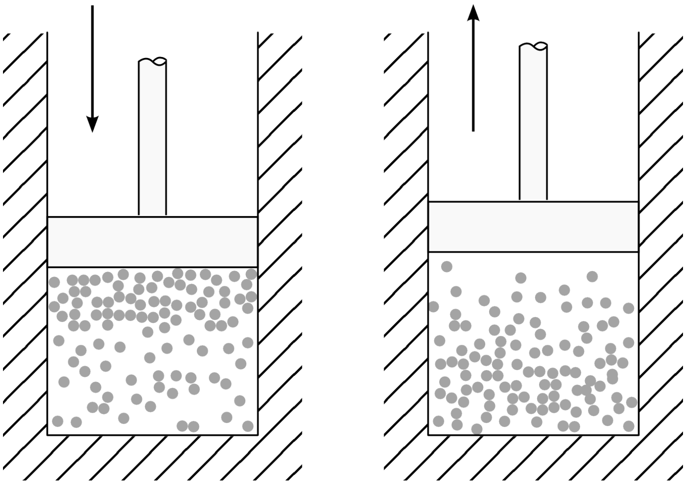


Figure 2.11: Irreversible compression and expansion. When a fluid is rapidly compressed (left), the pressure on the piston wall is increased. During a rapid expansion (right), this pressure is decreased.

Diagram CC-by-sa *Olivier Cleynen*

In order to describe what happens inside the fluid, we can take the example of water in a bathtub being pushed with hands – like the object shown in figure 2.12 being moved in liquid water. When the object is moved away and brought closer abruptly, the pressure on its walls is not the same as when it is moved slowly.

In each case, the amount of work done on the fluid during compression is larger, and the amount of work done by the fluid during expansion is smaller.

We call this phenomenon *irreversibility*. It will be a great challenge in our quantitative study of thermodynamics and will make our conversions of work and heat even more difficult.

What happens in the cylinder filled with fluid when it is not compressed infinitely slowly? During a rapid compression, the pressure on the piston wall is greater than the average pressure inside the cylinder (figure 2.13). *More energy is expended than necessary* to carry out the displacement.

We could thus say that when compressed and expanded abruptly, a fluid behaves like a "fragile" spring, inside which something changes: it is not able to fully return all the mechanical energy it has stored.

If the received work is not equal to the work returned, then where did the excess energy go? This surplus of energy, supplied in the form of work by the piston, is *converted into heat inside the fluid* during the movements.

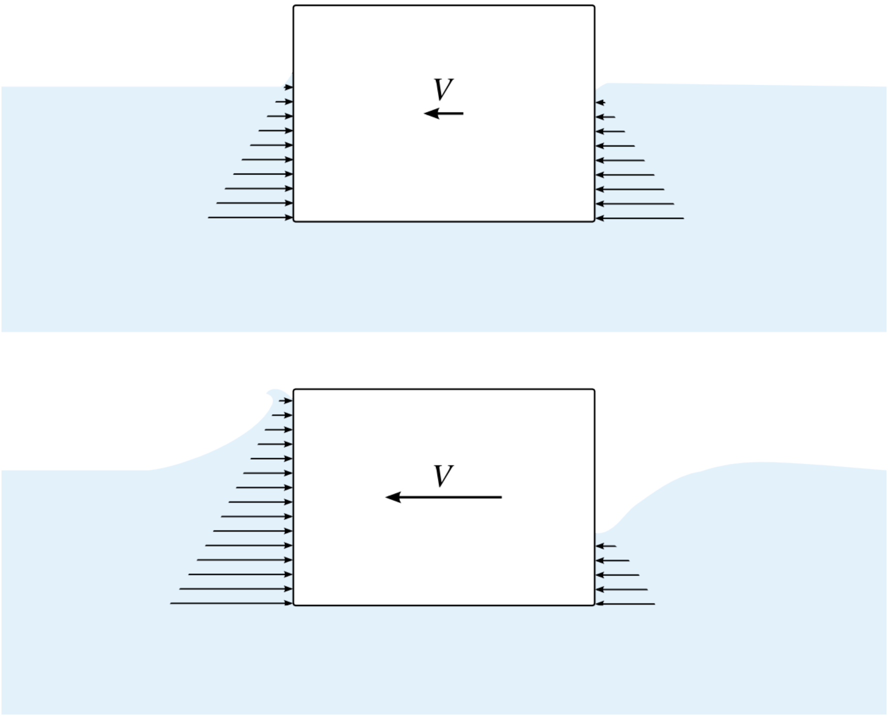


Figure 2.12: A solid object being moved in a water tank, with slow motion (above) and with fast motion (below). During fast motion, the pressure forces aiding the movement are weaker, and the forces opposing the movement are greater. In the limit of infinitely slow motion, these forces are equal.

Diagram CC-0 *Olivier Cleynen*

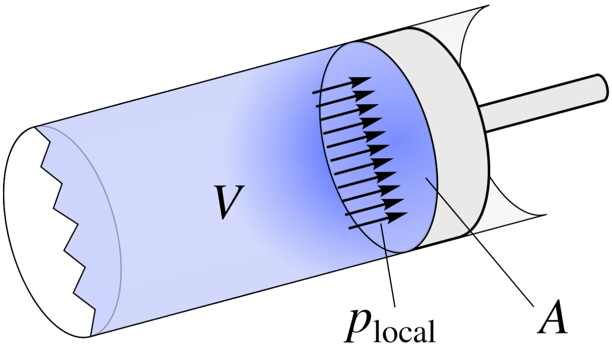


Figure 2.13: Compressed fluid abruptly. The local pressure at the piston surface is higher than it would have been with slow movement.

Diagram CC-BY-SA *Olivier Cleynen*

The process traced on a pressure-volume diagram (figure 2.14) is much more complex than in the case of an infinitely slow process. The average pressure inside the fluid increases more rapidly than it would in a slow motion.

During expansion, the opposite phenomenon occurs (figure 2.15): a zone of lower pressure forms in front of the piston wall, and the work done by the fluid on the piston is less than it would have been in the reversible case.

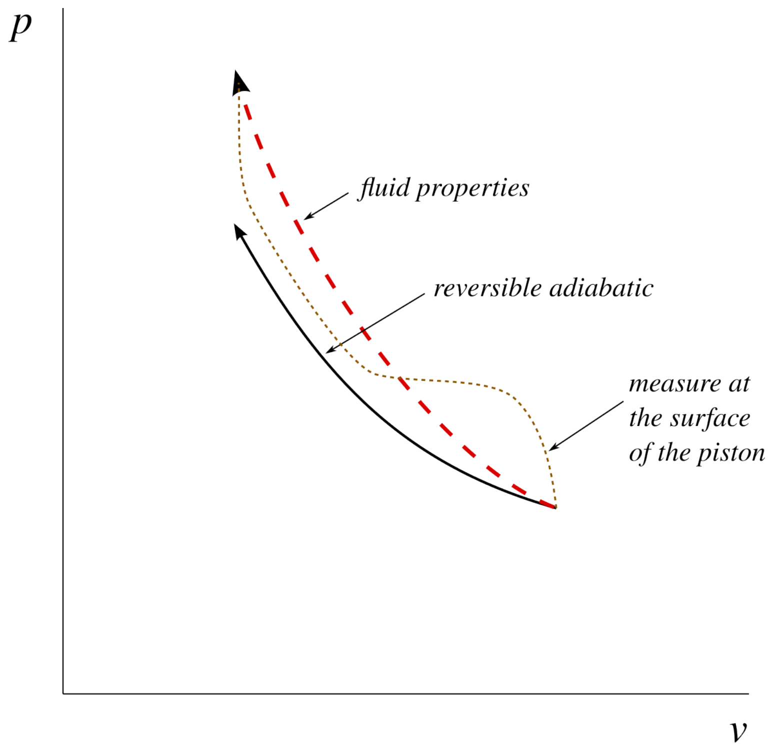


Figure 2.14: Irreversible adiabatic compression on a pressure-volume diagram. We draw the curve with dashes: it is not a continuous series of states because the fluid pressure is not homogeneous during the process. The path the fluid would have followed if the compression had been infinitely slow is represented with a solid line. During compression, the "surplus" of work supplied by the piston is converted into heat, even though the gas is perfectly isolated.

Diagram CC-0 *Olivier Cleynen*

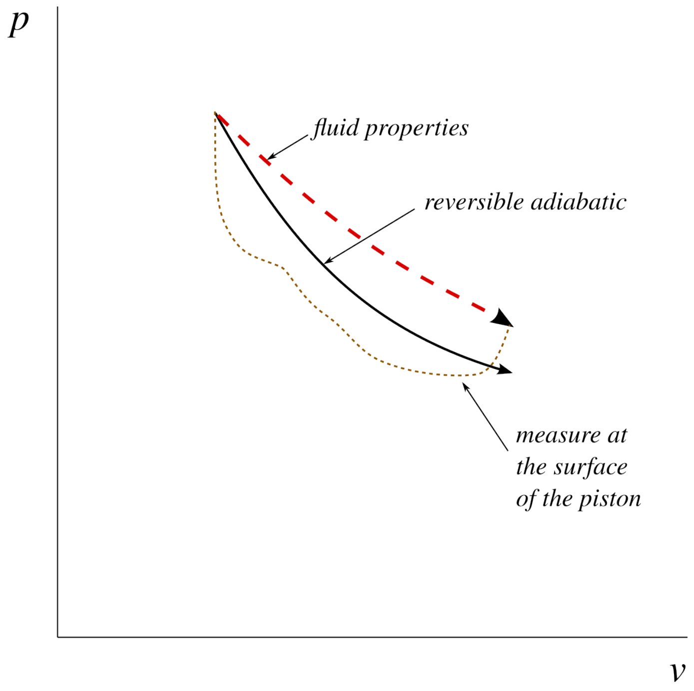


Figure 2.15: Irreversible adiabatic expansion on a pressure-volume diagram. The work received by the piston is less than it would have been with a slow motion. The path followed by the fluid is represented with dashes, because the pressure is not homogeneous during the movement.

Diagram CC-0 *Olivier Cleynen*

From a quantitative point of view, the more abrupt the movements on the fluid, the more the process will resemble one with heat input ("hardening" of the fluid and increase in the exponent k during compressions, decrease in the exponent k during expansions).

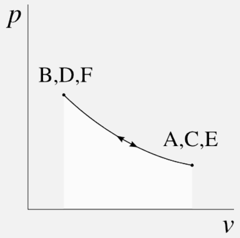
However, the work done on or by the fluid can no longer be simply calculated by integral, since the pressure inside the cylinder is not homogeneous at all. It is the pressure at the piston surface that would allow this work to be calculated. Unfortunately, no simple mathematical relationship describes this relationship between pressure and volume. An experimental measurement must be made each time.

Example 2.6

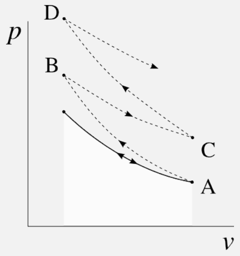
A gas is confined in a sealed cylinder and back-and-forth movements between two given volumes are performed with the piston, without transferring heat. Initially, the back-and-forth movements are very slow. Then, the back-and-forth movements are carried out very rapidly.

What will the processes look like on a pressure-volume diagram?

During slow processes, the pressure always passes through the same values during the back-and-forth movements:



However, when the movements are fast, at the end of each trip the final pressure is greater than it would have been with a slow movement:



Thus, the pressure gradually rises on the pressure-volume diagram: the excess work invested during compressions, and the lack of work recovered during expansion, result in an increase in the internal energy of the gas, whose temperature continuously rises.

2.4.4 Reversibility

Let's take a few moments to reflect on what we have just described. Every time we compress a fluid "too quickly," something happens that prevents us from recovering our work.

From an engineering perspective, a slow process is a limit case: one where dissipations are minimized. For example, the work required to compress a gas to 10 bar is minimal when the compression is reversible. Similarly, a turbine in which the expansion is reversible will extract the maximum work from a compressed fluid. On the contrary, in a car shock absorber, processes are made highly irreversible so that it does less work on the return path than was done on it on the outbound journey.

From a physics standpoint, the phenomenon of irreversibility is fascinating. Indeed, we start from collisions of molecules, a completely reversible phenomenon, to create an irreversible process: one that only goes in one direction! In order to bring the gas back to the state it was in before being abruptly compressed, we are forced to transfer heat away from it. It is surprising that without going against Newton's laws, we have created a situation where *we cannot go back by "doing the opposite"*. Are there other irreversible processes? Can we quantify irreversibility? We will attempt to answer these questions in chapters 7 (*the second law*) and 8 (*entropy*).

> « Where does irreversibility come from? It does not come from Newton's laws. If we claim that the behavior of everything is ultimately to be understood in terms of the laws of physics, and if it also turns out that all the equations have the fantastic property that if we put $t = -t$ we have another solution, then every phenomenon is reversible. How then does it come about in nature on a large scale that things are not reversible? »
>
> Richard Feynman, 1963 [30, 35]
> *The Feynman Lectures on Physics*

In the meantime, we shall agree that three conditions must be met for a process to be reversible:

1. The process must occur without friction. There should be no friction in the mechanical elements (for example, between piston and cylinder).
2. The pressure in the fluid must be homogeneous. The movement of the walls must therefore be infinitely slow, and the fluid must move without turbulence or internal friction.
3. The temperature difference between the fluid and its environment must be infinitely small. If heat is supplied or rejected, it must be transferred infinitely slowly.

These three conditions obviously exclude any real process—and in particular, any practical application in an engine! However, we will use them to establish an ideal theoretical limit for all of the real processes that we will study.

2.5 Quantifying Heat with a Closed System

At the risk of frustrating the student, we must immediately admit that *we cannot directly quantify heat transfers.* We will always proceed by deduction: by quantifying the change in energy and subtracting the work transfers, we obtain the amount of heat that has been transferred. Mathematically, in a closed system, we simply reuse equation 2/1 to obtain:

$$Q_{1\to2} = \Delta U \; - \; W_{1\to2} \tag{2/16}$$

$$q_{1\to2} = \Delta u \; - \; w_{1\to2} \tag{2/17}$$

for a closed system.

The entire difficulty in quantifying a heat transfer is now to predict and quantify the change in internal energy, ΔU. For gases, U is simply proportional to temperature; for liquids and vapors, the relationship is more complex. We will learn to quantify energy in fluids in chapters 4 (*the ideal gas*) and 5 (*liquids and vapors*).

A Bit of History: the Compound Engine

*

In the 1830s, the steam engine had just revolutionized the landscape and the economic network of Great Britain. Almost everything traveled by rail: passengers, crops, coal, industrial products. These trains were hauled by steam engines, of monumental dimensions and deplorable efficiency — ninety-seven percent of the energy released by coal was lost in the chimneys. This was not a big issue: coal and water were abundant, and it was sufficient to make punctual stops along the railway lines to replenish the machines.

At sea, however, wind was still being used for propulsion. In order to connect two continents by engine power (meaning without tacking!), two problems had to be solved.

The first problem is that the engines consumed a lot of water. Sea water, although abundant, was unusable in its natural state because the salt and limestone deposits resulting from its boiling clogged the boilers and posed a serious risk of explosion. In order to use it in boilers, it was therefore necessary to desalinate it, a very energy-intensive operation.

The problem was solved with the use of *condensers*, which locomotives had done away with because of space constraints. Now, when steam had done its work in the cylinders, it was no longer simply discharged into the atmosphere, but instead cooled in large condensers before being compressed and reintroduced into the boiler. The water circulated cyclically throughout the engine – one would only need to compensate for leaks.

The second problem was more serious and more difficult to solve: how to increase efficiency? It was not just a financial question: the first steam-powered transatlantic ship, the *SS Savannah*, was so inefficient that it completed its crossing under sail, even though it was carrying *only* the coal for its engine!

In order to increase the efficiency of an engine of given capacity, we seek to increase the amount of work generated by each kilogram of steam, which can be approximated by the relation 2/15:

Figure 2.16: *SS Savannah*, in the first steam-powered transatlantic crossing in 1819, which was completed under sail.

Image by Hunter Wood (public domain, 1819)

$$w_{A\to B} = -\int_{A}^{B} p \, dv$$

The first thing to do is to increase the pressure p_A of the steam, that is, its pressure before it begins to expand in the cylinders. This is not an easy task: raising the boiler pressure increases the structural stresses it undergoes, hence its cost, and reduces its efficiency as the walls must be thickened and strengthened.

Next, one can try to increase Δv, the total change in volume during the piston movement. In other words, it is necessary to increase the volume swept by the cylinders. Once again, this is not an easy task.

On one hand, increasing the cylinder diameter (in order to increase the area A) subjects the pistons to a greater force F_A for a given pressure p_A (2/5):

$$p \equiv \frac{F}{A}$$

By increasing the transmitted force, the structural limits of the engine mechanics are quickly reached.

On the other hand, increasing the piston travel lengthens the pistons and makes the connecting rod and crankshaft mechanisms significantly heavier. Additionally, the pressure and volume of the steam are linked: they roughly follow a relationship of the form $pv^{k_1} = k_2$ during expansion. In other words, pressure decreases when the volume is increased: as the cylinder is lengthened, the work gains become increasingly small.

The *compound* engine addresses this issue by using multiple cylinders *in series* (see figure 2.17). The high-pressure steam first drives a piston of small diameter (thus limiting the force exerted on the mechanism). It is then transferred to another cylinder of larger diameter. This cylinder allows for the same force to be obtained with a lower pressure; it sweeps a larger volume.

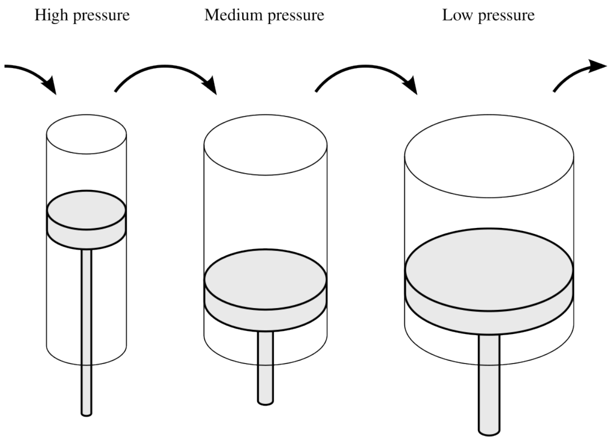


Figure 2.17: Cylinders in series, known as *compound*.

Diagram CC-BY-SA *Olivier Cleynen*

By increasing the total volume swept by the expanding steam, more work can be extracted from the compressed steam without oversizing the crankshaft or overloading the pistons.

With such an engine, the merchant navy was able to look beyond coastal shipping: it seized upon this new technology which experienced immediate success. From two cylinders in series (*double compound*) it moved to three, and sometimes even four (*quadruple compound*!), in order to extract ever more energy from steam, in the form of work.

Enthusiastic shipowners could now boast that they now only needed to burn one sheet of paper to move one ton of cargo one mile. Even though it was understood that the said paper is very thick, progress had been made. Soon, the tea from the East Indies would arrive in the Londonian teacups – the British Empire now had the machinery required to power its formidable economic network.

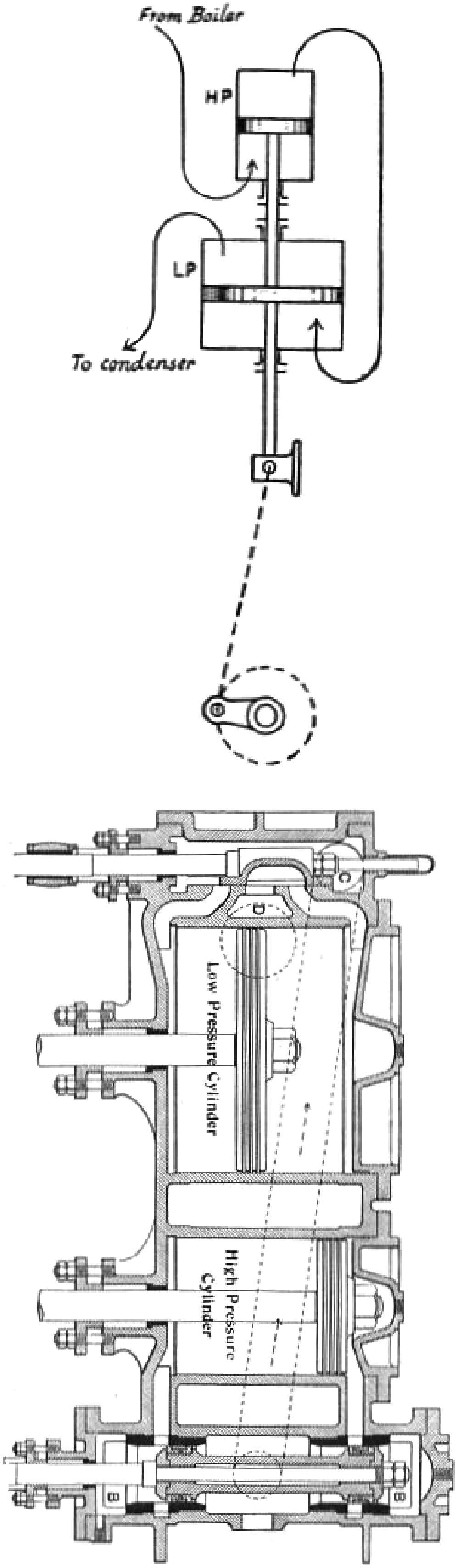


Figure 2.18: Different steam compound systems.

Images by Prof. William Ripper, 1889 (public domain)

Problems

2.1 Simple Processes

(An exercise simply designed to practice the sign conventions and vocabulary of the chapter.)

A mass of 400 g of water is placed in a sealed reservoir. It undergoes a process during which it receives $50\,\mathrm{kJ\,kg^{-1}}$ of heat, and so its internal energy increases by 4 kJ.

1. Did it receive or supply work, and how much?

This same mass is then supplied with 800 J of work in an adiabatic manner.

2. What is the change in its specific internal energy?

2.2 Arbitrary Processes of a Gas in the Laboratory

A mass of 80 g of helium is contained in a cylinder of $0.04\,\mathrm{m^3}$. The gas is first cooled reversibly at constant pressure until $0.02\,\mathrm{m^3}$ and 2 bar; then heated at constant volume until 4 bar.

1. Plot the process on a pressure-volume diagram.
2. What is the work supplied or received by the gas?

2.3 Truck Pneumatic Suspension

The pneumatic suspension system of a truck trailer can be modeled with an air cylinder. When the trailer is loaded, the piston attached to the trailer descends inside the cylinder attached to the wheel axle, compressing the air trapped inside (figure 2.19).

Initially, the truck is loaded very gradually. The air inside the cylinder neither loses nor receives heat. Its characteristics then change according to the relationship $pv^{1.4} = 5.438 \times 10^4$ (in SI units).

The compression starts at $p_A = 2.5$ bar (36.26 psi). Once the loading has been completed, the pressure has risen to $p_B = 10$ bar (146 psi).

1. Draw the process qualitatively (that is, without showing numerical values) on a pressure-volume diagram.
2. The work W done by a force $\vec{F}$ over a displacement $\vec{l}$ is expressed as

$$W \equiv \vec{F} \cdot \vec{l} \qquad (1/11)$$

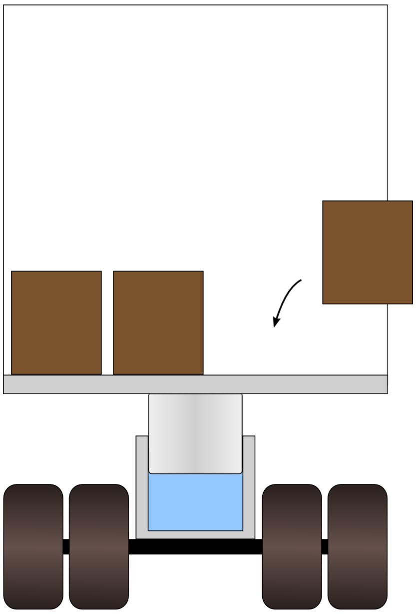


Figure 2.19: Schematic modeling of a truck pneumatic suspension system. The piston, at the center, compresses a mass of air (in blue) when the trailer is loaded.

Diagram CC-0 *Olivier Cleynen*

From this equation, express the work done on a body of fixed mass in terms of its specific volume and internal pressure.

3. How much energy did the gas receive during loading?
4. How much energy would the gas give back if the truck were unloaded very gradually?

The truck is unloaded abruptly and the piston rises quickly until the final pressure p_C drops back to its initial value $p_C = p_A = 2.5$ bar.

4. Draw the process qualitatively on the previous pressure-volume diagram.
5. What can be done to bring the gas back to the exact state it was in before loading?

2.4 Air Compressor

In a small air compressor (figure 2.20), a piston compresses a fixed mass of air slowly and without friction. The cylinder is equipped with fins, which are designed for heat dissipation. Thus, the compression is done at constant internal energy.

We spend $150\,\mathrm{kJ\,kg^{-1}}$ of work in order to compress the air.

1. What is the heat transfer during compression?

Before starting the compression, the air is at atmospheric pressure and density (1 bar; $1.2\,\mathrm{kg\,m^{-3}}$). The diameter of the cylinder is 5 cm (1.97 in) and its inner depth is 15 cm (5.91 in).

2. What is the mass of air included in the cylinder?

During compression, it is observed that pressure and specific volume are related by the relation $pv = k$ (where k is a constant).

3. To which pressure can the air be compressed at the end of the compression?

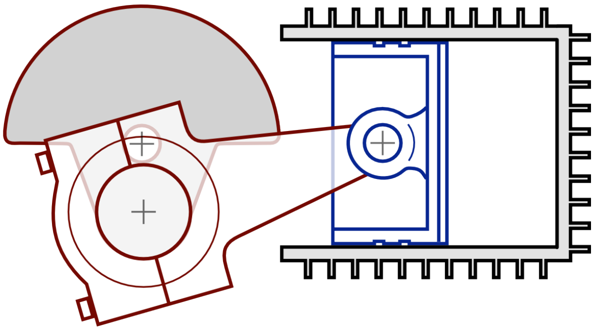


Figure 2.20: Cross-sectional diagram of a small piston air compressor. The intake and exhaust valves are not shown.

Diagram CC-BY-SA *Christophe Dang Ngoc Chan & Olivier Cleynen*

2.5 Cycle of a Gasoline Engine

We want to study the operation of a four-cylinder gasoline engine (figure 2.21). Like all reciprocating heat engines, it supplies work by varying the pressure and volume of small amounts of air trapped in its cylinders. Here, we simplify the details of its operation to reduce it to the ideal case, where all processes are reversible.

The engine has a displacement of 1.1 L; it is equipped with four cylinders of diameter 7 cm and has a compression ratio (ratio between maximum and minimum volumes in a cylinder) of 7.9. Air enters the engine under atmospheric conditions (14.5 psi or 1 bar, $0.84\,\mathrm{m^3\,kg^{-1}}$).

We can describe a cycle inside a cylinder with the following four steps:

From A to B the air is adiabatically compressed from the bottom dead center to the top dead center. During this process, we know that its properties are related by the relation $p\,v^{k_1} = k_2$. At B, the pressure has reached 246.1 psi (16.97 bar).

From B to C it is heated at constant volume (as if the piston were stationary) until the pressure reaches 1087.8 psi (75 bar). By measuring temperature, it is found that its specific internal energy increases by $1543.3\,\mathrm{kJ\,kg^{-1}}$.

From C to D the air is adiabatically expanded from the top dead center to the bottom dead center. Its properties are related by the relation $p\,v^{k_1} = k_3$.

Figure 2.21: Cutaway view of pistons and cylinders of an automobile engine.

Photo CC-BY-SA *by Commons User:Mj-bird*

From D to A it is cooled at constant volume (as if the piston were stationary) until it returns to its state at A.
(In practice, this cooling phase takes place outside the engine, in the atmosphere. However, it can be modeled this way without introducing errors.)

1. Draw the processes undergone by the air qualitatively on a pressure-volume diagram.
2. What is the mass of air present in a cylinder?
   hint: $V_{\text{displacement}} = 4(V_{\text{max. cylinder}} - V_{\text{min. cylinder}}) = 4(V_{\text{bottom dead center}} - V_{\text{top dead center}})$
3. What is the specific work done to the air during compression (from A to B)?
4. What is the specific heat received by the air during combustion (from B to C)?
5. What is the specific work done by the air during expansion (from C to D)?
6. What is the specific heat transferred away from the air during the cooling phase?
7. What is the engine efficiency, that is, the ratio of the net work output during the cycle to the heat input during combustion?
8. How many cycles must be performed each second for the engine to produce a power of 80 hp (58.84 kW)?

2.6 Work in a Diesel Engine

We are studying the operation of a four-cylinder reciprocating engine by modeling its operation in the most favorable case, in other words, with very slow (perfectly reversible) processes.

Inside the engine block schematized in figure 2.22, four pistons linked to the engine shaft by a crankshaft (not shown) are in motion. The process is different in each cylinder:

Cylinder A: compression (the air remains trapped in the cylinder).
The air compression starts at 0.8 bar and its properties are related by the equation $pV^{1.3} = k_1$.

Cylinder B: intake.
Air is taken in at constant pressure of 0.8 bar.

Cylinder C: exhaust.
Air is expelled at constant pressure of 1.1 bar.

Cylinder D: expansion.
The high-pressure, high-temperature air is trapped in the cylinder; its properties are also related by the equation $pV^{1.3} = k_2$.

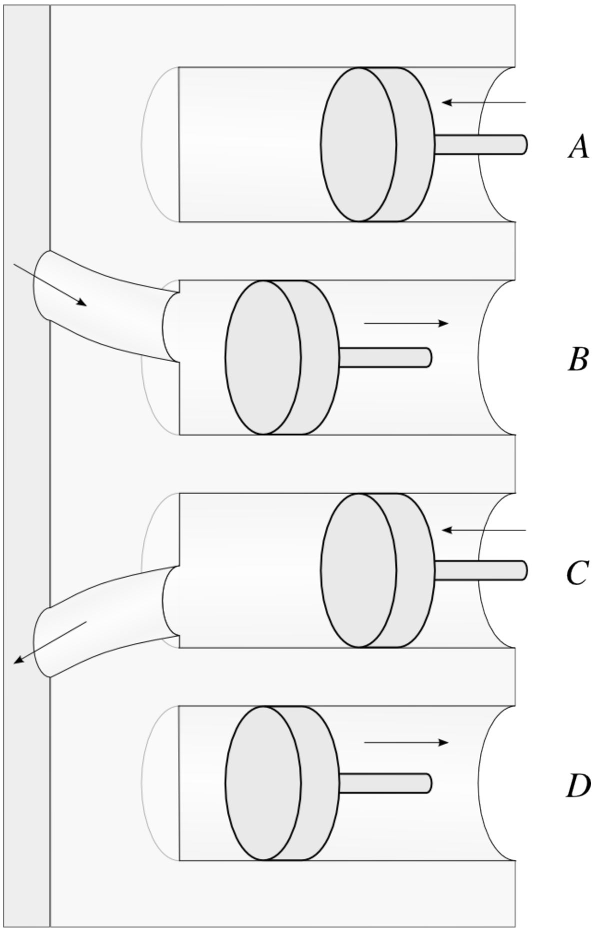


Figure 2.22: Schematic representation of the operation of a four-cylinder engine. Pistons A and C are going up, and pistons B and D are going down. They are all connected to the same motor shaft, not shown here.

Diagram CC-by-sa *Olivier Cleynen*

Of course, the role of each cylinder changes twice per revolution. Here, we are studying the work transfers over one half revolution.

Even though cylinders B and C are not closed systems, for the purposes of this problem, we can model their processes as if they were, without introducing errors.

The atmospheric conditions are 1 bar and 1.225 kg m^{-3}. The engine displacement is 1.5 L and the compression ratio (that is, the ratio of the minimum to maximum volumes within each cylinder) is 22.

1. Draw the process in each of the cylinders on the same pressure-volume diagram, qualitatively.
2. What is the energy required to move cylinders B and C?
3. What is the energy received by the gas in cylinder A?

We want the engine to deliver a power of 30 kW at a speed of 2000 revolutions/min. Its mechanical losses are around 15 %.

4. What is the work that must be done by cylinder D during expansion?
5. What should be the pressure generated by combustion in cylinder D, so that the expansion may release enough energy to operate the engine?

2.7 Taking Heat From Where it is Cold

A student is conducting an experiment with a bit of air in a cylinder, controlling its volume with a piston. The goal is to extract heat from the outside, where the temperature is low, in order to reject it inside the room.

The mass of air trapped in the cylinder is 6×10^{-3} kg.

Initially, the air in the cylinder occupies a volume of 0.5 L (0.132 US gal). The pressure and temperature are room conditions (1 bar; 18 °C or 64.4 °F).

From A to B The student isolates the cylinder well with a thermal insulator, and slowly expands the gas by increasing its volume to 4.5 L (1.189 US gal). We know that during this type of expansion, pressure and volume are related by the equation

$$pv^{1.4} = k_2$$

where k_2 is a constant (we will see where this relationship comes from and learn how to calculate the temperature T_B in chapter 4).

The gas temperature drops dramatically during the expansion: at B, the thermometer finally reads $T_B = 121$ K.

From B to C The student ensures the cylinder volume remains constant by mechanically locking the piston, removes the thermal insulator, and places the cylinder outside the building (outside temperature: $-5\,^{\circ}\mathrm{C}$ or $23\,^{\circ}\mathrm{F}$). The gas temperature and pressure slowly rise.

From C to A When the pressure reaches $1\,\mathrm{bar}$ precisely, the cylinder air temperature is indicated as $T_{\mathrm{C}} = 262\,\mathrm{K}$.

The student wishes to return to the initial conditions (by reducing the volume to the initial volume) while keeping the pressure constant at $1\,\mathrm{bar}$.

1. Sketch the process on a pressure-volume diagram.
2. Show that during a reversible process undergone by a closed system whose properties are related by a relation of the form $pv^{k_1} = k_2$ (where $k_1 \neq 1$ and k_2 are constants), the specific work done is:

$$w_{\mathrm{A}\to\mathrm{B}} = \frac{p_{\mathrm{B}}v_{\mathrm{B}} - p_{\mathrm{A}}v_{\mathrm{A}}}{k_1 - 1}$$

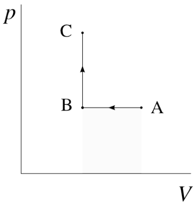
3. What is the work done by the gas during the expansion?
4. The thermal capacity of air when its volume is fixed is $718\,\mathrm{J\,kg^{-1}\,K^{-1}}$. How much heat was transferred to or from the outside air?
5. How much work will be needed to perform the C → A return at constant pressure?
6. Over the entire cycle, will the student have done or received work?
7. [difficult question] The return path at constant pressure requires heat transfer. In which direction and to what extent? Why can't (unfortunately) this transfer be entirely done inside the building?

Answers

2.1 1) $W_{\text{A}\to\text{B}} = \Delta U - mq_{\text{A}\to\text{B}} = -16\,\text{kJ}$ (so the work is done)

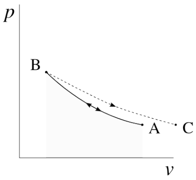
2) $\Delta u = q_{\text{B}\to\text{C}} + w_{\text{B}\to\text{C}} = 0 + \frac{W_{\text{B}\to\text{C}}}{m} = +2\,\text{kJ}\,\text{kg}^{-1}$.

2.2



2) $W_{\text{A}\to\text{C}} = W_{\text{A}\to\text{B}} + W_{\text{B}\to\text{C}} = -p_{\text{cst.}}\,[V]_{V_\text{A}}^{V_\text{B}} + 0 = +4\,\text{kJ}$

2.3



2) see §1.3 p. 16 & §2.4.1 p. 36

3) $v_\text{A} = 0.336\,\text{m}^3\,\text{kg}^{-1}$ and $v_\text{B} = 0.125\,\text{m}^3\,\text{kg}^{-1}$; so $w_{\text{A}\to\text{B}} = -k\left[\frac{1}{-0.4}v^{-0.4}\right]_{v_\text{A}}^{v_\text{B}} = +102.1\,\text{kJ}\,\text{kg}^{-1}$.

4) $w_{\text{B}\to\text{A}} = -w_{\text{A}\to\text{B}}$

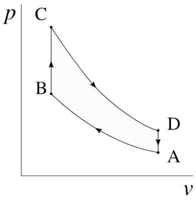
5) Cooling at constant pressure, for example.

2.4 1) $q_{\text{A}\to\text{B}} = \Delta u - w_{\text{A}\to\text{B}} = -w_{\text{A}\to\text{B}}$.

2) $m = \frac{V_\text{A}}{v_\text{A}} = 3.534 \times 10^{-4}\,\text{kg}$

3) $\frac{v_\text{B}}{v_\text{A}} = \exp\left[-\frac{w_{\text{A}\to\text{B}}}{k}\right]$; so $p_\text{B} = p_\text{A}\frac{v_\text{A}}{v_\text{B}} = 6.05\,\text{bar}$

2.5



2) $V_\text{A} = 3.149 \times 10^{-4}\,\text{m}^3$; so $m_\text{A} = \frac{V_\text{A}}{v_\text{A}} = 3.748 \times 10^{-4}\,\text{kg}$

3) $k_1 = 1.3699$ and $k_2 = 7.8753 \times 10^4\,\text{u.SI}$; so $w_{\text{A}\to\text{B}} = -k_2\left[\frac{1}{-k_1+1}v^{-k_1+1}\right]_{v_\text{A}}^{v_\text{B}} = +260.7\,\text{kJ}\,\text{kg}^{-1}$.

4) $q_{\text{B}\to\text{C}} = \Delta u - w_{\text{B}\to\text{C}} = \Delta u - 0 = +1543.3\,\text{kJ}\,\text{kg}^{-1}$

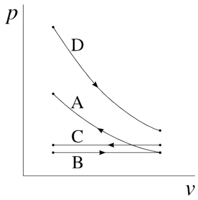
5) $w_{\text{C}\to\text{D}} = -k_3\left[\frac{1}{-k_1+1}v^{-k_1+1}\right]_{v_\text{C}}^{v_\text{D}} = -\frac{k_3}{k_2}w_{\text{A}\to\text{B}} = -\frac{p_\text{C}}{p_\text{B}}w_{\text{A}\to\text{B}} = -1152.2\,\text{kJ}\,\text{kg}^{-1}$

6) $q_{\text{D}\to\text{A}} = -w_{\text{A}\to\text{B}} - q_{\text{B}\to\text{C}} - w_{\text{C}\to\text{D}} = -651.8\,\text{kJ}\,\text{kg}^{-1}$

7) $\eta_\text{engine} = \left|\frac{w_{\text{A}\to\text{B}} + w_{\text{C}\to\text{D}}}{q_{\text{B}\to\text{C}}}\right| = 57.8\,\%$ (very honorable)

8) $f = \frac{\dot{W}_\text{engine}}{m_\text{A}\,(w_{\text{A}\to\text{B}} + w_{\text{C}\to\text{D}})} = 176.1\,\text{Hz}$ (176 combustions per second), so approximately 5300 rotations per minute with a four-stroke, four-cylinder engine.

2.6



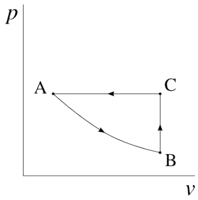
2) $W_{\text{cyl. B}} = -p_\text{B}\,[V]_{V_\text{min.}}^{V_\text{max.}} = -p_\text{B}\left(\frac{V_\text{displacement}}{4}\right) = -30\,\text{J}$; $W_{\text{cyl. C}} = +41.3\,\text{J}$

3) $V_{\text{A1}} = \frac{22}{21\times4}V_\text{swept} = 3.9286 \times 10^{-4}\,\text{m}^3$ and $V_{\text{A2}} = \frac{V_{\text{A1}}}{22} = 1.7857 \times 10^{-5}\,\text{m}^3$. So, $k_1 = 2.9895\,\text{u.SI}$, and finally $W_{\text{cyl. A}} = \frac{k_1}{0.3}\left(V_{\text{A2}}^{-0.3} - V_{\text{A1}}^{-0.3}\right) = +160\,\text{J}$.

4) $\dot{n} = 2000\,\text{rpm} = 33.3\,\text{rps}$: there are therefore 66.7 processes per second ($f = 66.7\,\text{Hz}$). We obtain $W_{4\text{ cylinders}} = \frac{1}{f}\frac{1}{\eta_\text{mech}}\dot{W}_\text{engine}$; and finally $W_{\text{cyl. D}} = W_{4\text{ cylinders}} - W_{\text{cyl. A}} - W_{\text{cyl. B}} - W_{\text{cyl. C}} = -700.7\,\text{J}$.

5) We calculate k_2 based on $W_{\text{cyl. D}}$, and we obtain $p_{\text{D1}} = 194.8\,\text{bar} = 2825\,\text{psi}$.

2.7



3) After obtaining $p_\text{B} = 4.61 \times 10^{-2}\,\text{bar}$, we calculate $W_{\text{A}\to\text{B}} = \frac{p_\text{B}V_\text{B} - p_\text{A}V_\text{A}}{k_1 - 1} = -73.14\,\text{J}$.

4) $Q_{\text{B}\to\text{C}} = mc_v\Delta T = +607.4\,\text{J}$

5) $W_{\text{C}\to\text{A}} = +400\,\text{J}$ (easy!)

6) $W_\text{cycle} = +326.86\,\text{J}$, so work done by the student.

7) $Q_{\text{C}\to\text{A}} = -934.26\,\text{J}$

8) It is a matter of temperature…

CHAPTER 3
Open Systems

— *or* —

Measuring the Intangible in the Ungraspable

Chapter 3 – Open Systems

Executive summary for chapter 3

An open system is crossed by a mass flow. Heat and work transfers cause variations in the *enthalpy* of the fluid. For work to be reversible, the movement must be infinitely slow.

Introduction

In the previous chapter, we quantified energy exchanges within closed systems. This chapter 3 (*open systems*) aims to answer a similar question: how to quantify energy transfers within a system when it is crossed by a mass flow?

3.1 Why Use an Open System?

In many machines, the fluid used to transfer heat and work is continuously circulating. It can then be difficult to identify a particular amount of mass, making it a closed system, in order to quantify energy transfers to and from it. For example, in a jet engine nozzle, air expands and accelerates continuously: at any given moment, there is no identifiable volume that would have *one* specific speed or *one* particular pressure.

Using an open system is very useful to account for energy in flows. Rather than separating stages in time (for example before and after compression), we quantify work and heat transfers by separating stages in space (for example upstream and downstream of the compressor).

3.2 Accounting Conventions

3.2.1 The open system

We call an *open system* an arbitrary subject of study whose boundaries are permeable to mass (figure 3.3). In general, its volume can change, and it can have multiple inlets and outlets, each with a different flow rate and pressure.

« The constructive difficulties that have to be overcome in a large gas motor because of the immense piston pressures and heat expansion of the complicated cylinder heads (cracks galore!) are well known. A safe gas turbine would in this respect be an improvement. »

Aurel Stodola, 1904
Die Dampfturbinen [26, 27]

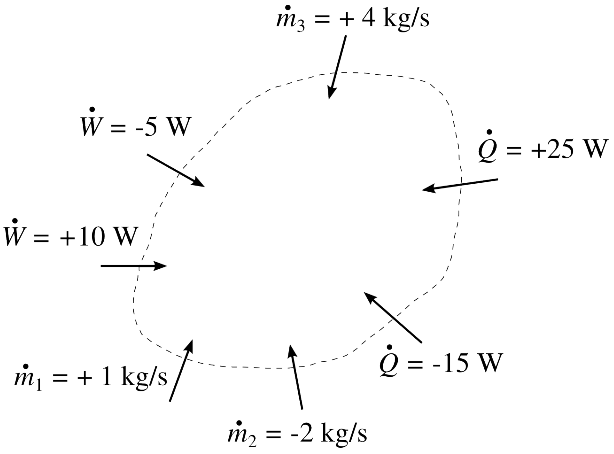


Figure 3.1: Sign conventions for an open system. Inflows are positive, outflows are negative; they are all represented with inward arrows.

Diagram CC-0 *Olivier Cleynen*

In our study of thermodynamics, we will only use open systems:

- with fixed volume;
- having only one inlet and one outlet;

- being crossed by a constant mass flow rate $\dot{m}$ (positive by convention).

These systems are said to be in *steady state* (sometimes called *steady flow* or *stationary flow* regime).

3.2.2 Sign conventions

Just like for closed systems, we will take the open system's point of view to quantify transfers:

- Receiving work, heat, or mass results in a *positive* transfer;
- Expending work, heat, or mass results in a *negative* transfer.

Thus, we add up all transfers as on a bank statement.

3.3 The First Law in an Open System

We have seen that in a closed system, the law of conservation of energy is expressed by the equation $q + w = \Delta u$ (2/2). In an open system, the situation is a little different and we must consider additional forms of energy.

3.3.1 Entering and exiting the system: flow work

Let's imagine an open system in steady flow, containing a small water pump. In order to insert water into the pump at a given pressure, energy must be supplied to the system. Conversely, to push the water outside (at a higher pressure), the system must supply energy. How can we quantify this energy?

Consider the case of a *fluid element* (namely, a small quantity of fluid in transit, with volume V_{element}) entering our system at pressure p_1 (figure 3.2).

The work $W_{\text{insertion}}$ received by the system when the element is pushed through the insertion is:

$$W_{\text{insertion}} = p_1 \, V_{\text{element}} \tag{3/1}$$

where $W_{\text{insertion}}$ is the insertion work (J),
and V_{element} is the volume of the fluid element (m^3).

If such a volume of fluid enters the system every second, then the system receives power in the form of work, which we call *insertion power*, $\dot{W}_{\text{insertion}}$.

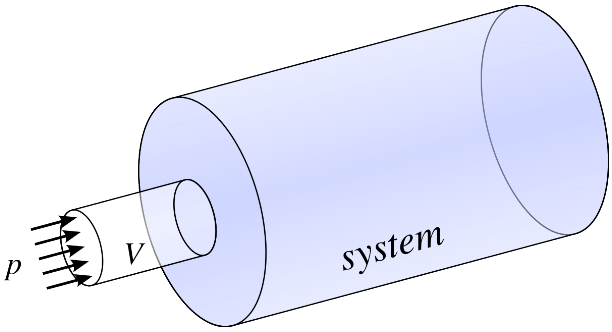


Figure 3.2: A fluid element of volume V_{element} entering at pressure p into the open system.

Diagram CC-0 *Olivier Cleynen*

We sometimes express it in specific form (§1.1.5):

$$\dot{W}_{\text{insertion}} = p_1 \, \dot{V}_1 = \dot{m}_1 \, p_1 \, v_1 = \dot{m} \, p_1 \, v_1 \tag{3/2}$$

$$w_{\text{insertion}} = p_1 \, v_1 \tag{3/3}$$

where $\dot{W}_{\text{insertion}}$ is the insertion power (W),
$w_{\text{insertion}}$ is the specific insertion power (J kg^{-1}),
$\dot{m}_1$ is the net mass flow rate at 1 (kg s^{-1}),
$\dot{m}$ is the mass flow rate crossing the system (always positive, kg s^{-1}),
$\dot{V}_1$ is the volumetric flow rate of fluid ($\text{m}^3\,\text{s}^{-1}$),
and v_1 is the specific volume of the fluid at the inlet ($\text{m}^3\,\text{kg}^{-1}$).

Similarly, for the fluid to exit the system at the other end, the system must continuously supply a power called *extraction power*:

$$\dot{W}_{\text{extraction}} = -p_2 \, \dot{V}_2 = \dot{m}_2 \, p_2 \, v_2 = -\dot{m} \, p_2 \, v_2 \tag{3/4}$$

$$w_{\text{extraction}} = -p_2 \, v_2 \tag{3/5}$$

where the outgoing mass flow rate $\dot{m}_2$ (negative) is expressed in terms of the mass flow rate $\dot{m}$ crossing the system (always positive, kg s^{-1}).

The net sum of these two powers at the boundaries is called *flow power*, $\dot{W}_{\text{flow}} \equiv \dot{W}_{\text{insertion}} + \dot{W}_{\text{extraction}}$. Its sign depends on the operating conditions – the student is encouraged to visualize and formulate the conditions under which the flow power can be negative, zero, or positive.

3.3.2 Energy balance

Let us try to design an open system in steady flow in the most general way possible, as represented in figure 3.3. We will now account for all energy transfers within it.

When entering the system, the fluid already has an internal energy u_1; therefore, the system sees its own internal energy increase with power $\dot{U}_1$:

$$\dot{U}_1 = \dot{m} \, u_1 \tag{3/6}$$

Similarly, the fluid has a specific mechanical energy e_{mech1} (1/9), and the system also receives power $\dot{E}_{\text{mech1}}$:

$$\dot{E}_{\text{mech1}} = \dot{m} \, e_{\text{mech1}} = \dot{m} \left(\frac{1}{2} \, C_1^2 + g \, z_1 \right) \tag{3/7}$$

These expressions 3/6 and 3/7 have the opposite sign at the system's outlet, where we assign them the index 2.

At this point, we have covered all of the energy forms that can be observed crossing the boundaries of an open system together with the fluid: flow work, internal energy, and mechanical energy. Since the first law states that energy is indestructible (§1.1.2), the addition of power $\dot{Q}$ in the form of heat or $\dot{W}$ in the form of work can only vary these three forms. This results in the equation:

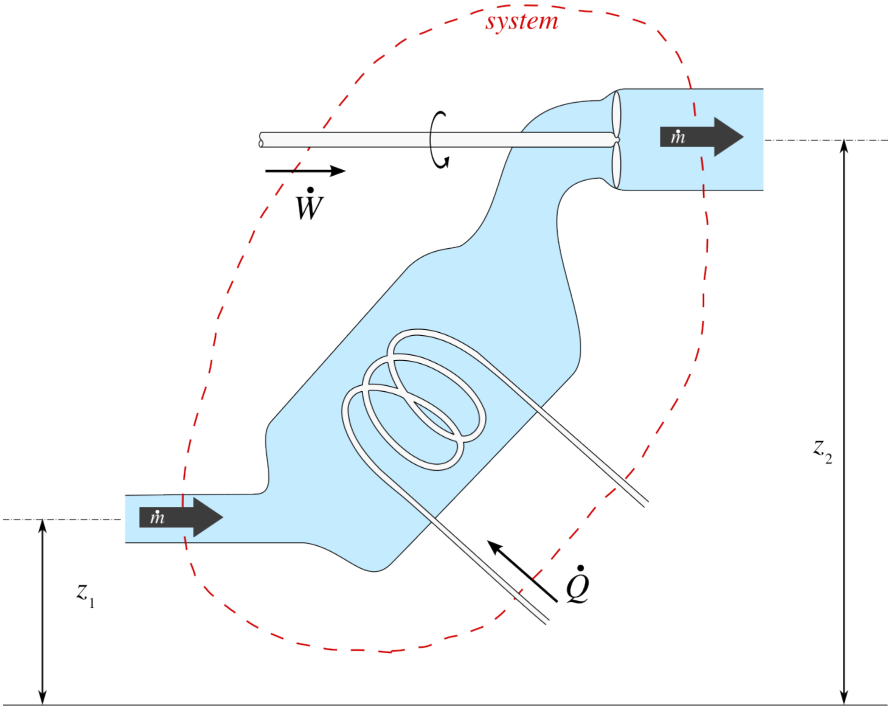


Figure 3.3: An arbitrary open system. The system (whose boundaries are dashed lines, in red) is crossed from left to right by the fluid flowing with a constant mass flow rate $\dot{m}$. It receives power $\dot{W}_{1\to2}$ in the form of work and power $\dot{Q}_{1\to2}$ in the form of heat.

Diagram CC-0 *Olivier Cleynen*

$$\dot{Q}_{1\to2} + \dot{W}_{1\to2} + \left(\dot{W}_{\text{insertion}} + \dot{U}_1 + \dot{E}_{\text{mech1}}\right) + \left(\dot{W}_{\text{extraction}} + \dot{U}_2 + \dot{E}_{\text{mech2}}\right) = 0 \tag{3/8}$$

where all terms are expressed in watts.

We can re-express equation 3/8 in terms of directly measurable quantities:

$$\dot{Q}_{1\to2} + \dot{W}_{1\to2} + \dot{m}\left(p_1\,v_1 + u_1 + \frac{1}{2}C_1^2 + g\,z_1\right) = \dot{m}\left(p_2\,v_2 + u_2 + \frac{1}{2}C_2^2 + g\,z_2\right) \tag{3/9}$$

or:

$$\dot{Q}_{1\to2} + \dot{W}_{1\to2} = \dot{m}\left[\Delta u + \Delta(pv) + \frac{1}{2}\Delta\left(C^2\right) + g\,\Delta z\right] \tag{3/10}$$

$$q_{1\to2} + w_{1\to2} = \Delta u + \Delta(pv) + \Delta e_{\text{mech.}} \tag{3/11}$$

where the symbols Δ indicate the change of properties between points 1 and 2 within the system.

Equations 3/9 and 3/11 are extremely useful in thermodynamics, because they allow us to quantify by deduction the powers involved in flows. They allow us, in particular, to predict the properties of the fluid at the outlet of a device for which we know the mechanical power and heat emissions. For example, we can determine the remaining energy in the air at the outlet of a turbine for which we know the power.

Example 3.1

The compressor of a turbofan admits $1.5\,\mathrm{kg\,s^{-1}}$ of air at a pressure of $0.8\,\mathrm{bar}$, internal energy of $192.5\,\mathrm{kJ\,kg^{-1}}$, and specific volume of $0.96\,\mathrm{m^3\,kg^{-1}}$. It compresses the air to $30\,\mathrm{bar}$, releasing it with an internal energy of $643.1\,\mathrm{kJ\,kg^{-1}}$ and a specific volume of $8.57 \times 10^{-2}\,\mathrm{m^3\,kg^{-1}}$. The velocity and altitude of the air remain unchanged.

What is the power of the compressor if its heat transfers are negligible?

We apply equation 3/10 to obtain:

$$\dot{W}_{1\to2} = -\dot{Q}_{1\to2} + \dot{m}\left[\Delta u + \Delta(pv) + \tfrac{1}{2}\Delta\left(C^2\right) + g\,\Delta z\right]$$
$$= 0 + \dot{m}\left[\Delta u + \Delta(pv) + 0 + 0\right] = 1.5\left[(643.1 \times 10^3 - 192.5 \times 10^3)\right.$$
$$\left. + (30 \times 10^5 \times 8.57 \times 10^{-2} - 0.8 \times 10^5 \times 0.96)\right]$$
$$= +9.464 \times 10^5\,\mathrm{W} = +946.4\,\mathrm{kW}.$$

☞ The only difficulty in applying this equation concerns the proper conversion of units. Pressure and energy should always be converted from their usual units to SI units.

☞ The power is positive, as expected since the air *receives* the work. In a turbine, the work would be negative.

3.3.3 Enthalpy

In many cases, the terms u and pv vary in the same manner together with the state of the fluid (in fact, we will even see in the next chapter that in the case of an ideal gas, they are both proportional to the temperature). In order to simplify their use in calculations, they are often combined into a single term.

We call the sum of the terms u and pv the *specific enthalpy*, and assign it the symbol h:

$$h \equiv u + p\,v \tag{3/12}$$

where the terms are expressed in $\mathrm{J\,kg^{-1}}$.

Of course, the *enthalpy* H is simply defined as:

$$H \equiv m\,h \tag{3/13}$$

where H is measured in joules (J).

In practice, the term *enthalpy* is often used even if it refers to specific enthalpy; the symbol and context help determine which variable is being referred to.

« The decrease of the heat contents is equal to the heat value of the gained “useful work” plus the heat carried away to the outside plus the increase of kinetic energy per pound (or kilogram) of steam. »

Aurel Stodola, 1904
(where the “heat contents” are not yet named *enthalpy*)
Die Dampfturbinen [26, 27]

By using the concept of enthalpy, equations 3/9 and 3/11 are simplified to become:

$$\dot{Q}_{1\to2} + \dot{W}_{1\to2} = \dot{m}\,(\Delta h + \Delta e_{\text{mech.}}) \tag{3/14}$$

$$q_{1\to2} + w_{1\to2} = \Delta h + \Delta e_{\text{mech.}} \tag{3/15}$$

Thus, in an open system, we see that heat and work transfers change the *enthalpy* of the fluid, and not only its internal energy as in a closed system.

Example 3.2

In a nozzle, air expands without any work or heat transfer. It enters with a specific enthalpy of $776\,\mathrm{kJ\,kg^{-1}}$ and a velocity of $30\,\mathrm{km/h}$ ($18.6\,\mathrm{mph}$) and exits at the same altitude, with an enthalpy of $754\,\mathrm{kJ\,kg^{-1}}$.

What is the air ejection velocity?

We start from equation 3/15:

$$\begin{aligned} q_{1\to 2} + w_{1\to 2} &= \Delta h + \Delta e_{\text{mech.}} \\ \Delta e_{\text{mech.}} &= -\Delta h + 0 + 0 \\ \frac{1}{2}\left(C_2^2 - C_1^2\right) &= -\Delta h \\ C_2 &= \left[-2\,\Delta h + C_1^2\right]^{\frac{1}{2}} \end{aligned}$$

So $C_2 = \left[-2 \times (754 \times 10^3 - 776 \times 10^3) + \left(\frac{30}{3.6}\right)^2\right]^{\frac{1}{2}} = 209.9\,\mathrm{m\,s^{-1}} = 755.7\,\mathrm{km/h} = 470\,\mathrm{mph}$.

☞ Care must be used with conversions: in the equations, velocities and energies are always in SI units.

3.4 Quantifying Work with an Open System

3.4.1 Work of a fluid in a slow process

We have seen that when the fluid undergoes a slow process, the work done by a closed system can be quantified by carrying out the integral $-\int p\,\mathrm{d}v$ (2/15). With an open system, the expression is slightly different. In order to develop it, we propose to study the steady compression of a fluid passing through a compressor.

To this end, let us first observe the process undergone by a fixed mass quantity m_A circulating in the compressor (figure 3.4). As it passes between the moving blades, its pressure varies by $\mathrm{d}p$ and its volume by $\mathrm{d}v$. This is simply a moving closed system: since the process is very slow (reversible), the work δw_{m_A} received by the system will be:

$$\delta w_{m_\mathrm{A}} = -p\,\mathrm{d}v \tag{3/16}$$

during a reversible process,
and where the notation δ is used to denote the infinitesimal transfer of work (work being a path quantity, see Appendix A4 p. 318).

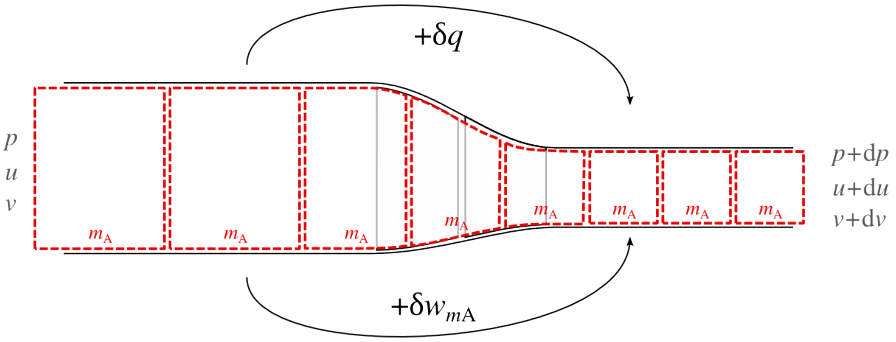


Figure 3.4: A fixed mass quantity m_A flows from left to right through a compressor. It is compressed: its properties change from p and v to $p + \mathrm{d}p$ and $v + \mathrm{d}v$. If we consider the point of view of a closed system in transit, the work transfer is $\delta w_{m_\mathrm{A}} = -p\,\mathrm{d}v$.

Diagram CC-0 *Olivier Cleynen*

Now, let's observe the course of this *same* phenomenon from the point of view of an open system (figure 3.5). What specific power $\delta w_{\mathrm{O.S.}}$ must be supplied to the compressor so that each fluid particle receives work δw_{m_A}?

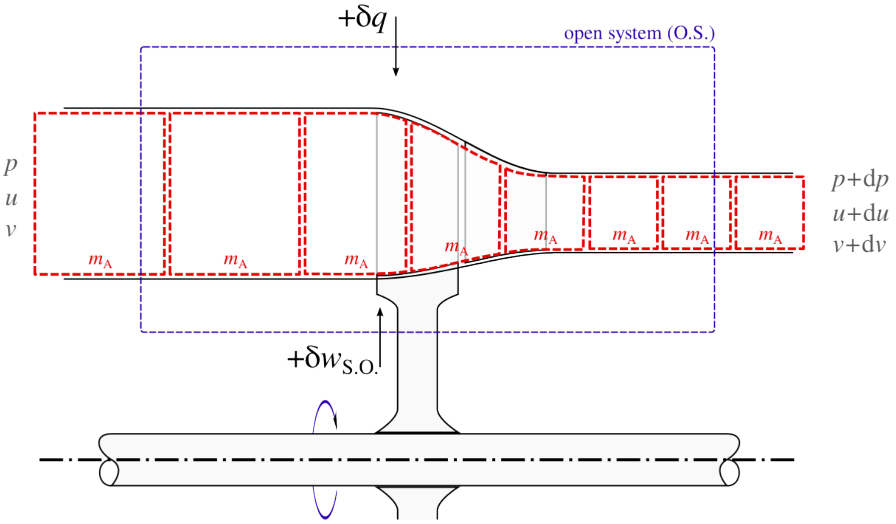


Figure 3.5: The same flow as in figure 3.4, now observed from the viewpoint of a stationary open system crossed from left to right by a steady flow. We seek to quantify the work $\delta w_{\mathrm{O.S.}}$ to be supplied to the system so that each mass quantity m_A receives work δw_{m_A}.

Diagram CC-0 *Olivier Cleynen*

The open system has four work transfer forms:

The specific insertion power $w_{\text{insertion}}$ (3/3) is due to the permanent arrival of the fluid at the system's inlet. From the viewpoint of the open system, we have:

$$w_{\text{insertion}} = +p\,v \tag{3/17}$$

The specific compression power $-\delta w_{m_\mathrm{A}}$ is the specific work that the open system must transfer to each mass quantity m_A to effectively compress it:

$$-\delta w_{m_\mathrm{A}} = -(-p\,\mathrm{d}v) \tag{3/18}$$

The specific extraction power $w_{\text{extraction}}$ is spent by the open system to continuously remove the fluid.

At the outlet, the fluid properties have become $p + \mathrm{d}p$ for pressure, and $v + \mathrm{d}v$ for volume. Thus, we have:

$$w_{\text{extraction}} = -(p + \mathrm{d}p)(v + \mathrm{d}v) \tag{3/19}$$

The specific power received from the outside $\delta w_{\text{O.S.}}$ is the power that feeds the compression: this is the quantity we aim to quantify.

These four powers cancel each other out, since the total work transfer involved in the flow does not depend on the adopted viewpoint:

$$\delta w_{\text{O.S.}} + w_{\text{insertion}} + (-\delta w_{m_{\text{A}}}) + w_{\text{extraction}} = 0 \tag{3/20}$$

Therefore, we can quantify the specific power $\delta w_{\text{O.S.}}$ that must be supplied to the compressor:

$$\begin{aligned}
\delta w_{\text{O.S.}} &= -w_{\text{insertion}} + \delta w_{m_{\text{A}}} - w_{\text{extraction}}\\
\delta w_{\text{O.S.}} &= -p\,v + (-p\mathrm{d}v) + (p + \mathrm{d}p)(v + \mathrm{d}v)\\
&= -p\,v - p\,\mathrm{d}v + p\,v + p\,\mathrm{d}v + \mathrm{d}p\,v + \mathrm{d}p\ \mathrm{d}v\\
&= \mathrm{d}p\,v + \mathrm{d}p\ \mathrm{d}v
\end{aligned}$$

And since the product $\mathrm{d}p \times \mathrm{d}v$ tends to zero when using infinitesimal quantities, we obtain the surprising expression:

$$\delta w_{\text{O.S.}} = v\,\mathrm{d}p \tag{3/21}$$

The terms $\mathrm{d}p$ and $\mathrm{d}v$ in our study are not necessarily positive: this expression applies equally to expansions and compressions, as long as they are reversible.

By integrating this expression 3/21 to apply it to the general case in steady flow, we obtain:

$$w_{\text{O.S.}} = \int v\,\mathrm{d}p \tag{3/22}$$

$$\dot{W}_{\text{A}\to\text{B}} = \dot{m}\int_{\text{A}}^{\text{B}} v\,\mathrm{d}p \tag{3/23}$$

in steady flow,
when the process is reversible,
and regardless of the heat input.

Thus, when we want to quantify reversible work in an open system, it is the integral $+\int v\,\mathrm{d}p$ that needs to be calculated, and not $-\int p\,\mathrm{d}v$.

On a pressure-volume diagram, we can visualize this work by adding the insertion work and extraction work to the compression work, as shown in figure 3.6.

The reversible work done in steady, reversible flow is thus visualized by the area enclosed *to the left* of the curve, as shown in figure 3.7.

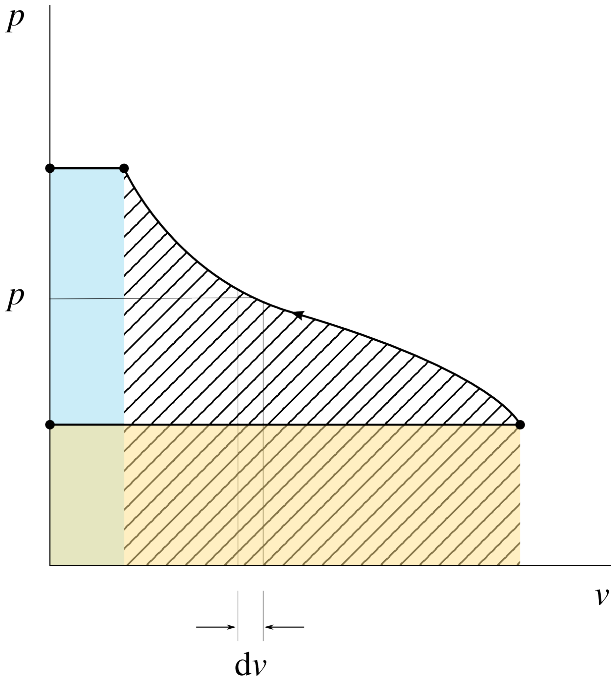


Figure 3.6: Work received by an open system crossed by a fluid, during a slow process.
The system first receives the insertion work ($p_{\text{ini.}} v_{\text{ini.}}$, in orange, positive) to enter the system, then it spends compression work (hatched area, negative), and finally, it spends extraction work ($p_{\text{fin.}} v_{\text{fin.}}$, in blue, negative).
The net sum of these three areas is the specific power to be supplied to the open system.

Diagram CC-0 *Olivier Cleynen*

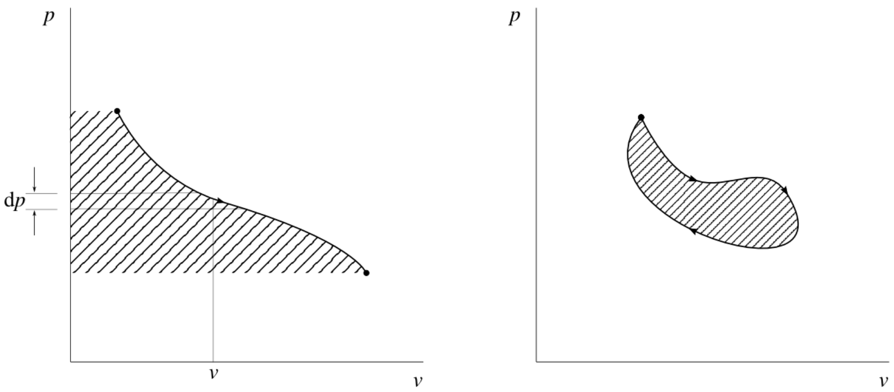


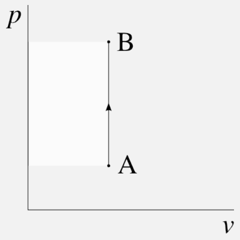
Figure 3.7: Work measured in an open system, during a reversible process.
The integral of $v \, \mathrm{d}p$ is visualized by the area to the left of the curve.
If the fluid returns to its initial state (having completed a *thermodynamic cycle*), the work done is visualized by the area enclosed within the curve. In this case, the quantification is the same for closed and open systems.

Diagram CC-0 *Olivier Cleynen*

Example 3.3

A pump slowly compresses $2\,\mathrm{kg\,s^{-1}}$ (4.41 lb/s) of water from 1 to 20 bar (from 14.5 to 290 psi). During compression, the specific volume of water remains constant at $v_L = 10^{-3}\,\mathrm{m^3\,kg^{-1}}$. What is the power required in the form of work?

The process can be drawn qualitatively (that is, without showing numerical values) on a pressure-volume diagram as follows:



We use equation 3/23, being cautious with the units. Since v is independent of p, integration is straightforward: $\dot{W}_{\mathrm{A}\to\mathrm{B}} = \dot{m} \int_{\mathrm{A}}^{\mathrm{B}} v \,\mathrm{d}p = \dot{m}\, v_L \int_{\mathrm{A}}^{\mathrm{B}} \mathrm{d}p = \dot{m}\, v_L\, [p]_{p_\mathrm{A}}^{p_\mathrm{B}} = 2 \times 10^{-3} \left(20 \times 10^5 - 1 \times 10^5\right) = +3.8 \times 10^3\,\mathrm{W} = +3.8\,\mathrm{kW}$.

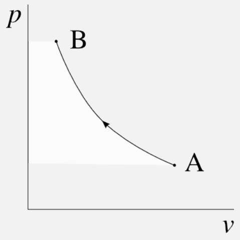
☞ This power is indeed positive, since the fluid in the system is receiving the work.

☞ Here the specific volume v_L is constant (as always with liquid water). If it were the pressure that was constant, then the work would be zero even if v were to vary.

Example 3.4

A compressor slowly compresses an air flow of $2\,\mathrm{kg\,s^{-1}}$ from 1 bar to 20 bar. During compression, the specific volume and pressure of the air are related by the expression $p\,v^{1.35} = k$. At the inlet, the specific volume of the air is $v_\mathrm{A} = 0.8\,\mathrm{m^3\,kg^{-1}}$.
What is the power required in the form of work?

The process can be drawn qualitatively on a pressure-volume diagram as follows:



Here the specific volume is a function of pressure: we have $v = \left(\frac{k}{p}\right)^{\frac{1}{1.35}} = k^{\frac{1}{1.35}} p^{-\frac{1}{1.35}}$. We start from equation 3/23 :

$$\dot{W}_{\text{A}\to\text{B}} = \dot{m} \int_{\text{A}}^{\text{B}} v \, \mathrm{d}p = \dot{m}\, k^{\frac{1}{1.35}} \int_{\text{A}}^{\text{B}} p^{-\frac{1}{1.35}} \, \mathrm{d}p = \dot{m}\, k^{\frac{1}{1.35}} \left[\frac{1}{-\frac{1}{1.35}+1} p^{-\frac{1}{1.35}+1}\right]_{p_\text{A}}^{p_\text{B}}$$
$$= \dot{m} \, \left(p_\text{A} v_\text{A}^{1.35}\right)^{\frac{1}{1.35}} \frac{1}{0.259\,26} \left[p^{0.259\,26}\right]_{p_\text{A}}^{p_\text{B}}$$
$$= 2 \left(1 \times 10^5 \times 0.8^{1.35}\right)^{\frac{1}{1.35}} \frac{1}{0.259\,26} \left[\left(20 \times 10^5\right)^{0.259\,26} - \left(1 \times 10^5\right)^{0.259\,26}\right]$$
$$= +7.247 \times 10^5 \,\text{W} = +724.7 \,\text{kW}.$$

☞ Here the key is to correctly describe the function $v_{(p)}$ before proceeding with the integration.

☞ The power of the compressor is 190 times larger than that of the pump in the previous example. Additionally, the specific volume of the air at the inlet is 800 times larger: a much larger machine will be required (it must handle a volumetric flow rate $\dot{V}_\text{A} = \dot{m}\, v_\text{A} = 1.6 \,\text{m}^3\,\text{s}^{-1} = 1600 \,\text{L}\,\text{s}^{-1} = 423$ US gal/s at the inlet).

3.4.2 Work of a fluid in a fast process

When the process is carried out rapidly (as is always the case in practice), we encounter the phenomena described in the previous chapter (§2.4.3): the pressure exerted on the moving walls no longer corresponds to the "average" pressure inside the fluid. The work required in compressions is greater and the work received during expansions is less than during slow processes.

Using an open system to account for energy transfers does not change the problem, of course. We do not have the means to predict analytically the work required for compression at a given speed. The problem −calculating the spatial distribution of pressure inside the fluid over time− falls within the scope of fluid mechanics, and will be solved on a case-by-case basis.

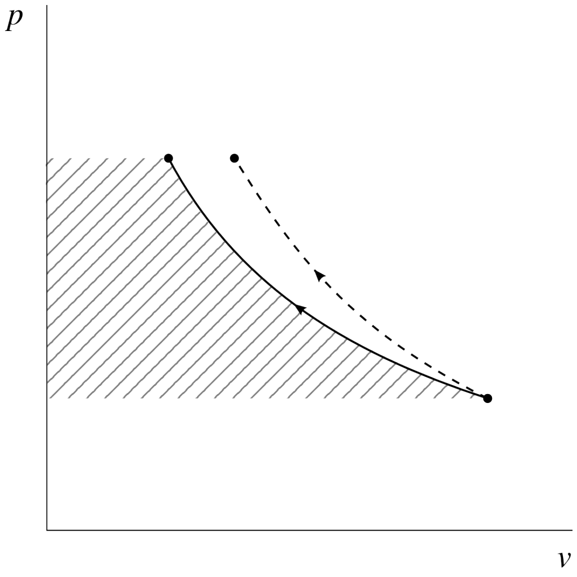


Figure 3.8: Reversible (solid line) and irreversible (dashed line) compressions represented on a pressure-volume diagram. In an open system, work transfers can be visualized with the area to the left of the curve, but only when the processes are reversible. *Diagram* CC-0 *Olivier Cleynen*

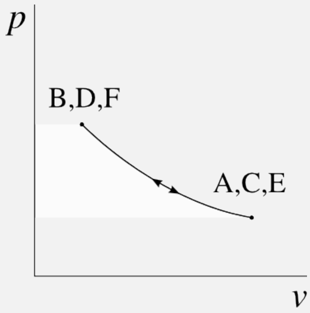
On our pressure-volume diagrams, we represent irreversible processes with a dashed line, to clearly differentiate them from reversible processes, as shown in figure 3.8.

Example 3.5

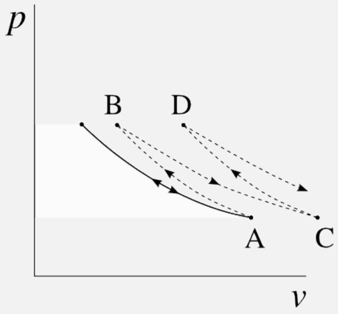
Air is continuously compressed from 1 to 20 bar in a compressor. Just after leaving the compressor, the air enters a turbine that expands it from 20 to 1 bar. After leaving the turbine, the air is again inserted into the compressor.

What will be the shape of the processes on a pressure-volume diagram?

If the processes occur infinitely slowly, the pressure and specific volume always pass through the same values during the back-and-forth movements:



However, if the processes are carried out with realistic speed, at each trip the final specific volume is larger than it would have been during a slow trip:



Thus, the properties gradually shift on the pressure-volume diagram. Unless the processes are infinitely slow, running the compressor requires more energy than the turbine is able to provide. This excess energy is absorbed by the air, increasing its internal energy and temperature.

3.5 Quantifying Heat with an Open System

With an open system, we will use the same method as with a closed system: since we cannot quantify heat transfers directly, we will always proceed by deduction. Mathematically, we simply reuse equation 3/14 to obtain:

$$\dot{Q}_{1\to2} = \dot{m}\left(\Delta h + \Delta e_{\text{mech.}}\right) - \dot{W}_{1\to2} \tag{3/24}$$

$$q_{1\to2} = \Delta h + \Delta e_{\text{mech.}} - w_{1\to2} \tag{3/25}$$

for an open system.

Once again, the main challenge in quantifying a heat transfer is predicting and quantifying the change in enthalpy, Δh. For gases, h is almost proportional to temperature; for liquids and vapors, the relationship is more complex. We will learn how to quantify enthalpy in fluids in chapter 4 (*the ideal gas*) and chapter 5 (*liquids and vapors*).

A Bit of History: Temperature and Amount of Heat

*

By Philippe Depondt
Pierre and Marie Curie University, Paris

In the second half of the 17th century, we started to concern ourselves with the matter of distinguishing between the *degree* of heat and the *quantity* of heat. This was a new question that could only arise when it became possible to reliably measure the temperature, or the "degree of heat," of a body.

Our understanding of heat is largely tied to the sensation of hot or cold: we get burned from contact with boiling water, we feel cold when holding an ice cube in our hand. We can "feel" more or less hot, or more or less cold, an object containing "heat" would "be" more or less hot, all of this was a bit the same… In this context, the idea of a materiality of heat, the idea that a measurable quantity of heat could be transferred from one body to another and induce predictable and measurable temperature changes, remained far off!

Joseph Black (1728-1799), in Edinburgh, Scotland, then took a decisive step forward: he observed that snow does not instantly melt even though the ambient temperature can be well above the melting temperature of ice. Piles of snow, even in full sunlight, can take several days to disappear… He then posed the question: what is the temperature of the water capable of completely melting its own weight of snow at $0\,^{\circ}\mathrm{C}$ ($32\,^{\circ}\mathrm{F}$) without changing its temperature? The experiment provided the answer: $78\,^{\circ}\mathrm{C}$ ($172.4\,^{\circ}\mathrm{F}$)! The lukewarm water cooled down, while the snow remained at the same temperature: the 78 degrees of heat brought by the lukewarm water were incorporated into the ice. The conclusion drawn was that the melting of the snow required the supply of these 78 degrees of heat by the liquid water. He called this heat *latent heat* [39], to differentiate it from *sensible heat* associated with measurable temperature changes. Beyond a clever experiment and a brilliant interpretation, a new concept had emerged.

Black continued his activities by conducting a whole series of calorimetry experiments, mixing water at different temperatures, mixing bodies of different natures at different temperatures; each time he demonstrated that the specific heat depends on the nature of the body:

> We must, therefore, conclude that different bodies, although they be of the same size, or even of the same weight, when they are reduced to the same temperature or degree of heat, whatever that be, may contain very different quantities of the matter of heat; which different quantities are necessary to bring them to this level, or equilibrium, with one another.
>
> Joseph Black, 1807 [2]

He did not make any assumptions about the nature of this heat that he characterized. For most of his contemporaries, however, its apparent conservation indicated that it was a material fluid devoid of mass which Lavoisier later named *caloric*. In this view, this substance could be transferred from one body to another to raise the temperature of the receiving body and lower that of the giving body: some bodies (such as water) contained more of it at a given temperature than others (like oil), resulting in different specific heats. On the other hand, Black believed that the fusion of a solid or the vaporization of a liquid could be a kind of chemical combination of the caloric fluid with the matter: the caloric would then disappear as such and become "latent".

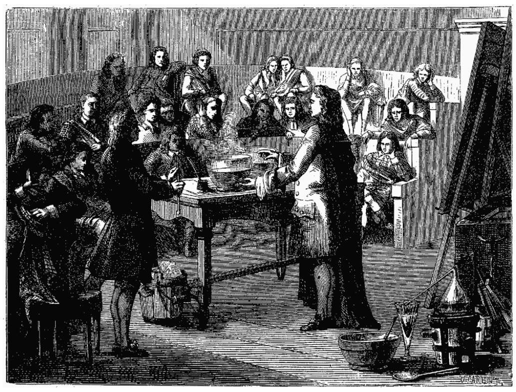


Figure 3.9: Joseph Black conducting an experiment on latent heat during a university lecture in Edinburgh. One can imagine that he had no difficulty engaging the students, since it was about thermodynamics…

Engraving by unknown author published by Louis Figuier in 1867 (public domain)

Problems

3.1 Steam Turbine

A steam turbine (figure 3.10) is used in a small power plant fueled by the combustion of biomass.

At the inlet of the turbine, the steam has the following properties:

- Pressure: 45 bar (652.7 psi)
- Temperature: 400 °C (932 °F)
- Specific volume: $0.064\,77\ \mathrm{m^3\,kg^{-1}}$
- Internal energy: $2914.2\ \mathrm{kJ\,kg^{-1}}$

At the outlet of the turbine, the following properties are measured:

- Pressure: 0.75 bar (10.8 psi)
- Temperature: 91.61 °C (196.9 °F)
- Specific volume: $2.122\ \mathrm{m^3\,kg^{-1}}$
- Internal energy: $2316.3\ \mathrm{kJ\,kg^{-1}}$

The heat losses are negligible.

1. What is the specific power output released by the turbine in the form of work?
2. What steam flow rate is required to produce a power of 4 MW?

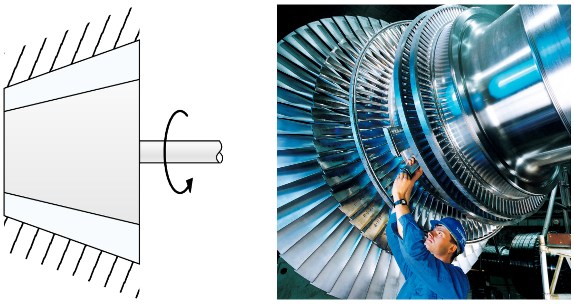


Figure 3.10: Schematic diagram and photo of a steam turbine.

Diagram CC-0 *Olivier Cleynen; Photo* CC-BY-SA *Siemens Pressebild*

3.2 Electric Power Generator

In a portable electricity-generating power plant, the electric generator is driven by a mechanical shaft. Along this shaft, there is an air compressor and a turbine (figure 3.11). This type of device, sometimes simply called a “gas turbine”, is particularly compact and efficient; however, it requires the use of refined fuels.

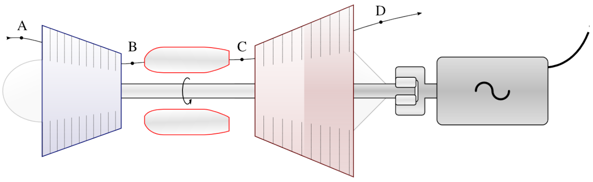


Figure 3.11: Schematic diagram of an electricity-generating turboshaft engine

Diagram CC-BY-SA *Olivier Cleynen*

The compressor brings the air from atmospheric conditions to a high pressure and temperature.

Compressor inlet:

- Pressure: 1 bar (14.5 psi)
- Specific volume: $0.751\ \mathrm{m^3\,kg^{-1}}$
- Internal energy: $206.78\ \mathrm{kJ\,kg^{-1}}$

Compressor outlet:

- Pressure: 35 bar (507.6 psi)
- Specific volume: $6.602 \times 10^{-2}\ \mathrm{m^3\,kg^{-1}}$
- Internal energy: $578.13\ \mathrm{kJ\,kg^{-1}}$

Between the compressor and the turbine, the combustion chamber raises the temperature. The combustion takes place at constant pressure; it brings the gases to a specific volume of $0.1168\ \mathrm{m^3\,kg^{-1}}$ and an internal energy of $1028.8\ \mathrm{kJ\,kg^{-1}}$.

At the turbine outlet, the gases are ready to be cooled in a catalytic exhaust system designed, among other things, to reduce noise emissions.

Turbine outlet:

- Pressure: 1.2 bar (17.4 psi)
- Specific volume: $1.526\ \mathrm{m^3\,kg^{-1}}$
- Internal energy: $460.88\ \mathrm{kJ\,kg^{-1}}$

The air flow rate admitted into the machine is $8\ \mathrm{kg\,s^{-1}}$; the changes in its mechanical energy are nearly zero. Heat losses through the walls of the machine are negligible. Mechanical losses are 2 % of the power transmitted to the generator. The electric generator itself has an efficiency of 85 %.

1. What power is received or rejected by the air in the compressor?
2. What power is received or rejected by the air in the turbine?
3. What is the electric power generated by the power plant?
4. Represent the processes undergone by the air as it passes through the engine on a pressure-volume diagram, qualitatively (that is, without showing numerical values).
5. What is the power lost in the form of heat together with the exhaust gases?
   [hint: it is the heat that the gases should lose to return to their state at the inlet of the compressor]

3.3 Steam Boiler

A power plant provides electricity as well as industrial and domestic heat from the combustion of household waste (known as a *cogeneration* plant). It is equipped with a water circuit that receives some of the heat released by the combustion, at constant pressure, in a boiler.

Water enters the boiler (figure 3.12) in a liquid state, pressurized at $71.89\,\mathrm{kg_f/cm^2}$ ($70.5\,\mathrm{bar}$). Its internal energy is then $1160.2\,\mathrm{kJ\,kg^{-1}}$. We want to feed the turbine with $317\,\mathrm{t\,h^{-1}}$ of steam at an enthalpy of $3595.9\,\mathrm{kJ\,kg^{-1}}$.

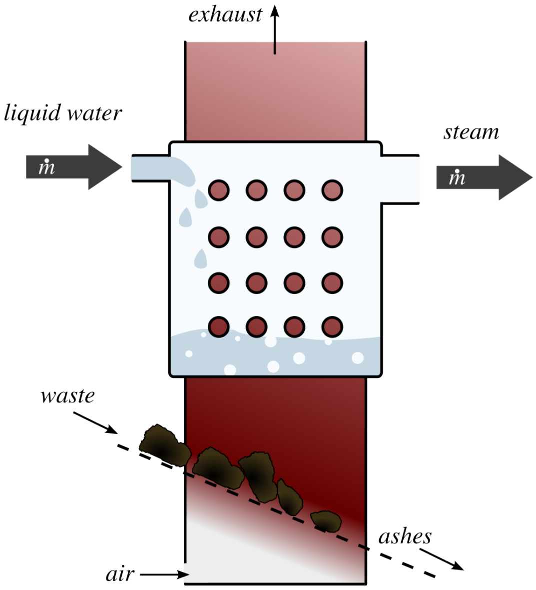


Figure 3.12: Schematic diagram of a boiler operating from the combustion of waste.

Diagram CC-0 *Olivier Cleynen*

The combustion of household waste generates between 9 and $11\,\mathrm{MJ\,kg^{-1}}$ of heat; the boiler efficiency is $76\,\%$.

What is the minimum flow rate of waste that the power plant must receive in order to produce the required amount of steam?

3.4 Turbine Engine Nozzle

In the nozzle of a small turbojet engine, the air pressure drops while its speed increases. The nozzle (figure 3.13) is a component without any moving parts: no work is done there. Heat losses are negligible, and the air flow rate is $26\,\mathrm{kg\,s^{-1}}$ ($57.22\,\mathrm{lb/s}$).

At the inlet, the following characteristics are measured:

• Specific enthalpy:	$1092\,\mathrm{kJ\,kg^{-1}}$
• Velocity:	$10\,\mathrm{m\,s^{-1}}$ ($32.8\,\mathrm{ft/s}$)
• Temperature:	$1250.33\,\mathrm{°F}$ ($950\,\mathrm{K}$)
• Specific volume:	$1.36\,\mathrm{m^3\,kg^{-1}}$
• Pressure:	$2.325\,\mathrm{kg_f/cm^2}$ ($2.28\,\mathrm{bar}$)
• Internal energy:	$781.85\,\mathrm{kJ\,kg^{-1}}$

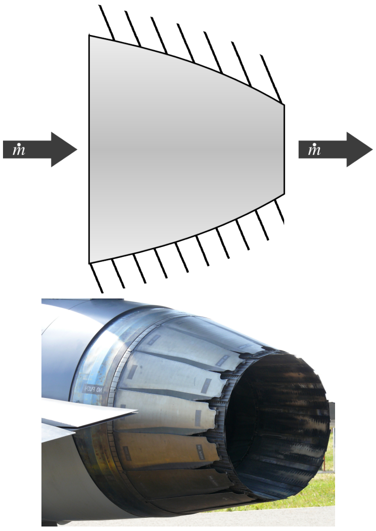


Figure 3.13: Schematic diagram of a nozzle and installation (with variable geometry) on the *Pratt & Whitney* F100 engine of a Lockheed Martin F-16.

Diagram CC-0 *Olivier Cleynen; Photo* CC-BY-SA *Ad Meskens (retouched)*

At the outlet, the air is brought back down to atmospheric pressure ($1\,\mathrm{bar}$). It is predicted (we will see how in chapter 4) that the air characteristics will reach:

• Temperature:	$944.7\,\mathrm{°F}$ ($780.2\,\mathrm{K}$)
• Specific volume:	$2.55\,\mathrm{m^3\,kg^{-1}}$
• Internal energy:	$642.1\,\mathrm{kJ\,kg^{-1}}$

1. What is the ejection speed of the gases?
2. What are the volumetric flow rates of air at the inlet and outlet of the nozzle?

3.5 Water Turbine

An engineer is working on a small hydroelectric power plant project. The objective is to harness the flow of a river ($3170\,\mathrm{US\,gal/s}$ or $12\,\mathrm{m^3\,s^{-1}}$) with a turbine connected to an electric generator (figure 3.14).

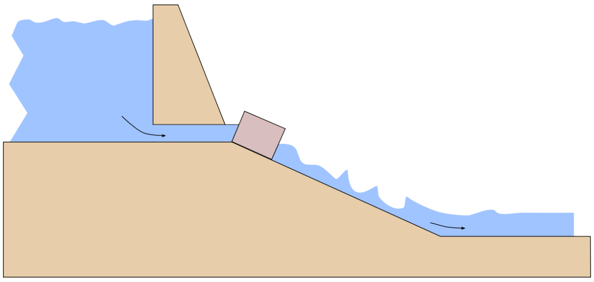


Figure 3.14: Schematic diagram of a hydroelectric power plant

Diagram CC-0 *Olivier Cleynen*

In its liquid state, water is essentially incompressible (meaning its density does not change when its pressure changes). Its internal energy also varies negligibly during adiabatic compressions and expansions.

The engineer first considers placing the turbine at the foot of a water reservoir, where the pressure is 4 bar and the velocity is nearly zero. The water falls through a height of 2 m (6.56 ft) through the turbine, and its ejection velocity is $4\,\mathrm{m\,s^{-1}}$ at atmospheric pressure (1 bar).

1. What power could the turbine transmit to the generator?

The engineer then studies a different configuration (figure 3.15). The turbine would keep the same characteristics but would be positioned further downstream from the water reservoir (shifted horizontally and vertically by 25 m, or 82 ft, each).

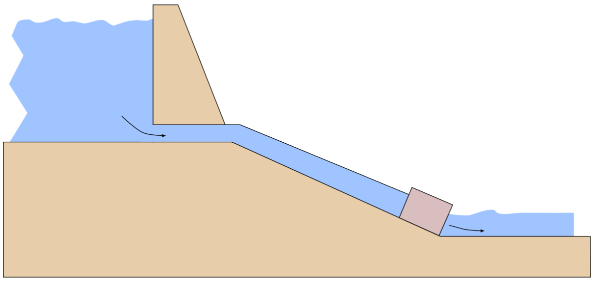


Figure 3.15: Schematic diagram of the modified power plant. A rigid pipe brings water to the turbine placed lower down.

Diagram CC-0 *Olivier Cleynen*

2. What would be the power transmitted in this case?

3.6 Afterburner System

In order to increase the thrust it generates, the nozzle from problem 3.4 is modified to add and *afterburning* device (later to be studied in §10.6.2 p. 293). It consists of a set of burners that allow a second combustion of fuel to occur in the engine, just before the air enters its expansion in the nozzle (figure 3.16). After the second combustion, the air goes through its expansion and acceleration until atmospheric pressure.

At the inlet, the conditions are the same as those indicated in problem 3.4.

The specific power added in the form of heat by the burners reaches $1322.5\,\mathrm{kJ\,kg^{-1}}$. The burnt fuel has a specific thermal capacity of $30\,\mathrm{MJ\,kg^{-1}}$. The combustion takes place at constant pressure, and it does not increase the gas's kinetic energy.

When the air completes its acceleration, its internal energy is predicted to be $1406.4\,\mathrm{kJ\,kg^{-1}}$ and its specific volume to be $5.59\,\mathrm{m^3\,kg^{-1}}$.

1. What is the increase in ejection speed (and thus thrust) generated by the afterburner?
2. How much fuel flow rate should be injected into the burners, in kg/h?
3. What is the volumetric flow rate of air after its final acceleration?
4. What is the efficiency of the afterburner, in other words, the ratio between the increase in gas kinetic energy and the increase in power to be supplied in the form of heat?

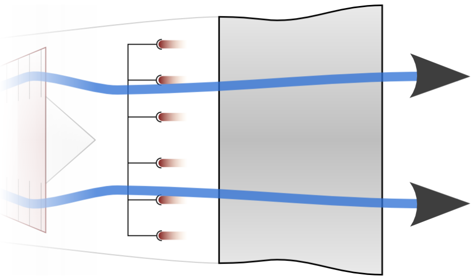


Figure 3.16: Schematic diagram of an afterburner system. Its operation is studied in §10.6.2 p. 293.

Diagram CC-by-sa *Olivier Cleynen*

3.7 Steam Turbine

A small steam turbine produces 500 kW of power, with a mass flow rate of $1.35\,\mathrm{kg\,s^{-1}}$ (~3 lb/s).

The average speed of the steam is $60\,\mathrm{m\,s^{-1}}$ at the inlet, $360\,\mathrm{m\,s^{-1}}$ at the outlet; it gains 3 m in altitude during the process. The heat loss is 3 kW.

1. What are the changes in kinetic energy, potential energy, and enthalpy of the steam as it passes through the turbine?
2. Since the heat loss is 3 kW, why not thermally isolate the turbine to be able to recover this power in the form of work?

3.8 Theoretical and Actual Turbines

In the free turbine of a helicopter's turboshaft engine, air is expanded to extract work which is transmitted to the two rotors. The characteristics are as follows:

• Mass flow rate:	$2\,\mathrm{kg\,s^{-1}}$
• Heat losses:	negligible
• Inlet:	4 bar and $0.41\,\mathrm{m^3\,kg^{-1}}$
• Outlet pressure:	1.1 bar

In the most favorable case, the expansion would take place reversibly, and the air would follow a relationship of the form $pv^{1.4} = k$ (where k is a constant).

1. Which conditions must be met for the expansion to be reversible?
2. What would be the power supplied by the turbine in this case?

In practice, it is observed that the power supplied by the turbine is 20 % lower than the value calculated above. An engineer installs probes at the inlet and outlet of the turbine and observes that the pressure there indeed reaches the theoretically expected values. S/he also measures the heat transfer from the air to the turbine and confirms that it is negligible.

3. Draw the processes undergone by the air in the reversible and real cases on a pressure-volume diagram, qualitatively.
4. In which form will the engineer find (and measure) the missing 20 % of power?

3.9 Compressor and Turbine of a Turboprop Engine

The compressor within a turboprop engine (figure 3.17) admits a constant flow of air at ambient conditions (0.8 bar/11.6 psi and 1 m^3 kg^{-1}). It must bring this air to a final pressure of 11 bar (159.5 psi), without any heat transfer.

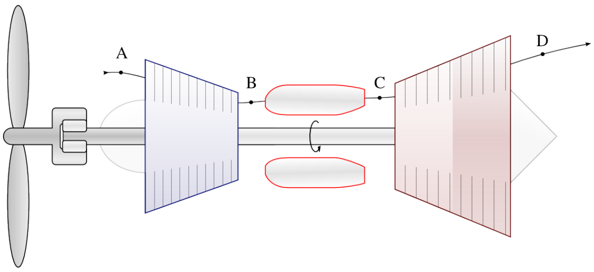


Figure 3.17: Schematic diagram of a turboprop engine. These engines are studied in more detail in §10.5.4 p. 288.

Diagram CC-BY-SA *Olivier Cleynen*

Within the compressor, the air behaves in such a way that its properties follow the relation $p\,v^{1.4} = k$, where k is a constant.

1. What is the minimum specific power to be supplied to the compressor?
2. Represent the properties of the gas as it passes through the compressor on a pressure-volume diagram, qualitatively.
3. On the above diagram, show the change the gas would undergo if the compressor were not reversible (real compressor, inducing internal friction in the gas) but nevertheless maintained its outlet pressure at 11 bar.

Within the same engine, the turbine, which is adiabatic, must power not only to the compressor but also the propeller at the front of the engine. It is equipped with numerous probes to measure the properties of the air.

At its inlet, the following properties are measured:

• Pressure	11 bar
• Velocity	12 m s^{-1}
• Specific volume	0.36 m^3 kg^{-1}
• Internal energy	985.8 kJ kg^{-1}

At the outlet, the properties of the air have become:

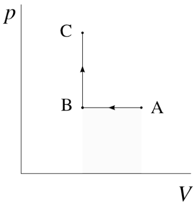
• Pressure	0.8 bar
• Velocity	12 m s^{-1}
• Specific enthalpy	652.5 kJ kg^{-1}

4. What is the specific power of the turbine?
5. What condition must be met within the engine to provide the propeller with a power of 600 kW?

Answers

3.1 1) $w_{\mathrm{A}\to\mathrm{B}} = -730.3\,\mathrm{kJ\,kg^{-1}}$
2) $\dot{m} = \frac{\dot{W}_{\mathrm{A}\to\mathrm{B}}}{w_{\mathrm{A}\to\mathrm{B}}} = 5.477\,\mathrm{kg\,s^{-1}}$

3.2



1) $\dot{W}_{\mathrm{A}\to\mathrm{B}} = +4.219\,\mathrm{MW}$
2) $\dot{W}_{\mathrm{C}\to\mathrm{D}} = -6.349\,\mathrm{MW}$
3) $\dot{E}_{\mathrm{generator}} = \eta_{\mathrm{generator}}\eta_{\mathrm{transmission}}\left(\dot{W}_{\mathrm{A}\to\mathrm{B}} + \dot{W}_{\mathrm{C}\to\mathrm{D}}\right) = -1.774\,\mathrm{MW}$
5) $\dot{Q}_{\mathrm{cooling}} = -\dot{W}_{\mathrm{A}\to\mathrm{B}} - \dot{Q}_{\mathrm{combustion}} - \dot{W}_{\mathrm{C}\to\mathrm{D}} = -2.897\,\mathrm{MW}$ (so more than half of the combustion heat…)

3.3 1) $\dot{m}_{\mathrm{waste}} \geq 92.1\,\mathrm{t/h} \approx 203\,000\,\mathrm{lb/h}$

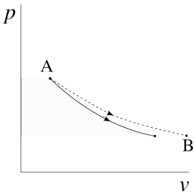
3.4 1) $C_2 = 624\,\mathrm{m\,s^{-1}}$ (about $2250\,\mathrm{km/h}$ or $1400\,\mathrm{mph}$…)
2) $\dot{V}_1 = \dot{m}\,v_1 = 35.4\,\mathrm{m^3\,s^{-1}}$ & $\dot{V}_2 = 66.3\,\mathrm{m^3\,s^{-1}}$.

3.5 1) $\dot{W}_{\mathrm{A}\to\mathrm{B}} = -3.74\,\mathrm{MW}$
2) $\dot{W}_{\mathrm{A}\to\mathrm{B2}} = -6.68\,\mathrm{MW}$

3.6 1) $+52\,\%$ compared to dry thrust ($C_{3b} = 950.1\,\mathrm{m\,s^{-1}}$)
2) $\dot{m}_{\mathrm{fuel}} = \frac{\dot{m}_{\mathrm{air}}\ q_{1\to 2b}}{q_{\mathrm{fuel}}} = 4126\,\mathrm{kg/h} = 9096\,\mathrm{lb/h}$
3) $\dot{V}_{3b} = 145.3\,\mathrm{m^3\,s^{-1}}$
4) $\eta_{\mathrm{afterburning}} = \frac{\frac{1}{2}\left(C_{3b}^2 - C_2^2\right)}{q_{1\to 2b}} = 19.2\,\%$ (one reason why it is never used on civilian aircraft)

3.7 1) $\Delta e_c = +63\,\mathrm{kJ\,kg^{-1}}$, $\Delta e_p = +0.0294\,\mathrm{kJ\,kg^{-1}}$ (!), $\Delta h = -435.6\,\mathrm{kJ\,kg^{-1}}$.
2) In equation 3/15, setting $q_{1\to 2} = 0$ does not guarantee that $w_{1\to 2}$ will increase. It is probable that the outlet conditions are modified: the $3\,\mathrm{kW}$ will at least partly be found in the Δ values calculated above.

3.8



1) see §2.4.4 p. 49
2) $\dot{W}_{\mathrm{A}\to\mathrm{B}} = \dot{m}k^{\frac{1}{1.4}}\left[\frac{1}{-\frac{1}{1.4}+1}p^{-\frac{1}{1.4}+1}\right]_{p_{\mathrm{A}}}^{p_{\mathrm{B}}} = -354.1\,\mathrm{kW}$
4) In the form of Δh – the outlet air will have higher specific volume and temperature (internal energy), and maybe also higher kinetic energy.

3.9 1) $w_{\mathrm{A}\to\mathrm{B}} \geq \int_{\mathrm{A}}^{\mathrm{B}} v\,\mathrm{d}p = +312\,\mathrm{kJ\,kg^{-1}}$
2) & 3) see fig. 3.8 p. 71;
4) $w_{\mathrm{C}\to\mathrm{D}} = -729.3\,\mathrm{kJ\,kg^{-1}}$
5) $\dot{m}_{\mathrm{air}} = \frac{\dot{W}_{\mathrm{propeller}}}{w_{\mathrm{propeller}}} = \frac{\dot{W}_{\mathrm{propeller}}}{w_{\mathrm{A}\to\mathrm{B}} + w_{\mathrm{C}\to\mathrm{D}}} = 1.438\,\mathrm{kg\,s^{-1}}$.

CHAPTER 4
The Ideal Gas

or

The Improbable Thermometer of Scholar Clapeyron

Chapter 4 – The Ideal Gas

Executive summary for chapter 4

The ideal gas is a mathematical model that allows us to predict the properties (internal energy, enthalpy) of a gas simply based on its temperature. By approximation, air is an ideal gas.

Introduction

In chapters 2 and 3 we learned to quantify energy transfers — but we can only do so when we know the values of u or h, which are quantities that are impossible to measure directly in practice.

This chapter 4 (*the ideal gas*) aims to answer two questions:

- How can we describe the behavior of air when it is heated or compressed?
- How can we predict the values of u and h when using air?

This chapter is incompatible with chapter 5 (*liquids and vapors*), where we will have to forget everything learned here.

4.1 Definition

4.1.1 The manometer as a thermometer

Let us start with the most important point:

> The ideal gas is a *mathematical model*,
> allowing to predict the temperature of a gas
> as a function of its pressure.

The ideal gas model defines by itself a temperature scale. It is proposed to measure the absolute temperature T very simply with a manometer, stating that it is directly proportional to the pressure p and inversely proportional to the density ρ.

We can thus say that the ideal gas does not describe the reality of things, that it is not a physical principle, but only a simplified model of gas behavior. Its range of validity is limited and fuzzy.

« The change in temperature occasioned in gases by a change in volume may be regarded as one of the most important facts of physics, owing to the numerous consequences it entails, and at the same time as one of the most difficult to elucidate and to measure by decisive experiments. It appears in several circumstances to present singular anomalies. »

Sadi Carnot, 1824 [4]

4.1.2 Definition: the equation of state

We will call *ideal gas* a fluid in the gaseous state whose product of pressure and volume, pv, remains proportional to its temperature. The proportionality constant is called the *gas constant*, denoted as R; it depends on the nature of the gas.

$$pv = RT \tag{4/1}$$

by definition for an ideal gas,
where p is the pressure (Pa),
v is the specific volume ($\text{m}^3\,\text{kg}^{-1}$),
T is the temperature (K),
and R is the gas constant of the considered gas ($\text{J}\,\text{K}^{-1}\,\text{kg}^{-1}$).

Equation 4/1 is called the *equation of state of ideal gases*. It can also be expressed in terms of mass:

$$pV = mRT \tag{4/2}$$

where V is the volume (m^3),
and m is the mass of the gas considered (kg).

« M. S. Carnot, avoiding the use of mathematical analysis, arrives through a series of subtle and hard to grasp reasonings at results which are without difficulty deduced from a more general law, which I shall endeavor to establish. »

Émile Clapeyron, 1834 [5]

It is also possible to express Equation 4/2 in terms of the amount of substance in moles.[1] Because it is inseparable from the concept of absolute temperature, it took 150 years for this equation to take its final form: the one given by Émile Clapeyron in 1834 [5].

Example 4.1

A mass of $2\,\mathrm{kg}$ of gas with a constant $R = 100\,\mathrm{J\,K^{-1}\,kg^{-1}}$ is contained in a reservoir of $200\,\mathrm{L}$ ($52.83\,\mathrm{US\ gal}$) at a pressure of $3\,\mathrm{bar}$ ($43.51\,\mathrm{psi}$). What is its temperature?

We start from Equation 4/2 to express the temperature: $T = \frac{p\,V}{m\,R} = \frac{3\times10^5\times0.2}{2\times100} = 300\,\mathrm{K} = 26.85\,^\circ\mathrm{C} = 80.33\,^\circ\mathrm{F}$.

☞ Admirable Émile! The simplicity of this calculation will be missed in the next chapter.

☞ The only thorn in this equation concerns the units, which must be properly converted to SI: $200\,\mathrm{L} = 0.2\,\mathrm{m^3}$ and $3\,\mathrm{bar} = 3\times10^5\,\mathrm{Pa}$. The temperature is always in kelvins.

Example 4.2

Atmospheric air can be modeled as an ideal gas with $R_\mathrm{air} = 287\,\mathrm{J\,kg^{-1}\,K^{-1}}$. Under ambient conditions ($1\,\mathrm{bar}$, $20\,^\circ\mathrm{C}$, that is, $14.5\,\mathrm{psi}$ and $68\,^\circ\mathrm{F}$), what are the specific volume and the density of the atmosphere?

We start from Equation 4/1 to express the specific volume: $v = \frac{R\,T}{p} = \frac{287\times(20+273.15)}{1\times10^5} = 0.841\,\mathrm{m^3\,kg^{-1}}$. The density then follows simply: $\rho = \frac{1}{v} = 1.189\,\mathrm{kg\,m^{-3}}$.

☞ Once again, forgetting to convert the temperature units to kelvins would be fatal (try using $0\,^\circ\mathrm{C}$ or $0\,^\circ\mathrm{F}$).

☞ In one cubic meter ($220\,\mathrm{gal\ imp}$), we have only $1.2\,\mathrm{kg}$ ($2.6\,\mathrm{lb}$) of air. This is very little, especially when compared to liquid water ($1000\,\mathrm{kg}$ for the same volume). Air machines typically operate with large volumetric flow rates.

---

[1] Sometimes in other books, the constant in $\mathrm{J\,K^{-1}\,kg^{-1}}$ is denoted as r. The quantity then denoted $R = 8.3143\,\mathrm{J\,K^{-1}\,mol^{-1}}$ is universal, and gases adopt different values of r depending on their molar mass $M \equiv \frac{m}{n}$. In this book, we do not quantify amounts of substance.

4.1.3 What does an ideal gas represent?

The ideal gas is the simplest model one can imagine to represent the behavior of a gas.

According to this model, molecules behave like spheres bouncing off each other (figure 4.1). One can imagine a large number of very small billiard balls in chaotic motion, colliding and bouncing off each other without attracting one another or dissipating their energy through friction.

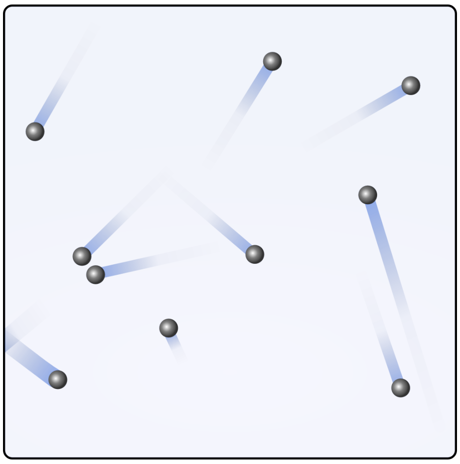


Figure 4.1: An ideal gas can be visualized as a set of balls in random motion. They collide without friction and without mutual attraction. The speed of each ball changes with each collision.

Diagram CC-BY-SA *by Commons User:Sharayanan & Olivier Cleynen*

In this chaos, temperature is a measure of the kinetic energy of the molecules. It is quantified by measuring the time-averaged force resulting from the impact of the molecules on a wall of the container – that is, with pressure. With this model, we can propose a temperature scale such that $T \propto p$.

The fewer molecules impacting the surface, and the more forcefully they must impact it in order to generate a given pressure. Thus, when the density ρ decreases at a given pressure, it means that the temperature is increasing: we can also propose $T \propto \frac{1}{\rho}$.

If these two proposals are combined into a single equation, we obtain a simple model to quantify temperature: $T \propto pv$.

4.1.4 What does an ideal gas *not* represent?

The behavior of molecules when they are close to each other is actually very complex, since the attractive forces then play a decisive role. The influence of these forces is all the more significant when the molecules are slow and structurally complex (the interaction between two hydrocarbon molecules, for example, is more difficult to model than the interaction between two helium molecules).

The macroscopic consequences of these interactions, and the conditions under which they should no longer be neglected, are addressed in chapter 5 (*liquids and vapors*).

> « Anyone who wants to analyze the properties of matter in a real problem might want to start by writing down the fundamental equations and then try to solve them mathematically. Although there are people who try to use such an approach, these people are the failures in this field; the real successes come to those who start from a *physical* point of view, people who have a rough idea where they are going and then begin by making the right kind of approximations, knowing what is big and what is small in a given complicated situation. »
>
> Richard Feynman, 1963 [30, 35]
> *The Feynman Lectures on Physics*

For now, we will remember that the ideal gas model works better:

- When molecules collide at high speed, that is, when the gas temperature is high;
- When the average space between molecules is large, that is, when the specific volume of the gas is large.

These conditions ensure that the attractive forces between molecules can only play a minor role in the overall behavior of the gas. They are met for air in the vast majority of engineering applications. We will use the value $R_{\text{air}} = 287\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$ for pure air in our machines.

4.1.5 Model limitations

It will not take long for the student to find the limits of Equation 4/2, which indicates that a non-zero mass of ideal gas occupies *zero volume* at zero temperature. Strictly speaking, the ideal gas cannot exist – the mathematical model loses its meaning at very low temperatures since it does not take into account the volume of the molecules themselves.

Several other equations of state can be used to better match real gases over a wider range of properties.

Thus, the *Van der Waals equation*, proposed as early as the late 18th century, suggests:

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT \tag{4/3}$$

where a and b are two constants.

This equation has the advantage of taking into account two factors ignored in the equation of state 4/1: the attractive force between molecules (the term a/v^2, which becomes part of the pressure expression) and the volume occupied by the molecules themselves (the term b which is subtracted from the available volume).

Despite the difficulties inherent in quantifying the terms a and b, these modifications have significantly extended the range of application of equations of state. They earned their author, Johannes Diderik Van der Waals, the Nobel Prize in Physics in 1910.

Building mathematical models to describe the state of real gases is an important research area in fluid mechanics. The curious student can refer to equations of state such as the Beattie-Bridgeman, Benedict-Webb-Rubin, or the Strobridge models, in order get an overview of their increasing complexity. As for us, we will stick to equation 4/1.

4.2 Properties of Ideal Gases

4.2.1 Two important thermal capacities

We have already discussed the concept of thermal capacity in the first chapter (1/16). It is defined as the amount of heat required to increase the temperature of one kilogram of the substance by one Kelvin (or one degree Celsius, since these temperature differences are equal). Thus, we have:

$$c = \frac{\delta q}{\mathrm{d}T} \tag{4/4}$$

where c is the specific thermal capacity ($\mathrm{J\,K^{-1}\,kg^{-1}}$),
δq the (specific) infinitesimal heat transfer ($\mathrm{J\,kg^{-1}}$),
and $\mathrm{d}T$ the produced infinitesimal change in temperature (K).

Since the temperature of a gas also varies when work is done on it or by it, there are an infinite number of different ways to change its temperature by one degree, by combining heat and work (figure 4.2). Each of these requires a unique amount of heat; thus, there are *infinitely many thermal capacities* associated with it.

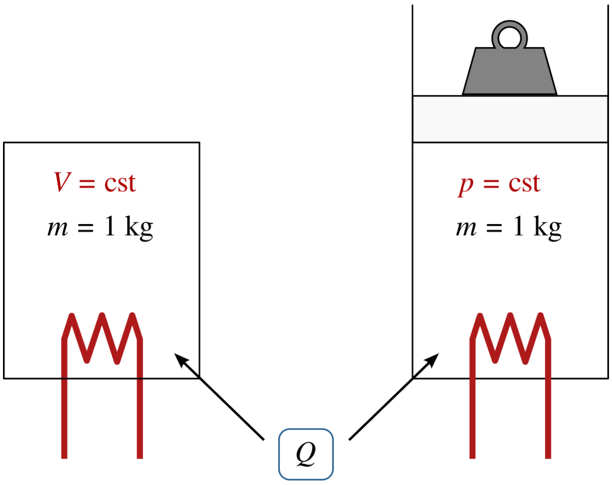


Figure 4.2: Two identical quantities of gas receive the same amount of heat Q. The temperature increase will be lower on the right due to the work done on the piston.

Diagram CC-0

Among these, two particular values (figure 4.3) serve as references for describing the behavior of an ideal gas:

the thermal capacity at constant volume: c_v ,

the thermal capacity at constant pressure: c_p .

These two quantities are properties (or state quantities, see Appendix A4 p. 318), and we will soon use them to quantify energy in gases. In an ideal gas, c_v and c_p are independent of temperature. In real gases, these capacities vary with temperature (figure 4.4), but for most hand-written engineering applications, it is reasonable to use average values. For air, we will take $c_{v\,\text{(air)}} = 718\,\mathrm{J\,kg^{-1}\,K^{-1}}$ and $c_{p\,\text{(air)}} = 1005\,\mathrm{J\,kg^{-1}\,K^{-1}}$.

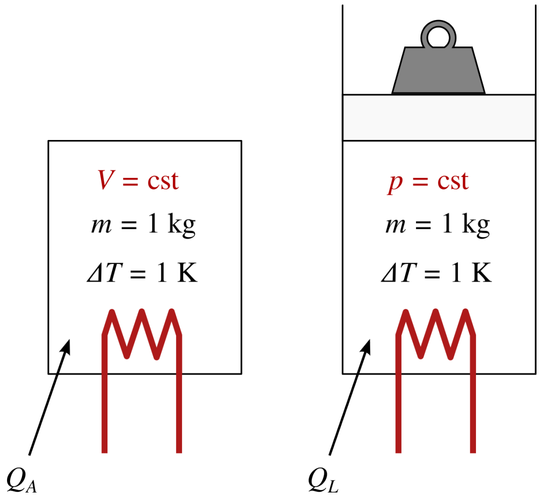


Figure 4.3: Definitions of heat capacities. On the left, the volume is fixed and the specific thermal capacity will be c_v. On the right, the pressure is constant and the capacity will be c_p.

Diagram CC-0

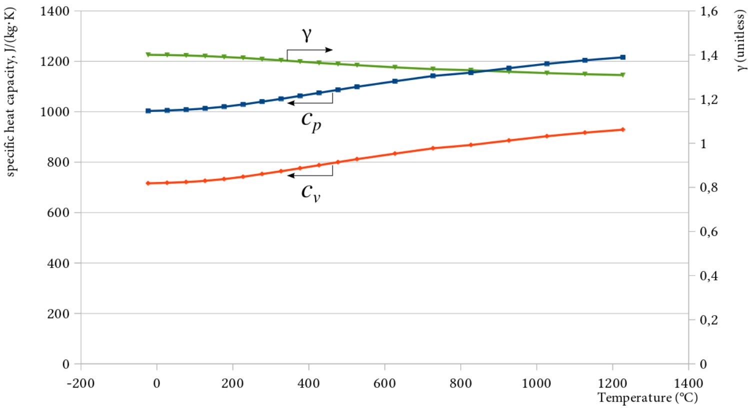


Figure 4.4: Specific thermal capacity of air as a function of temperature. There is a noticeable change in values in the temperature range used in engineering, which we will neglect in the scope of this book.

Data from NBS Circular 564 "Tables of Thermal Properties of Gases" (1955) up to 1000 K, *calculated according to the model of B. G. Kyle in "Chemical and Process Thermodynamics" (1984) above* 1000 K, *and published by Israel Urieli*

4.2.2 Difference of thermal capacities

The large number of equations we are discussing makes this short section 4.2.2 useful, but not essential. For the engineer, its only interest is to simplify the writing of the equations in the section that comes after it.

Let us observe the amounts of energy involved in the experiment described in figure 4.3. We supply each body with a different amount of heat to achieve the same temperature change. The difference between the two required amounts of heat comes from the fact that the gas at constant pressure (on the right) has done work during the process.

What is the difference between the thermal capacities of each? In both cases, we have $q_{1\to2} + w_{1\to2} = \Delta u$ (2/2). For body A on the left, since no work is done and the process is at constant volume, we can write:

$$\begin{cases} q_{\mathrm{A}} &= c_v \, \Delta T \\ \Delta u &= q_{\mathrm{A}} \end{cases} \tag{4/5}$$

For body B on the right, with constant pressure $p_{\text{cst.}}$, we can write:

$$\begin{cases} q_{\text{B}} &= c_p \, \Delta T \\ \Delta u &= q_{\text{B}} + (-p_{\text{cst.}} \, \Delta v) \end{cases} \tag{4/6}$$

By combining the two systems 4/5 and 4/6, we obtain

$$c_v \, \Delta T = c_p \, \Delta T - p_{\text{cst.}} \, \Delta v \tag{4/7}$$

which simply states that the difference between the two amounts of heat supplied to the gas is found in the work done by the gas on the right. (To be truly rigorous, in order to assert that Δu_A and Δu_B are equal, we would need to wait for equation 4/11 which comes in the next section.)

A little algebra leads to:

$$\begin{aligned} (c_p - c_v)\Delta T &= p_{\text{cst.}} \, \Delta v \\ (c_p - c_v) &= \frac{pv_2 - pv_1}{\Delta T} = \frac{RT_2 - RT_1}{\Delta T} = \frac{R \, \Delta T}{\Delta T} \\ c_p - c_v &= R \end{aligned} \tag{4/8}$$

This expression only serves to simplify equation 4/13 that we will write below.

4.2.3 Ratio of thermal capacities

The ratio of thermal capacities at constant pressure and constant volume is named γ. Thus:

$$\gamma \equiv \frac{c_p}{c_v} \tag{4/9}$$

By returning to figure 4.3 it quickly appears that c_p must be greater than c_v; thus γ is always greater than 1. We take $\gamma_{\text{air}} = 1.4$.

4.3 Energy and Temperature

4.3.1 Historical context

The first research undertakings aimed at exploring the concept of temperature took place at the very beginning of the 19th century. The scientific community was then very interested in gases – it was noticed that there are *two* ways to increase their temperature: by heating them, but also by compressing them.

Frenchman Joseph Louis Gay-Lussac sought to understand why the temperature of a gas drops when it expands (he actually sought, according to the concepts of the time, to identify the source of the *caloric* and the reasons why it flows). He thus endeavored to produce gas expansions that were as simple as possible, and to measure the temperature. Thirty years later, the Englishman James Prescott Joule resumed and deepened these experiments, but this time, by quantifying heat as *work equivalence*. These experiments with gas balloons and thermometers are anything but spectacular – but they would play a pivotal role in thermodynamics, because they allowed for the first time to distinguish heat, work, energy, and temperature. Joule's meticulous work lead to the first formal expression of the first law of thermodynamics, and to the end of the caloric theory according to which heat was a very low-density and invisible fluid. Our modern unit for energy is named after him as a tribute to these results.

4.3.2 Joule's law

In their most remarkable experiment, Joule and Gay-Lussac were seeking to vary the pressure and volume of a gas *without transferring heat or work to it*. For this, they let a compressed gas in a container expand into a second, empty container (figure 4.5). The work done was zero, since no surface had been moved – the process was entirely irreversible. The temperature was measured and… nothing happened! Joule and Gay-Lussac measured neither heat transfer nor temperature variation.

Joule carried out a multitude of different experiments during which he observed that regardless of the supplied work, the relationship between internal energy (which varies only with work and heat) and temperature remained essentially the same – and he suggested that for an ideal gas, it always remains identical.

> « I have taken two two-tubulure balloons, each with a capacity of twelve liters. To one of the tubulures of each balloon was adapted a faucet, and to the other a very-sensitive alcohol thermometer, whose centigrade degrees could easily be divided into hundredths[…] The vacuum having being made in both balloons, and having assured myself that they retained it exactly, I filled one of them with the gas upon which I wished to operate. About twelve hours later, I established communication between them by means of a lead pipe, and upon opening the faucets, the gas then precipitated itself into the empty balloon until the pressure equilibrium was re-established on both sides. During this time, the thermometer experienced variations which I carefully noted. »
>
> Louis Joseph Gay-Lussac, 1807 [3]

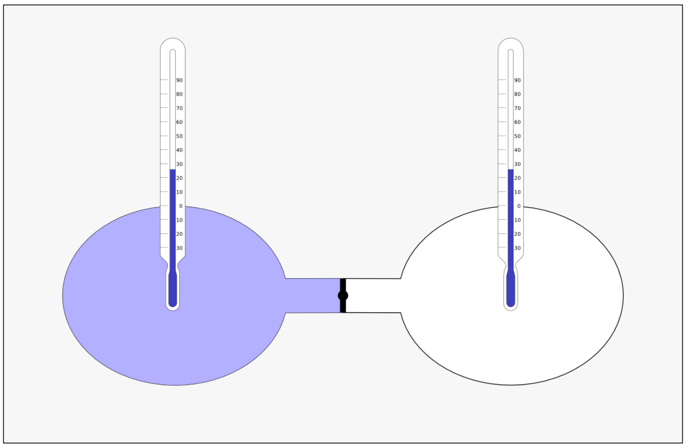


Figure 4.5: The expansion of Joule and Gay-Lussac.
A gas is initially trapped in a reservoir on the left; it is allowed to expand by opening the valve (in the center) which separates it from a completely empty reservoir on the right.
Joule and Gay-Lussac are interested in the temperature changes measured in each reservoir. The closer the gas properties resemble the behavior of ideal gases model (§4.1.4), the smaller the temperature changes they measure, becoming undetectable for some simple gases at high temperatures.

Diagram CC-0 *Olivier Cleynen*

This postulate is known as *Joule's law* and is posited as true for any ideal gas. It can be summarized as follows:

> The temperature of an ideal gas
> only varies with its internal energy.

Mathematically, we can write it as:

$$u = f(T) \tag{4/10}$$

The function f can be evaluated with an experiment in which the change of u is quantified. For example, during a process at constant volume $q = \Delta u$ and $q = c_v \, \Delta T$. We can thus assert that the function f is a simple proportional relation. With the internal energy arbitrarily set to zero at zero temperature ($u = 0\,\mathrm{J\,kg^{-1}}$ when $T = 0\,\mathrm{K}$), we obtain:

> « The difference between the means of tile experiments and interpolations being exactly such as was found to be due to the increased effect of the temperature of the room in the latter case, we arrive at the conclusion, that *no change of temperature occurs when air is allowed to expand in such a manner as not to develope mechanical power.* »
>
> James Prescott Joule, 1845 [8]

$$u = c_v \, T \tag{4/11}$$

for any ideal gas,
regardless of the process (reversible or not),
where u is the specific internal energy ($\mathrm{J\,kg^{-1}}$);
T is the temperature (K);
and c_v is the specific thermal capacity at constant volume ($\mathrm{J\,kg^{-1}\,K^{-1}}$).

For a mass m of ideal gas, we have of course:

$$U = m \, c_v \, T \tag{4/12}$$

As long as our fluid behaves as an ideal gas, this relation 4/11 remains true. It works for any process, reversible or not, and regardless of volume, pressure, or temperature constraints.

On the other hand, it should be noted that this equation 4/11, which results from Joule's law, does not work all for liquids and vapors. For example, we can add energy to a mass of boiling water without its temperature increasing. We will study liquids and vapors in chapter 5.

Example 4.3

The specific thermal capacity at constant volume for air is measured at $c_{v\,\text{(air)}} = 718\,\mathrm{J\,kg^{-1}\,K^{-1}}$.
We take a mass of 0.5 kg of air at 20 °C and transfer +15 kJ as heat and −10 kJ as work. What is its final temperature?

We know that the energy has varied with the transfers: $\Delta U = W_{\mathrm{A}\to\mathrm{B}} + Q_{\mathrm{A}\to\mathrm{B}} = m \, c_v \, \Delta T$. Thus, the temperature has varied proportionally: $T_\mathrm{B} = T_\mathrm{A} + \frac{\Delta U}{m \, c_v} = T_\mathrm{A} + \frac{W_{\mathrm{A}\to\mathrm{B}}+Q_{\mathrm{A}\to\mathrm{B}}}{m \, c_v} = 20 + \frac{-10\times10^3+(+15\times10^3)}{0.5\times718} = 33.92\,^\circ\mathrm{C}$.

☞ Good old James! We just only need to quantify the energy changes to know the temperature, and vice versa.

☞ Here the temperatures in degrees Celsius are only added and a conversion to kelvins would not have changed the result. In case of doubt, it is better not to take this shortcut.

4.3.3 Enthalpy of an ideal gas

Because we have just linked the internal energy u to the temperature, and because and the product pv also depends on the temperature, we can now easily express the enthalpy h of an ideal gas in terms of temperature only.

Indeed, we have $h \equiv u + pv$ (3/12); with a quick insertion of equations 4/1 and 4/11 we can write, for any ideal gas:

$$h = u + pv = c_v T + RT = (c_v + R)\, T \tag{4/13}$$

Using the equation 4/8 that we developed earlier, we can simplify this expression to obtain:

$$h = c_p\, T \tag{4/14}$$

For any ideal gas,
regardless of the process (reversible or not),
where h is the specific enthalpy ($\mathrm{J\,kg^{-1}}$);
T is the temperature (K);
and c_p is the specific thermal capacity at constant pressure ($\mathrm{J\,kg^{-1}\,K^{-1}}$).

Example 4.4

The specific thermal capacity at constant pressure for air is measured at $c_{p\,\text{(air)}} = 1005\,\mathrm{J\,kg^{-1}\,K^{-1}}$.
A flow rate of $2\,\mathrm{kg\,s^{-1}}$ of air passes through a compressor, where its temperature increases by $150\,^\circ\mathrm{C}$. What is the power supplied by the compressor to the air?

We know that the change energy is directly proportional to the change in temperature. Using equations 3/14 and 4/14, we obtain: $\dot{W}_{\mathrm{A}\to\mathrm{B}} + \dot{Q}_{\mathrm{A}\to\mathrm{B}} = \dot{m}\,\Delta h = \dot{m}\,c_p\,\Delta T = 2 \times 1005 \times (+150) = +301.5\,\mathrm{kW}$.

☞ With an ideal gas, a simple thermometer is enough to quantify energy…

☞ …but not to differentiate work and heat. Here we cannot separate $\dot{W}_{\mathrm{A}\to\mathrm{B}}$ from $\dot{Q}_{\mathrm{A}\to\mathrm{B}}$. We also cannot predict the state of the gas at the outlet, that is its pressure p and specific volume v (only their product). For this, a precise description of what happens between the inlet and the outlet would be needed.

4.3.4 Interlude: what to remember so far

The ideal gas is a model for quantifying the temperature of a gas. According to this model, the three main forms of energy we have used so far — internal energy u, enthalpy h, and the term pv — are directly proportional to the temperature T:

$$\begin{aligned} u &= c_v T \qquad &(\mathrm{J\,kg^{-1}}) \\ h &= c_p T \qquad &(\mathrm{J\,kg^{-1}}) \\ pv &= RT \qquad &(\mathrm{J\,kg^{-1}}) \end{aligned}$$

If we measure the absolute temperature of a gas, then we can immediately quantify these three forms of energy.

4.4 Elementary Reversible Processes

Here we intend to calculate the properties of an ideal gas, as well as the energy transfers involved, when it is compressed or expanded under completely arbitrary constraints of volume, pressure, or temperature.

4.4.1 What is this chapter section for?

The gas processes we study here are very hypothetical and not necessarily exciting, but they deserve the attention of the student for two reasons:

1. The behavior of gases is inherently complex, even when we use the ideal gas model. These elementary processes serve as small exercises for us, to help us learn step by step;
2. These elementary processes are conceptual tools that we will later assemble, first to quantify the theoretical limits of machines (in chapter 7), and then to describe the behavior of gases inside real machines (in chapter 10).

4.4.2 Processes at constant pressure

It is possible to heat or cool a gas while maintaining its pressure constant (figure 4.6). A process at constant pressure is called *isobaric*. In order to generate such a process, we can:

- with a closed system, heat or cool the gas while maintaining a constant force on the walls;
- with an open system, heat or cool the gas by simply letting it flow in a duct, without any moving parts. This is the case, for example, in the combustion chamber of a jet engine.

When the pressure is constant, the properties of the gas vary according to the relation

$$\frac{T}{v} = \text{constant} \qquad (4/15)$$

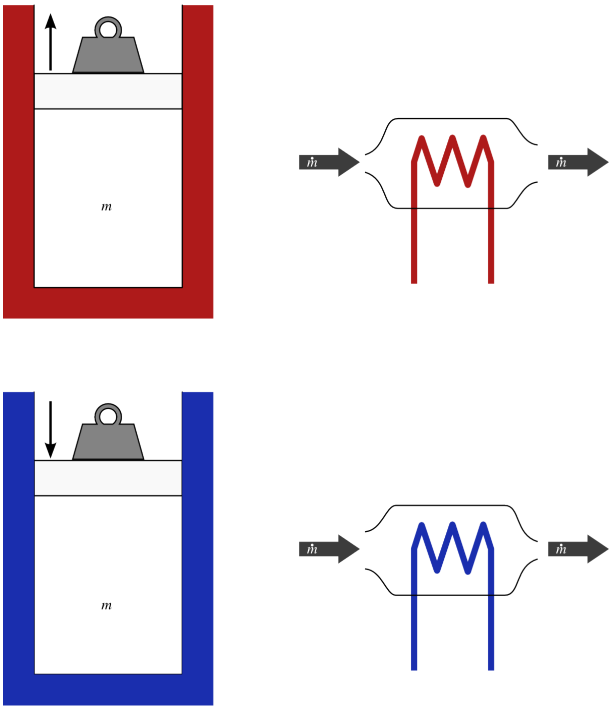


Figure 4.6: A constant-pressure (isobaric) process undergone by an ideal gas. In a closed system (left), the piston exerts a constant force throughout the process. In an open system (right), no work is done.

Diagram CC-0 *Olivier Cleynen*

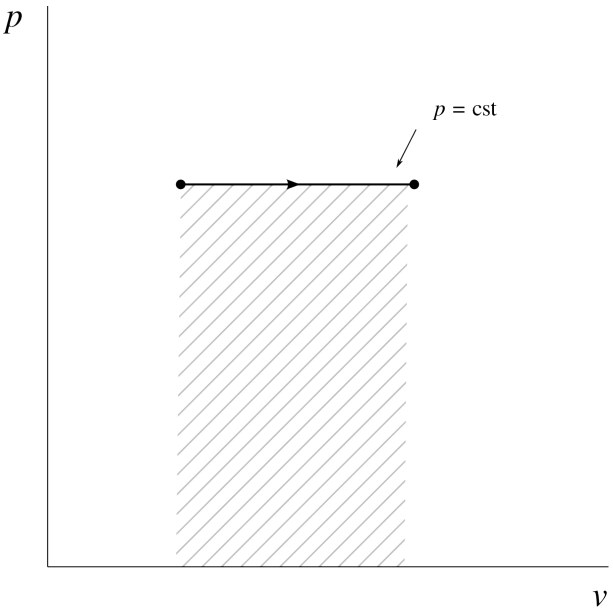


Figure 4.7: Heating at constant pressure of an ideal gas, represented on a pressure-volume diagram.

Diagram CC-0 *Olivier Cleynen*

In a closed system, we have $q_{1\to2} + w_{1\to2} = \Delta u$ (2/2) and, if the process is reversible, heat and work can be easily related to the temperature:

$$w_{1\to2} = -\int_1^2 p \, \mathrm{d}v = -p_{\text{cst.}} \int_1^2 \mathrm{d}v = -p_{\text{cst.}} \, \Delta v$$
$$w_{1\to2} = -R \, \Delta T \qquad (4/16)$$

during a reversible process at constant pressure $p_{\text{cst.}}$, in a closed system.

and we notice that the work is of opposite sign to the change in temperature.

Heat can be easily quantified:

$$q_{1\to2} = \Delta u - w_{1\to2} = \Delta u + p_{\text{cst.}} \, \Delta v = \Delta h$$
$$q_{1\to2} = c_p \, \Delta T \qquad (4/17)$$

during a reversible process at constant pressure, in a closed system.

When the process takes place in an open system, we have $q_{1\to2} + w_{1\to2} = \Delta h$ (3/15), and, if the process is reversible, heat and work can be easily quantified:

$$w_{1\to2} = \int_1^2 v \, \mathrm{d}p$$
$$w_{1\to2} = 0 \qquad (4/18)$$

during a reversible process at constant pressure, in an open system.

The work is of course zero, since there are no moving parts present to mechanically extract energy from the gas.

Heat is then responsible for the entire change in temperature:

$$q_{1\to2} = \Delta h - w_{1\to2} = \Delta h$$
$$q_{1\to2} = c_p \, \Delta T \qquad (4/19)$$

during a reversible process at constant pressure, in an open system.

Example 4.5

For air, we have $c_{p\,(\text{air})} = 1005\,\mathrm{J\,kg^{-1}\,K^{-1}}$, $c_{v\,(\text{air})} = 718\,\mathrm{J\,kg^{-1}\,K^{-1}}$, $R_{\text{air}} = 287\,\mathrm{J\,kg^{-1}\,K^{-1}}$. How much energy is needed to heat the air in a $30\,\mathrm{m^2}$ ($322.9\,\mathrm{ft^2}$) apartment from 10 °C to 20 °C (50 °F to 68 °F)?

The heating will likely take place at constant pressure (unless the apartment is hermetically sealed, the pressure will be atmospheric everywhere and the air will "leak" under the doors). We assume a pressure of 1 bar and a ceiling height of 2.5 m. We use a closed system encompassing all the heated air.

We have a volume of $75\,\mathrm{m^3}$, which leads to the total mass of air as $m_{\mathrm{A}} = \frac{p_{\mathrm{A}} V_{\mathrm{A}}}{R\,T_{\mathrm{A}}} = \frac{1\times10^5\times75}{287\times(10+273.15)} = 92.29\,\mathrm{kg} = 203.5\,\mathrm{lb}$.
The heat required to heat this amount of air at constant pressure can be quantified with equation 4/17: $Q_{\mathrm{A}\to\mathrm{B}} = m\,c_p\,\Delta T = 93.29 \times 1005 \times (20 - 10) = +9.28 \times 10^5\,\mathrm{J} = +928\,\mathrm{kJ}$.

☞ This result represents the final *net* heat transfer to the air (after losses to walls and windows, as well as transfers that compensate for them).

☞ Conversely, cooling would cause outside air to come in, which would need to be taken into account in the mass calculation.

4.4.3 Processes at constant volume

It is possible to heat or cool a gas while maintaining its volume constant (figure 4.8). A process at constant volume is called *isochoric*.

- With a closed system, we can heat or cool a gas in a fixed and closed reservoir. This is the case, for example, during the combustion phase in a gasoline engine.
- With an open system, the manipulation is more complex. When heating the gas, we must compress it, in order to prevent its volume from increasing; conversely, while cooling it, we must expand it so as to prevent its volume from decreasing. This manipulation has no common practical application.

When the specific volume of a perfect gas is constant, its properties vary according to the relation

$$\frac{T}{p} = \text{constant} \tag{4/20}$$

In a closed system, we have $q_{1\to2}+w_{1\to2} = \Delta u$ and, if the process is reversible, heat and work can be easily related to temperature.

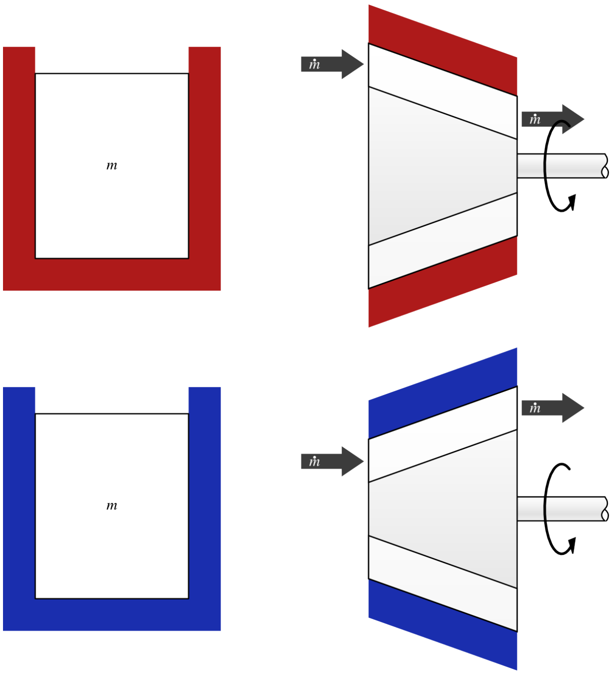


Figure 4.8: A constant-volume (isochoric) process undergone by an ideal gas. In a closed system (left), the volume is fixed and no work is done. In an open system (right), the gas must be compressed while heating and expanded while cooling, to maintain the specific volume constant.

Diagram CC-0 *Olivier Cleynen*

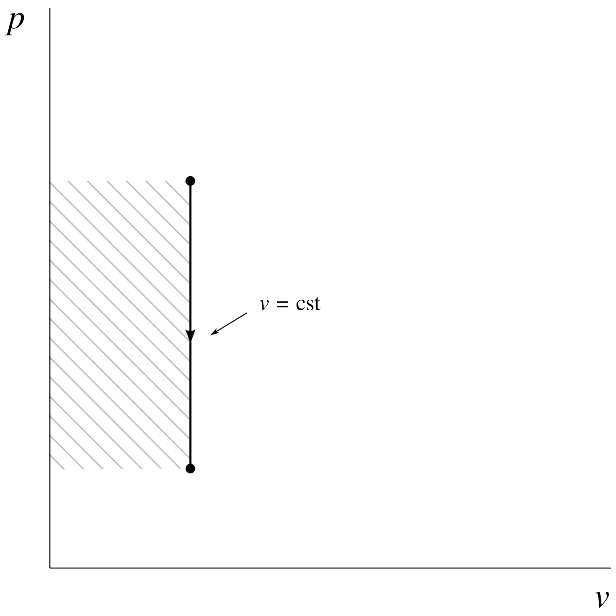


Figure 4.9: Cooling of an ideal gas at constant volume , represented on a pressure-volume diagram.

Diagram CC-0 *Olivier Cleynen*

Since the volume does not change, the work is of course zero:

$$w_{1\to2} = -\int_1^2 p \, \mathrm{d}v$$
$$w_{1\to2} = 0 \tag{4/21}$$

during a reversible process at constant volume, in a closed system.

The heat transfer can be easily quantified:

$$q_{1\to2} = \Delta u - w_{1\to2} = \Delta u$$
$$q_{1\to2} = c_v \, \Delta T \tag{4/22}$$

during a reversible process at constant volume, in a closed system.

When the process occurs in an open system, we have $q_{1\to2} + w_{1\to2} = \Delta h$ and, if the process is reversible, heat and work can be quantified, although with a little more difficulty:

$$w_{1\to2} = \int_1^2 v \, \mathrm{d}p = v_{\text{cst.}} \int_1^2 \mathrm{d}p = v_{\text{cst.}} \int_1^2 \frac{R}{v_{\text{cst.}}} \mathrm{d}T = R \int_1^2 \mathrm{d}T$$
$$w_{1\to2} = R \, \Delta T \tag{4/23}$$

during a reversible process at constant volume, in an open system.

One can then easily quantify the heat to be supplied:

$$q_{1\to2} = \Delta h - w_{1\to2} = c_p \, \Delta T - R \, \Delta T$$
$$q_{1\to2} = c_v \, \Delta T \tag{4/24}$$

during a reversible process at constant volume, in an open system.

Example 4.6

For air, we measure $c_{p\,(\text{air})} = 1005\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$, $c_{v\,(\text{air})} = 718\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$, $R_{\text{air}} = 287\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$.

In a cylinder of a gasoline engine, air is at a pressure of 17 bar (246.6 psi) with a density of $9.4\,\text{kg}\,\text{m}^{-3}$. The combustion of the fuel (so fast that the volume does not have time to vary) results in the supply of $1450\,\text{kJ}\,\text{kg}^{-1}$ of heat. What are the temperature and pressure values reached?

Initially, the temperature is $T_\text{A} = \frac{p_\text{A} v_\text{A}}{R} = \frac{p_\text{A}}{\rho_A R} = \frac{17\times10^5}{9.4\times287} = 630.1\,\text{K} = 357\,^\circ\text{C} = 674.5\,^\circ\text{F}$.

We use a closed system consisting of the mass of air. Since the volume does not change, the work is zero (4/21) and it is the heat transfer which changes the internal energy (4/22): $q_{\text{A}\to\text{B}} = \Delta u - w_{\text{A}\to\text{B}} = \Delta u = c_v\,\Delta T$; thus $T_\text{B} = T_\text{A} + \frac{q_{\text{A}\to\text{B}}}{c_v} = 630.1 + \frac{1450\times10^3}{718} = 2649.6\,\text{K} = 2376.5\,^\circ\text{C} = 4309.6\,^\circ\text{F}$.

The final pressure is obtained by comparing the final condition and the initial condition (4/20): $\frac{RT_\text{A}}{p_\text{A}} = v_\text{A} = v_\text{B} = \frac{RT_\text{B}}{p_\text{B}}$; thus $p_\text{B} = \frac{T_\text{B}}{T_\text{A}} p_\text{A} = \frac{2649.6}{630.1} \times 17 \times 10^5 = 7.148 \times 10^6\,\text{Pa} = 71.5\,\text{bar} = 1037\,\text{psi}$.

☞ Care must be taken with temperatures in the fractions, where they must be expressed in kelvins.

☞ The maximum temperature, exceeding $2300\,^\circ\text{C}$, surpasses the melting temperature of most metals. In a gasoline engine, this temperature is only reached sporadically, with each combustion.

☞ The data in this example mimic those of problem 2.5 p. 54. This time, we can predict the final conditions without having to make any measurements.

4.4.4 Processes at constant temperature

> « It is easy to imagine a small quantity of gaseous or liquid combustible, or dust coal, gradually introduced into a volume of compressed and highly heated air, and burning by spontaneous or separate ignition. The piston is forced out at the same time in such a way that no increase of temperature takes place, because the heat developed by each particle of combustible is instantly absorbed by the cooling due to expansion. Therefore the whole of the heat developed will be transformed into work. »
>
> Rudolf Diesel, 1893
> *Theorie und Konstruktion eines rationellen Wärmemotors zum Ersatz der Dampfmaschinen und der heute bekannten Verbrennungsmotoren*
> [23, 24]

It is possible to heat or cool a gas while maintaining its temperature constant (figure 4.10). A process at constant temperature is called *isothermal*.

For an ideal gas, a process at constant temperature always occurs at constant energy. For each joule of heat supplied to the gas, one joule of work must be extracted from it; conversely, every heat withdrawal must be compensated by an equal amount of work input.

In practice, this complexity makes it so that isothermal heat transfers are rarely used in industry. However, they have crucial theoretical importance, which we will explore in chapter 7 (*the second law*).

When the temperature of an ideal gas remains constant, its properties vary according to the relation

$$p\,v = \text{constant} \qquad (4/25)$$

In a closed system, we have $q_{1\to2}+w_{1\to2} = \Delta u$, and if the process is reversible, heat and work can be related to the properties of the gas, although not

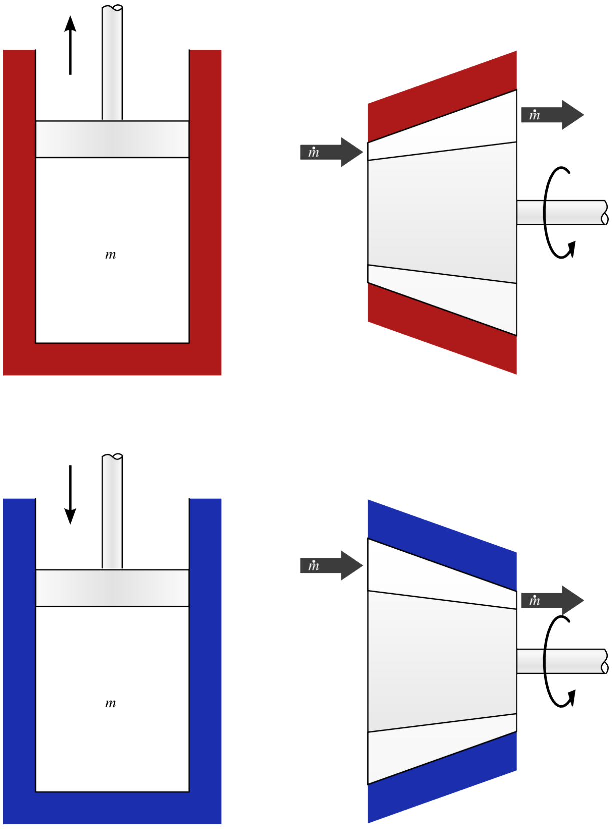


Figure 4.10: A constant-temperature (isothermal) process undergone by an ideal gas. In a closed system (left), the gas is allowed to do work on a piston while being heated, and conversely, work is done on the gas when it is cooled. In an open system (right), the same manipulations are carried out continuously.

Diagram CC-0 *Olivier Cleynen*

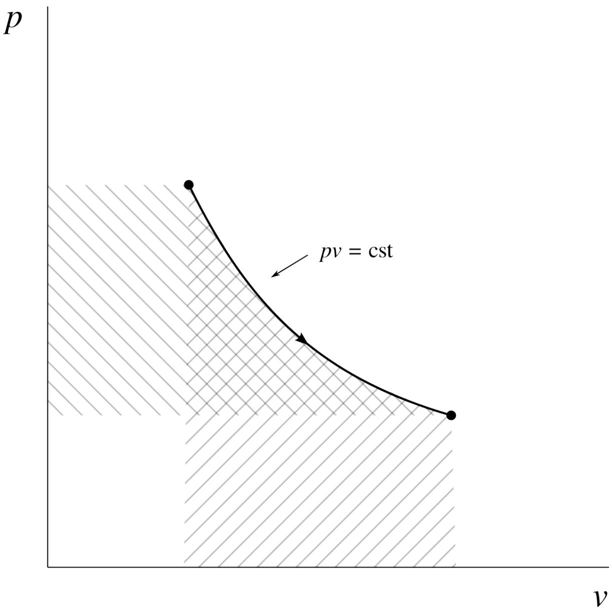


Figure 4.11: Expansion (heating) at constant temperature of an ideal gas, represented on a pressure-volume diagram.

Diagram CC-0 *Olivier Cleynen*

without some difficulty.

$$w_{1\to2} = -\int_1^2 p\,\mathrm{d}v = -\int \frac{R\,T_{\text{cst.}}}{v}\,\mathrm{d}v = -R\,T_{\text{cst.}}\int_1^2 \frac{1}{v}\,\mathrm{d}v = -R\,T_{\text{cst.}}[\ln v]_{v_1}^{v_2}$$

$$w_{1\to2} = R\,T_{\text{cst.}}\ln\left(\frac{v_1}{v_2}\right) \tag{4/26}$$

during a reversible process at constant temperature, in a closed system.

> « *When a gas varies in volume without changing temperature, the quantities of heat absorbed or released by this gas are in arithmetic progression, if the increases or reductions in volume happen to be in geometric progression.*
> When one compresses a liter of air maintained at the temperature of 10°, and it is reduced to 1/2 liter, a certain quantity of heat is released. This quantity will always be the same if one again reduces the volume from 1/2 liter to 1/4 liter, from 1/4 liter to 1/8, and so on. »
>
> Sadi Carnot, 1824 [4]

One can also express work in terms of pressure, since with equation 4/25, we have:

$$\frac{v_1}{v_2} = \frac{p_2}{p_1}$$

thus:

$$w_{1\to2} = R\,T_{\text{cst.}}\ln\left(\frac{p_2}{p_1}\right) \tag{4/27}$$

during a reversible process at constant temperature, in a closed system.

The heat transfer can be easily quantified. Indeed, the internal energy does not change:

$$q_{1\to2} = \Delta u - w_{1\to2} = 0 - w_{1\to2}$$

$$q_{1\to2} = -w_{1\to2} \tag{4/28}$$

during a reversible process at constant temperature, in a closed system.

When the process occurs in an open system, we have $q_{1\to2} + w_{1\to2} = \Delta h$ and, if the process is reversible, heat and work can be quantified in the same way:

$$w_{1\to2} = \int_1^2 v\,\mathrm{d}p = \int_1^2 \frac{R\,T_{\text{cst.}}}{p}\,\mathrm{d}p = R\,T_{\text{cst.}}\int_1^2 \frac{1}{p}\,\mathrm{d}p = R\,T_{\text{cst.}}[\ln p]_{p_1}^{p_2}$$

$$w_{1\to2} = R\,T_{\text{cst.}}\ln\left(\frac{p_2}{p_1}\right) = R\,T_{\text{cst.}}\ln\left(\frac{v_1}{v_2}\right) \tag{4/29}$$

during a reversible process at constant temperature, in an open system.

This relation, identical to equation 4/27, should not surprise the insightful student, since the pv = constant relationship ensures that for two given points in figure 4.11, the area under the curve is always equal to the area to the left of the curve.

The heat transfer can be quantified without difficulty, of course:

$$q_{1\to2} = \Delta h - w_{1\to2} = 0 - w_{1\to2}$$

$$q_{1\to2} = -w_{1\to2} \tag{4/30}$$

during a reversible process at constant temperature, in an open system.

Example 4.7

For air, we measure $c_{p\,(\text{air})} = 1005\,\mathrm{J\,kg^{-1}\,K^{-1}}$, $c_{v\,(\text{air})} = 718\,\mathrm{J\,kg^{-1}\,K^{-1}}$, $R_{\text{air}} = 287\,\mathrm{J\,kg^{-1}\,K^{-1}}$.
A mass of 2.5 kg (5.512 lb) of air in a reservoir is at a pressure of 2 bar (29.01 psi) and a temperature of 800 °C (1472 °F). We want to supply it

with 100 kJ of heat without changing its temperature. What should be the transfer of work? What will be the volume and pressure in the end?

The work is easy to determine: $W_{A\to B} + Q_{A\to B} = \Delta U = 0$ here since the temperature does not change. Thus, $W_{A\to B} = -Q_{A\to B} = -100$ kJ (the gas must provide as much work as it receives as heat).
The final properties are obtained using equation 4/26:

$$
\begin{aligned}
w_{A\to B} &= R\,T_{\text{cst.}} \ln\left(\frac{v_A}{v_B}\right) = R\,T_{\text{cst.}} \ln\left(\frac{V_A}{V_B}\right)\\
\frac{V_A}{V_B} &= \exp\left[\frac{w_{A\to B}}{R\,T_{\text{cst.}}}\right] = \exp\left[\frac{W_{A\to B}}{m\,R\,T_{\text{cst.}}}\right]\\
&= \exp\left[\frac{-100\times10^3}{2.5\times287\times(800+273.15)}\right] = 0.878\,207
\end{aligned}
$$

And since $V_A = \frac{m\,R\,T_A}{p_A} = \frac{2.5\times287\times(800+273.15)}{2\times10^5} = 3.85\,\text{m}^3 = 1017$ US gal, we obtain a final volume $V_B = \frac{V_A}{0.878\,207} = 4.384\,\text{m}^3 = 1158$ US gal.
Finally, the final pressure is obtained by comparing the final and initial states: $p_A V_A = m\,R\,T_A = m\,R\,T_B = p_B V_B$ (or with equation 4/27): $p_B = p_A \frac{V_A}{V_B} = 2\times10^5\times0.878\,207 = 1.756\times10^5\,\text{Pa} = 1.756\,\text{bar} = 25.47\,\text{psi}$.

☞ The volume increases and the pressure decreases, since the gas is doing work while it receives heat.

☞ It is difficult to hide that this type of process is rarely used in practice, but it will serve us to develop an extraordinary absolute thermometer-engine-refrigerator, in chapter 7 (*the second law*).

4.4.5 Reversible adiabatic processes

An *adiabatic* process is one where there is no heat transfer (figure 4.12). This can be achieved by wrapping the gas container or duct with a thick layer of thermal insulation.

A *reversible adiabatic* process is carried out infinitely slowly. A piston in a cylinder will need to be moved infinitely slowly for this, and a steady-flow compressor will need to be infinitely long. Later, in chapter 8 (*entropy*), we will call these processes *isentropic*.

It must be noted that even though there is absolutely no heat transfer, the temperature must necessarily vary in such a process, since the work is non-zero. This temperature change is often the intended effect, as we will see in chapter 7 (*the second law*).

In a closed system, we have $q_{1\to2} + w_{1\to2} = \Delta u$ and, if the process is reversible, heat and work are quantified without any difficulty:

$$q_{1\to2} = 0 \tag{4/31}$$

during a reversible adiabatic process, by definition.

$$
\begin{aligned}
w_{1\to2} &= \Delta u - q_{1\to2} = \Delta u\\
w_{1\to2} &= c_v\,\Delta T \qquad (4/32)
\end{aligned}
$$

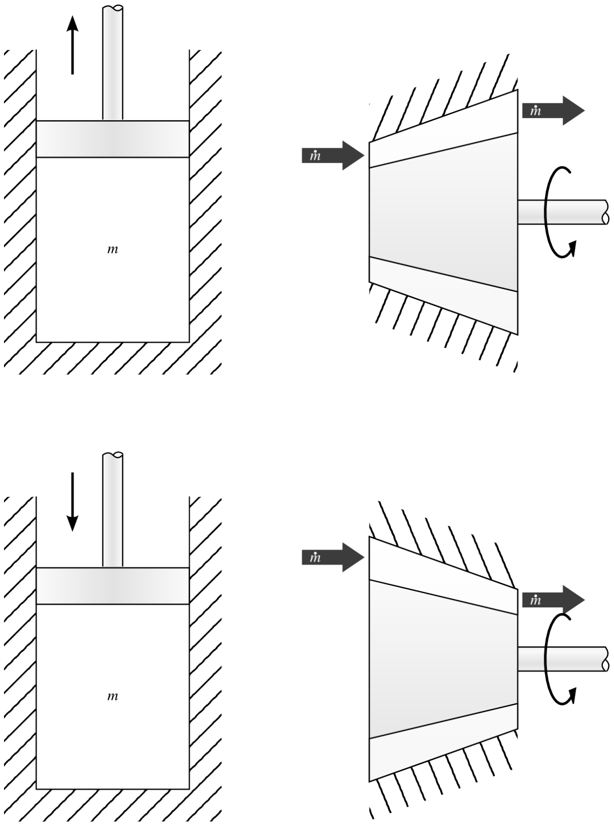


Figure 4.12: A reversible adiabatic (isentropic) process undergone by an ideal gas. In a closed system (left) as well as in an open system (right), the apparatus is perfectly insulated, so that there is no heat transfer, even if the gas temperature varies.

Diagram CC-0 *Olivier Cleynen*

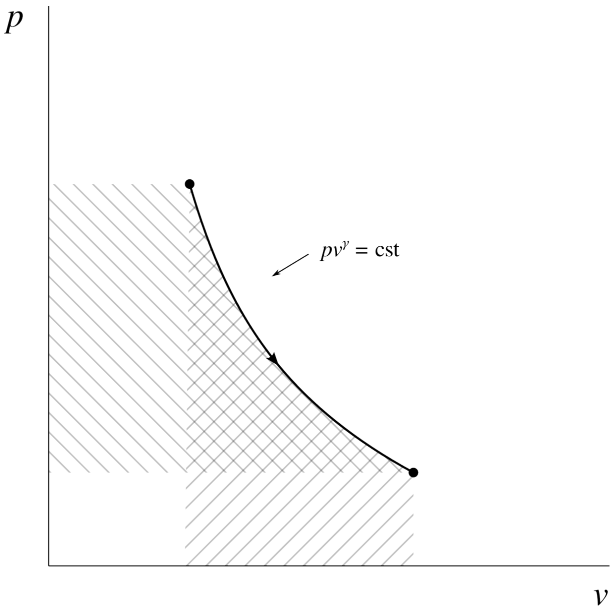


Figure 4.13: Reversible adiabatic expansion of an ideal gas, represented on a pressure-volume diagram.

Diagram CC-0 *Olivier Cleynen*

during a reversible adiabatic process in a closed system.

When the process occurs in an open system, we have $q_{1\to 2} + w_{1\to 2} = \Delta h$ and we can also write:

$$q_{1\to 2} = 0 \tag{4/33}$$

$$w_{1\to 2} = c_p \, \Delta T \tag{4/34}$$

during a reversible adiabatic process, in an open system.

Unfortunately, these two equations 4/32 and 4/34 are of no use until we have predicted the temperature T_2 at the end of the process. However, in a reversible adiabatic process, nothing remains constant: the specific volume, pressure, and temperature all vary. How can we quantify these properties?

Let us start with an infinitely small adiabatic process in a closed system. When the process is reversible, $\delta w = -p \, \mathrm{d}v$ and then:

$$\begin{aligned} \delta q = \mathrm{d}u - \delta w &= 0 \\ \mathrm{d}u + p \, \mathrm{d}v &= 0 \end{aligned}$$

By using $\mathrm{d}u = c_v \, \mathrm{d}T$ for an ideal gas and $p = RT/v$, we can rewrite this equation as:

$$\begin{aligned} c_v \, \mathrm{d}T + \frac{RT}{v} \, \mathrm{d}v &= 0 \\ \frac{1}{T} \, \mathrm{d}T + \frac{R}{c_v} \frac{1}{v} \, \mathrm{d}v &= 0 \end{aligned}$$

By integrating between two states 1 and 2:

$$\begin{aligned} \ln\left(\frac{T_2}{T_1}\right) + \frac{R}{c_v} \ln\left(\frac{v_2}{v_1}\right) &= 0 \\ \ln\left(\frac{T_2}{T_1}\right) + \ln\left(\frac{v_2}{v_1}\right)^{\frac{R}{c_v}} &= 0 \\ \ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{v_1}{v_2}\right)^{\frac{R}{c_v}} \end{aligned} \tag{4/35}$$

And since $R = c_p - c_v$ (4/8) and $\gamma \equiv c_p / c_v$ (4/9), we have $\frac{R}{c_v} = \gamma - 1$, which allows us to reformulate equation 4/35 above as:

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{v_1}{v_2}\right)^{\gamma - 1}$$

Thus, we have linked temperature and specific volume when the process is reversible adiabatic (devoid of heat transfer and infinitely slow).

Some algebraic manipulations, which are left to the student to revise, allow us to derive this expression in terms of pressure. We thus obtain the following three relations:

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{v_2}{v_1}\right)^{\gamma-1} \tag{4/36}$$

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \tag{4/37}$$

$$\left(\frac{p_1}{p_2}\right) = \left(\frac{v_2}{v_1}\right)^{\gamma} \tag{4/38}$$

for any reversible adiabatic process.

This last equation 4/38 is equivalent to the expression:

$$p v^{\gamma} = \text{constant} \tag{4/39}$$

for any reversible adiabatic process.

Example 4.8

For air, we have $c_{p\,(\text{air})} = 1005\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$, $c_{v\,(\text{air})} = 718\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$, $R_{\text{air}} = 287\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$, and $\gamma_{\text{air}} = 1.4$.

A compressed air tank of 200 L (52.83 US gal) contains air at 40 bar and 50 °C (580.2 psi and 122 °F). The ambient atmosphere is at 1 bar (14.5 psi). What is the maximum amount of work that can be extracted from the compressed air without supplying heat?

The maximum work will be obtained if the expansion is reversible. Since we are not allowed to supply heat, our best option here is to perform a reversible adiabatic expansion from 40 bar to 1 bar.

We want to calculate the final temperature, since it will give us the change in energy, thus the work done by the gas. Among the three daunting relations 4/36 to 4/38, it is the second one that interests us: $\left(\frac{T_A}{T_B}\right) = \left(\frac{p_A}{p_B}\right)^{\frac{\gamma-1}{\gamma}}$. Thus: $T_B = T_A \left(\frac{p_A}{p_B}\right)^{-\frac{\gamma-1}{\gamma}} = (50 + 273.15)\left(\frac{40}{1}\right)^{-\frac{1.4-1}{1.4}} = 112.6\,\text{K} = -160.5\,°\text{C} = -257\,°\text{F}$.

The work done by the closed system composed of the gas is therefore $w_{A \to B} = \Delta u - q_{A \to B} = c_v\,\Delta T - 0 = 718 \times (-160.5 - 50) = -1.5115 \times 10^5\,\text{J}\,\text{kg}^{-1} = -151.1\,\text{kJ}\,\text{kg}^{-1}$.

By calculating the mass $m_A = \frac{p_A V_A}{R\,T_A} = \frac{40 \times 10^5 \times 0.2}{287 \times (50+273.15)} = 8.626\,\text{kg}$, we obtain $W_{A \to B} = m_A w_{A \to B} = 8.626 \times -151.1 \times 10^3 = -1.3038 \times 10^6\,\text{J} = -1.304\,\text{MJ}$.

☞ This amount of energy is enough to accelerate, without friction, a vehicle weighing 1 t to a speed $C = \left[\frac{1.3038 \times 10^6}{\frac{1}{2} \times 1000}\right]^{0.5} = 51.1\,\text{m}\,\text{s}^{-1} \approx 180\,\text{km/h} \approx 114\,\text{mph}$.

☞ The final temperature, −160 °C (!), reminds us not to confuse “adiabatic” with “constant temperature”.

☞ In the fractions, the pressures can be left in bars or psi, but the temperatures cannot remain in °C or °F.

☞ The expansion corresponds to the maximum work because it is reversible. If it were not reversible, then the temperature of the gas would drop less and the work would be lower (we would still have $w = c_v \Delta T$). In the most extreme case, that of Joule and Gay-Lussac's expansion (§4.3.2), the temperature would remain at 50 °C and no work would be done.

4.4.6 Arbitrary processes

It is important to keep in mind that in practice, the properties of a gas can be *changed in any arbitrary manner* (figure 4.14).

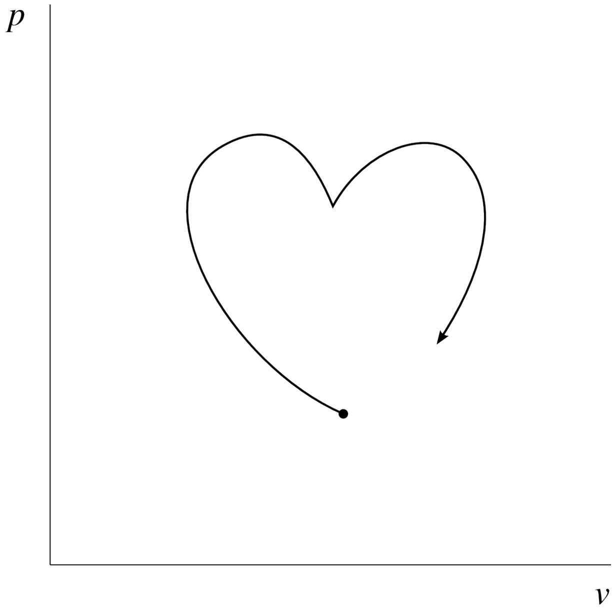


Figure 4.14: An entirely arbitrary process undergone by an ideal gas represented on a pressure-volume diagram. Such a process requires a complex combination of heat and work transfers, which the student is invited to conceptualize.

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We have focused on four specific processes of ideal gases, since each plays an important role for physicists and engineers in the design of thermal machines. However, this should not limit our way of thinking about a gas or the changes it may undergo. By cleverly controlling heat and work transfers, we can certainly cause any arbitrary process.

A Bit of History: Lavoisier and Laplace's Inquiries *

By Philippe Depondt
Pierre and Marie Curie University, Paris

The debates on the nature of heat continued until the end of the 19th century with the gradual acceptance of atomic theories. An important step in this reflection is succinctly and eloquently presented in the *Mémoire sur la chaleur* (Memoir on Heat), 1780 [1] by the French physicists Lavoisier and Laplace:

> “Physicists are divided on the nature of heat. Many among them regard it as a fluid spread throughout nature, and of which bodies are more or less penetrated, according to their temperature and their particular disposition to retain it; it can combine with them, and, in this state, it ceases to act on the thermometer and to transfer from one body to another; it is only in the state of freedom, which allows it to establish equilibrium within bodies, that it forms what we call *free heat*.
>
> Other physicists think that heat is merely the result of the imperceptible movements of the molecules of matter. It is known that bodies, even the densest ones, are filled with a great number of pores or small voids, whose volume can considerably surpass that of the matter they contain; these empty spaces allow their imperceptible parts the freedom to oscillate in all directions, and it is natural to think that these parts are in continual agitation, which, if it increases to a certain point, can disunite and decompose bodies; it is this internal motion which, according to the physicists we speak of, constitutes heat.
>
> To develop this hypothesis, we shall observe that, in all movements where there is no abrupt change, there exists a general law which geometers have designated under the name of the *principle of the conservation of vis viva*; this law consists in that, in a system of bodies acting upon each other in any manner, the vis viva, that is to say, the sum of the products of each mass by the square of its velocity, remains constant. If the bodies are driven by accelerating forces, the vis viva is equal to what it was at the origin of the movement, plus the sum of the masses multiplied by the squares of the velocities due to the action of the accelerating forces. In the hypothesis we are examining, heat is the vis viva resulting from the imperceptible movements of the molecules of a body; it is the sum of the products of the mass of each molecule by the square of its velocity.
>
> If one brings into contact two bodies whose temperatures are different, the quantities of motion they will mutually communicate will initially be unequal; the vis viva of the colder body will increase by the same amount by which the vis viva of the other will decrease, and this increase will continue until the quantities of motion communicated from one to the other are equal; in this state, the temperature of the bodies will have reached uniformity.
>
> This way of regarding heat easily explains why the direct impulse of solar rays is negligible, while they produce a great amount of heat. Their impulse is the product of their mass by their simple velocity; now, although this velocity is excessive, their mass is so small that this product is almost nil, whereas their vis viva, being the product of their mass by the square of their velocity, represents heat of an order much superior to that of their direct impulse. This impulse on a white body, which abundantly reflects light, is greater than on a black body, and yet the solar rays communicate less heat to the former because these rays, by being reflected, carry away their vis viva, which they communicate to the black body that absorbs them.
>
> We will not decide between the two preceding hypotheses; several phenomena seem to favor the latter; such is, for example, that of the heat produced by the friction of two solid bodies; but there are others that are more simply explained by the former; perhaps both occur simultaneously. In any case, since one can only form these two hypotheses regarding the nature of heat, one must accept the principles common to both; thus, according to both, *the quantity of free heat always remains the same in the simple mixing of bodies.* This is evident if heat is a fluid that tends to reach equilibrium, and if it is merely the vis viva resulting from the internal motion of matter, the principle in question follows from the principle of the conservation of vis viva. The conservation

> of free heat in the simple mixing of bodies is thus independent of any hypothesis regarding the nature of heat; it has been generally accepted by physicists, and we shall adopt it in the following research."

At the time when this text was written, the atomic hypothesis remained largely speculative due to the lack of adequate experimental means: the experiment by Jean Perrin that finally settled the issue only took place in the early years of the 20th century, and the X-ray diffraction experiments suggested by Max von Laue occurred in 1912.

Problems

Air is considered an ideal gas.

$c_{v\,(\text{air})} = 718\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$ $R_{\text{air}} = 287\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$

$c_{p\,(\text{air})} = 1005\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$ $\gamma_{\text{air}} = 1.4$

We assume that for a reversible adiabatic process (without heat transfer and infinitely slow), the properties of air are linked according to the following three relationships:

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{v_2}{v_1}\right)^{\gamma-1} \tag{4/36}$$

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \tag{4/37}$$

$$\left(\frac{p_1}{p_2}\right) = \left(\frac{v_2}{v_1}\right)^{\gamma} \tag{4/38}$$

We also assume that during a reversible isothermal process (at constant temperature and infinitely slow) of an ideal gas, the work done in an open or closed system is:

$$w_{1\to2} = R\,T_{\text{cst.}} \ln\left(\frac{p_2}{p_1}\right) = R\,T_{\text{cst.}} \ln\left(\frac{v_1}{v_2}\right) \tag{4/29}$$

4.1 Air Pressure

A mass of 5 kg (11 lb) of air is enclosed in a tank of $2\,\text{m}^3$ (528.3 US gal).

1. What are its specific volume and density?
2. What is the pressure if the temperature is 20 °C?

4.2 Heating of an Air Tank

A compressed air tank made out of sealed concrete has a fixed volume of $1.2\,\text{m}^3$. Air is stored in it at a pressure of 2 bar.
The tank is placed in the sun and solar heating raises the temperature from 5 °C to 60 °C (41 °F to 140 °F).

1. What are the mass, specific volume, density, and pressure inside the tank, before and after heating?

When the temperature reaches 60 °C, a valve opens and lets air escape to bring the pressure in the tank back down to the initial pressure of 2 bar (29 psi. During the release, the temperature of the air inside the tank remains constant.

2. How much air mass should be allowed to escape?

When the pressure has reached 2 bar, the valve closes and the tank, once again sealed, cools slowly at constant volume. The final temperature returns to 5 °C.

3. What is the final pressure in the tank?

4.3 Energy and Temperature

There is air in a flexible compartment at a pressure of 3 bar (43.51 psi). Its internal energy is $836\,\text{kJ}\,\text{kg}^{-1}$.

It is heated at constant pressure until 900 °C; then it is cooled and expanded while its properties vary according to the relation $pv^{1.1} = \text{const.}$ until its temperature reaches 25 °C.

[Trick question] How much energy has it received or rejected since the beginning of the process?

4.4 Power of an Air Pump

An air pump (figure 4.15) compresses air adiabatically, with a steady flow. The air temperature increases from 15 °C to 100 °C. What is the specific power input?

Figure 4.15: A small electric compressor mounted on a portable air tank

Photo by Commons User:Grikalmis (retouched, public domain)

4.5 Turbine of a Turbojet Engine

A student disassembles the *Turboméca Marboré* turbojet engine from a Fouga Magister in order to study and modify its operation. S/he operates the engine on a test bench.

At the inlet of the turbine, the conditions are measured at $110\,\text{m}\,\text{s}^{-1}$ (246 mph) and 1000 °C (1832 °F).
At the outlet of the turbine, these properties are measured at $125\,\text{m}\,\text{s}^{-1}$ (279.6 mph) and 650 °C (1202 °F).

The student also measures the heat losses from the turbine as $75\,\text{kJ}\,\text{kg}^{-1}$.

1. What is the specific power delivered by the turbine?
2. What condition must the student maintain to obtain a power of 1 MW?

4.6 Elementary Processes: Isothermal Compression

A mass of 3.5 kg (7.716 lb) of air is compressed reversibly in an isothermal manner (constant temperature) from 2 bar and 15 °C to 45 bar (from 29.01 psi and 59 °F to 652.7 psi)

1. Draw the process qualitatively (that is, without showing numerical values) on a pressure-volume diagram.
2. What are the amounts of work and heat involved?
3. If the compression were carried out in a reversible adiabatic manner, would the final volume be different?

4.7 Elementary Processes: Isobaric and Isochoric Coolings

A mass of 2 kg of air in a deformable reservoir is at a pressure of 4.5 bar (65.27 psi) and occupies a volume of 800 L (211.3 US gal). It is cooled, and the reservoir maintains constant pressure until the volume has been reduced by 40 %.

After this, the cooling is continued at constant volume until the temperature reaches 25 °C (77 °F).

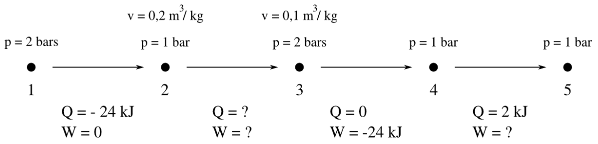
1. Sketch the process on a pressure-volume diagram..
2. What is the work done by the air?
3. What is the heat cost for the entire process?

4.8 Elementary Processes: Isentropic Compression

What is the minimum amount of work required to compress 5 kg of air at 1 bar and 20 °C to 50 bar without heat transfer? Plot the process undergone by the air on a pressure-volume diagram, qualitatively (that is, without showing numerical values).

4.9 Elementary Processes: Vocabulary

A fixed mass of perfect gas, with the sole purpose of exasperating a student in thermodynamics, slowly undergoes the following processes:



Among the processes above, which ones are:

1. at constant temperature (isothermal)?
2. at constant volume (isochoric)?

4.10 Elementary Processes: Pressure and Volume

Among the reversible processes described on each of the diagrams in figure 4.16, identify (without having to justify) the process at constant temperature, at constant pressure, reversible adiabatic, and at constant volume.

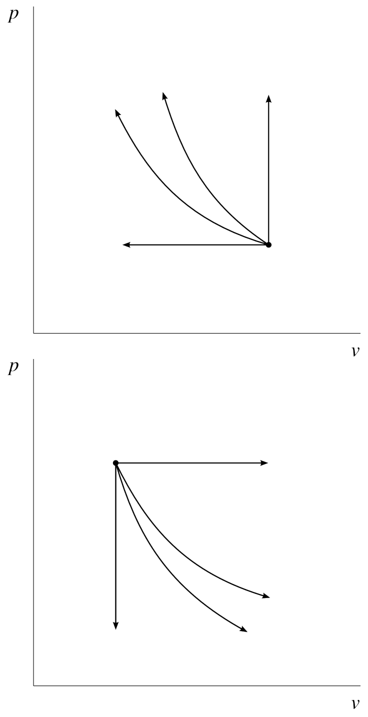


Figure 4.16: Elementary processes undergone by a perfect gas

Diagram CC-0 *Olivier Cleynen*

4.11 Compressor of a Turbofan

Inside one of the engines of a commercial aircraft, the compressor (figure 4.17) is almost adiabatic.

During cruise at 33 000 ft, the atmosphere is at −50 °C (−58 °F) and 0.25 bar (3.626 psi). The fan is driven by the turbine through a mechanical shaft. It receives 55 $\mathrm{kg\,s^{-1}}$ of air at atmospheric conditions, and compresses this flow up until 8 bar (116 psi).

1. Starting from the following relation,

$$\left(\frac{p_1}{p_2}\right) = \left(\frac{v_2}{v_1}\right)^{\gamma} \qquad (4/38)$$

Figure 4.17: Air intake of one of the four General Electric GEnx-2B turbofans equipping a Boeing 747-8. The two-color fan blades are visible in the foreground; behind, the air flow is divided between the compressor inlet (internal) and the cold flow rectifier stators (external).

Photo CC-by-sa *Olivier Cleynen*

valid for a reversible adiabatic process undergone by a perfect gas, show (without using equation 4/36) that:

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \tag{4/37}$$

2. What is the minimum theoretical power to be supplied to the compressor?
3. Under what conditions would this power be obtained?

In reality, the compressor requires significantly more power to operate. We model the actual process inside the compressor with two distinct phases:

- A heating at constant pressure, conducted by friction, with power representing 15 % of the theoretical power calculated earlier;
- Then, an ideal compression up to 8 bar.

4. Compare the theoretical compression from question 2 and this new process on a pressure-volume diagram. Graphically represent the work done on one of the processes.
5. What is the power supplied to the compressor in this new scenario?

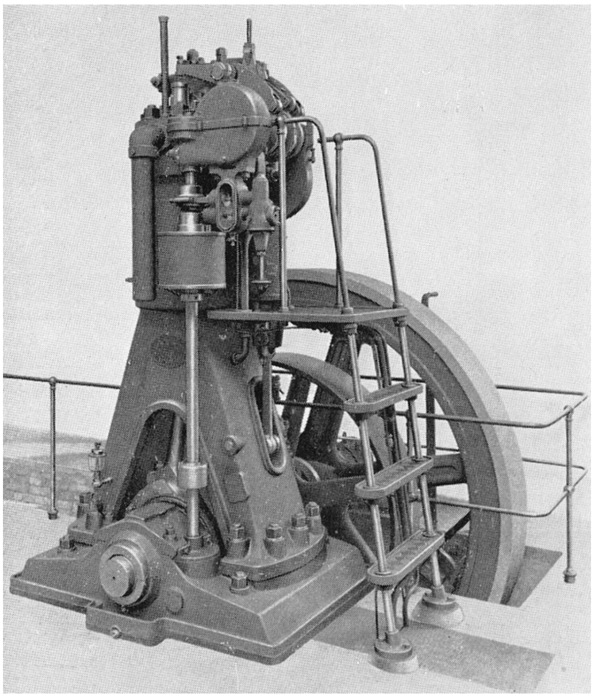


Figure 4.18: Diesel Engine from 1898, manufactured under license by Sulzer in Switzerland

Photo CC-by-sa *Sulzer AG*

4.12 Compression and Combustion in a Diesel Engine

In 1890, a young German engineer passionate about thermodynamics (§7.6) developed a low-power, low-speed, and high-efficiency engine in a laboratory (figure 4.18). The engine was designed to be robust and simple; it had only one cylinder. Here, we study part of its operating cycle.

The piston inside the cylinder periodically varies the volume between 3 L (*bottom dead center*, piston at the bottom of its stroke) and 0.3 L (*top dead center*, piston at the top of its stroke).

The engine starts its cycle at bottom dead center, when it is filled with air at 20 °C and 1 bar (68 °F and 14.5 psi). The piston compresses this air to top dead center.

The compression is done reversibly (very slowly), but non-adiabatically: the air receives heat through the walls throughout the process. The engineer predicted that its properties would vary according to the relation $p\, v^{1.5}$ = constant.

1. The work done by a force $\vec{F}$ along a displacement $\vec{l}$ is expressed as

$$W \equiv \vec{F} \cdot \vec{l} \tag{1/11}$$

Starting from this equation, express the work done on a fixed mass body in terms of its specific volume and internal pressure.

2. How much energy in the form of work will the gas compression have cost?

3. How much energy in the form of heat will the gas have received during compression?

When the piston reaches the top of its stroke, fuel is progressively injected into the cylinder to allow for combustion to occur. The amount of injected fuel provides for a total heat input of 2 kJ. The combustion takes place at constant pressure.

4. Draw the processes undergone by the gas during compression and combustion on a pressure-volume diagram, qualitatively (that is, without showing numerical values).
5. What will be the maximum temperature reached within the engine?
6. In order to avoid structural failure, the engineer must ensure that the force transmitted by the piston never exceeds 10 kN (2248 lbf). What constraint must be respected for this?

4.13 Turbojet Engine

An early 1960s military aircraft is equipped with a turbojet engine (figure 4.19). We wish to calculate the theoretical maximum speed at which it could accelerate the air at the nozzle outlet.

The engine is tested on a stationary test bench. When air passes through the turbojet engine, it goes through four components that we will model as if they were ideal:

The compressor (figure 4.20) compresses the air adiabatically and reversibly.
At the inlet, the air is at 0.9 bar and 5 °C; at the outlet, the pressure is increased to 19 bar.

The combustion chamber allows for the heating of air while maintaining its pressure constant.
At the outlet of the combustion chamber, the temperature has been increased to 1100 °C.

The turbine extracts energy from the air to power the compressor. In the turbine, the air expands adiabatically and reversibly.

The nozzle is a component in which no power is added or extracted from the air. As it flows across the nozzle, the air expands adiabatically and reversibly; its speed increases significantly. At the outlet of the nozzle, it has returned to atmospheric pressure and is expelled into the atmosphere.

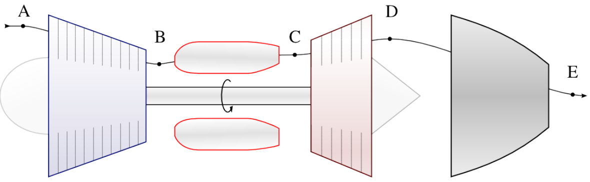


Figure 4.19: Schematic of a turbojet engine. Air flows through the machine from left to right.

Diagram CC-by-sa *Olivier Cleynen*

The goal of the problem is to calculate the speed at which the turbojet engine is capable of releasing the air.

1. Starting from the following relation,

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{v_2}{v_1}\right)^{\gamma-1} \tag{4/36}$$

valid for a reversible adiabatic process undergone by a perfect gas, show (without using equation 4/38) that:

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \tag{4/37}$$

2. What is the air temperature at the compressor outlet?
3. What is the specific power supplied to the compressor?
4. What is the specific power supplied in the form of heat in the combustion chamber?
5. What must be the temperature at the turbine outlet for it to power the compressor?
6. What will then be the pressure at the turbine outlet?
7. What will be the exhaust gas temperature at the nozzle outlet?
8. Finally, what will be the gas ejection speed at the nozzle outlet?
9. Draw the complete series of processes on a pressure-volume diagram, qualitatively.
10. On the same pressure-volume diagram, plot the processes that the gas would follow if the compressor could not compress the air reversibly (real compressor, compression with internal friction).

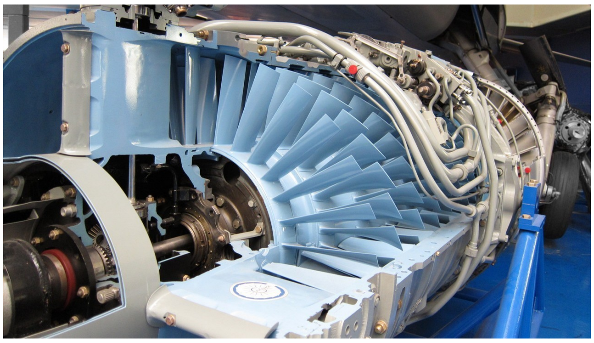


Figure 4.20: Compressor of a dissected SNECMA Atar turbojet engine (1948). Air flows from the left to the center of the image.

Photo CC-by-sa *Olivier Cleynen*

Answers

4.1 1) $v_1 = \frac{V_1}{m_1} = 0.4\,\mathrm{m^3\,kg^{-1}}$; $\rho_1 = \frac{1}{v_1} = 2.5\,\mathrm{kg\,m^{-3}}$
2) $p_1 = \frac{RT_1}{v_1} = 2.103\,\mathrm{bar}$.

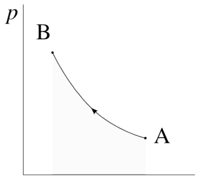
4.2 1) $m_1 = \frac{p_1 V_1}{RT_1} = 3.006\,\mathrm{kg}$; $v_1 = 0.3991\,\mathrm{m^3\,kg^{-1}}$; $\rho_1 = 2.505\,\mathrm{kg\,m^{-3}}$; $p_1 = 2\,\mathrm{bar}$;
$m_2 = m_1$; $v_1 = v_2$; $\rho_1 = \rho_2$; $p_2 = \frac{RT_2}{v_2} = 2.395\,\mathrm{bar}$.
2) $m_3 = 2.51\,\mathrm{kg}$, thus $m_{\text{exhaust}} = m_3 - m_2 0.4959\,\mathrm{kg}$;
3) $p_4 = \frac{RT_4 m_4}{V_4} = 1.67\,\mathrm{bar}$.

4.3 $\Delta u = c_v T_3 - u_1 = -622\,\mathrm{kJ\,kg^{-1}}$ (easily calculated with the final temperature and does not depend on the process or intermediate states).

4.4 $w_{1\to2} = \Delta h = c_p \Delta T = +85.4\,\mathrm{kJ\,kg^{-1}}$ (3/15 & 4/13)

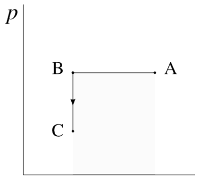
4.5 1) With equation 3/15, $w_{\text{turbine}} = c_p(T_B - T_A) + \frac{1}{2}(C_B^2 - C_A^2) - q_{A\to B} = -275\,\mathrm{kJ\,kg^{-1}}$
2) $\dot{m} = \frac{\dot{W}_{\text{turbine}}}{w_{\text{turbine}}} = 3.64\,\mathrm{kg\,s^{-1}}$

4.6



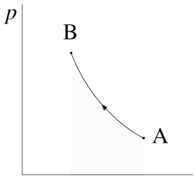
2) With equation 4/26, $w_{1\to2} = +257.58\,\mathrm{kJ\,kg^{-1}}$ (so, an amount of work received); $W_{1\to2} = +901.2\,\mathrm{kJ}$; $Q_{1\to2} = -W_{1\to2}$ (so, a an amount of heat expended)
3) Yes, we would have $v_{2\text{ad.rev.}} = v_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} > v_{2\text{isoth.}} = v_1 \left(\frac{p_1}{p_2}\right)$.

4.7



2) $W_{1\to3} = -\int_1^2 p\,\mathrm{d}V - \int_2^3 p\,\mathrm{d}V = -p_{\text{cst.}}\Delta V - 0 = +144\,\mathrm{kJ}$
3) $Q_{1\to3} = U_3 - U_1 - W_{1\to3} = mc_v\left(T_3 - \frac{p_1 V_1}{mR}\right) - W_{1\to3} = -616.5\,\mathrm{kJ}$

4.8



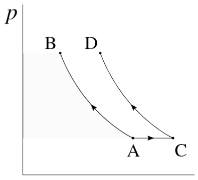
Optimal case: reversible adiabatic compression. Using equation 4/37, we calculate $T_B = 896.4\,\mathrm{K}$; $W_{\text{minimal}} = m\,c_v(T_B - T_A) = +2.166\,\mathrm{MJ}$.

4.9 Isothermal 2 → 3, isochoric 1 → 2.

4.10

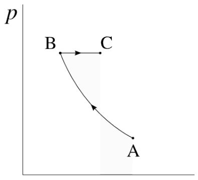
Clockwise, starting horizontally, on both diagrams: isobaric (p constant), isothermal (T constant), reversible adiabatic, isochoric (v constant).

4.11



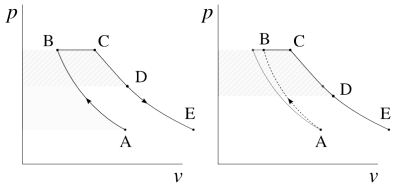
1) Replace v_2 with $\frac{RT_2}{p_2}$, do the same with v_1. Work out the algebra and the result will come naturally;
2) With eq. 4/37, we obtain $T_B = 600.7\,\mathrm{K}$, thus $\dot{W}_{A\to B} = +20.87\,\mathrm{MW}$;
3) see §4.4.5 p. 101;
5) $T_C = 279.8\,\mathrm{K}$; $T_D = 753.1\,\mathrm{K}$; thus $\dot{W}_{\text{real compressor}} = \dot{W}_{\text{friction losses A}\to\text{C}} + \dot{W}_{C\to D} = +29.29\,\mathrm{MW}$ (+40 %).

4.12



1) see §1.3 p. 16 & §2.4.1 p. 36;
2) $W_{A\to B} = -m\int_A^B p\,\mathrm{d}v = +1.298\,\mathrm{kJ}$
3) Wth $p_B = k v_B^{-1.5} = 31.6\,\mathrm{bar}$, we have $T_B = 926.3\,\mathrm{K}$. Then, $Q_{A\to B} = \Delta U - W_{A\to B} = +0.3254\,\mathrm{kJ}$.
5) With constant pressure, with equation 4/17, $T_C = \frac{Q_{B\to C}}{m\ c_p} + T_B = 1483.7\,\mathrm{K}$ (1211 °C).
6) $S < \frac{F_{\text{max.}}}{p_C} = 3.164 \times 10^{-3}\,\mathrm{m^2}$ (diameter $D_{\max} = 6.35\,\mathrm{cm}$).

4.13



2) With equation 4/37, $T_B = 664.83\,\mathrm{K}$

3) With equation 3/15, $w_{\text{compressor}} = w_{A\to B} = +388.61\,\mathrm{kJ\,kg^{-1}}$

4) $T_C = 1373.15\,\mathrm{K}$; thus $q_{\text{combustion}} = q_{B\to C} = +711.86\,\mathrm{kJ\,kg^{-1}}$

5) Since $w_{\text{turbine}} = -w_{\text{compressor}}$, we have $T_D = 986.47\,\mathrm{K}$

6) With equation 4/37, $p_D = 5.97\,\mathrm{bar}$

7) Idem, with equation 4/37, $T_E = 574.49\,\mathrm{K}$

8) With equation 3/15, $C_E = (-2\Delta h)^{\frac{1}{2}} = 909.98\,\mathrm{m\,s^{-1}}$

Of course, these values do not account for the irreversibilities in an actual turbojet. These effects are approached in problem 4.11 p. 109 and formalized in chapter 10 (*air-based power cycles*).

Chapter 5
Liquids and Vapors

or

The Dissociative Disorder of Liquefying Fluids

Chapter 5 – Liquids and Vapors

Executive summary for chapter 5

Water generally follows the same trends as an ideal gas, but when it condenses or evaporates, its properties change abruptly. They cannot be *calculated* like those of an ideal gas: they must be tabulated.

Introduction

This chapter 5 has exactly the same objectives as chapter 4 (*the ideal gas*), but applied to the study of fluids that liquefy and evaporate. It aims to answer two questions:

- How can we describe the behavior of a liquid or vapor they are heated or compressed?
- How can we predict the values of u and h when we use water in machines?

This chapter is incompatible with chapter 4 (*the ideal gas*) and we must forget everything that was learned there.

5.1 Evaporation and condensation

5.1.1 What is a liquid?

A liquid is a fluid (namely, a substance without a definite shape) whose molecules are very close together, but still free to move relative to each other.

In concrete terms, a liquid is obtained from a gas by slowing down and bringing its molecules closer together. There is no chemical reaction involved. Thus, liquid water and water vapor are made of the same material (the same molecules): this material is just assembled differently.

> « The steam here is merely a means of transporting the caloric; it serves the same function as in the heating of baths by steam, except that in the case which we are considering, its movement is made useful. »
>
> Sadi Carnot, 1824
> *Reflections on the Motive Power of Fire and on Machines Fitted to Develop that Power* [4]

Compared to gases, liquids have two important differences:

- They are practically *incompressible*, meaning that their specific volume v varies very little when they are compressed;[1]
- They are subject to the effects of surface tension, which delights aesthetes and fluid mechanicists (figure 5.1), but is inconsequential in thermodynamics.

Figure 5.1: Surface tension gives liquids fascinating visual properties but has no consequence in thermodynamics. For us, it is the "same water" whether in the gaseous or liquid state.

Photo CC-by *by Commons User:Fcb981*

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[1] The term *incompressible*, a true false friend for the student, does *not* mean that the pressure is constant or uniform. It merely means that the specific volume v, and thus with it the density ρ, remains constant.

5.1.2 Phase changes

Heating liquid water at ambient pressure (for example in a saucepan) makes it easy to see that the transition from the liquid state to the gaseous state occurs with a very large volume change. At 1 bar and 100 °C, the specific volume of water is multiplied by about a thousand before the temperature can increase again.

The abrupt change in one physical quantity while another changes is called a *phase change*. In this chapter, we will focus on the two phases: liquid and gaseous. In applications where it may change phase, we call a gas a *vapor*, and water vapor is called *steam*.

We will call the transition from liquid to vapor *boiling* or *evaporation*; conversely, the return from vapor to liquid is called *condensation* or *liquefaction*.

The concept of phase is difficult to define; there are many different phases (among which liquid, solid, gaseous, and plasma) and their boundaries are not always distinct. For example, we will see that it is possible to transform a liquid into vapor without ever observing boiling or abrupt property changes.

5.1.3 One way to approach evaporation and condensation

When we explored the model of the ideal gas in §4.1.3, we had pictured molecules as very small billiard balls in chaotic motion, colliding with each other without ever being attracted one to the other (figure 4.1). In reality, molecules are also subject to respective attractive forces that greatly affect their behavior.

Let's imagine, to start with, two very small billiard balls attracted to each other by a magnetic force and colliding without friction at very high speed (figure 5.2). The attracting force alters their trajectory when the balls are very close to each other; but once they move away, its influence becomes negligible.

Now, let's repeat the experiment with a lower initial speed (figure 5.3). There is a certain threshold speed below which the balls will not have enough kinetic energy to separate permanently. They will then form a pair, bouncing off each other periodically and occupying a significantly smaller average volume.

This simplistic model is a good initial approach to describe the phenomenon of condensation. When we reduce the kinetic energy of a vapor's molecules (by cooling the vapor), once a critical threshold is crossed, they assemble much more compactly while continuing to collide at the same average speed (unchanged temperature). The more energy is extracted from the gas, the greater the number of molecules in compact interaction becomes. They form small groups; if they are numerous enough, the groups of 10^{14} (one hundred trillion) molecules scatter light and form a suspension that is visible to the naked eye. Droplets with a diameter of 0.1 mm contain around 10^{16} molecules.

Any gas can thus be liquefied by cooling it and reducing its volume. The temperature and pressure required for liquefaction depend on the size and

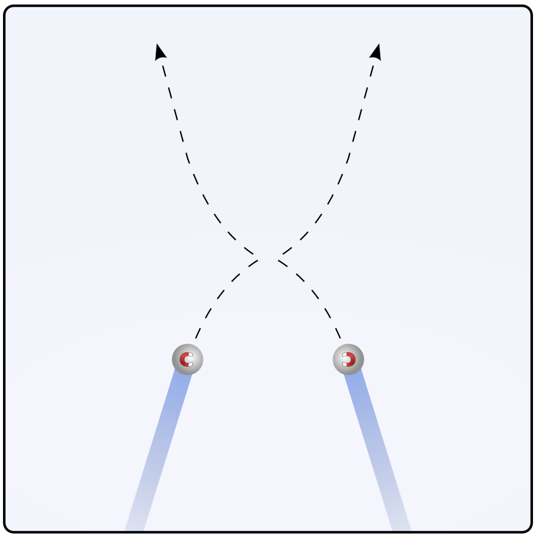

Figure 5.2: Two magnetized billiard balls colliding without friction at high speed. The mutual attractive force alters the trajectory and behavior of the two balls, but only for a brief moment and over a short distance.

Diagram CC-BY-SA *by Commons User:Sharayanan & Olivier Cleynen*

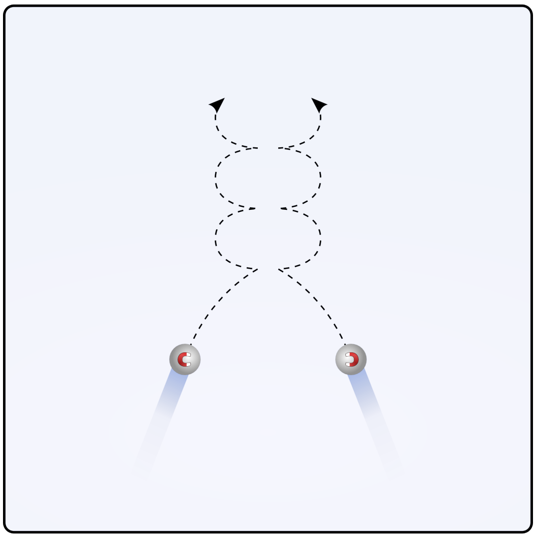

Figure 5.3: Two magnetized billiard balls colliding without friction at low speed. Below a threshold speed, the two balls will continue their trajectory together.

Diagram CC-BY-SA *by Commons User:Sharayanan & Olivier Cleynen*

geometry of the molecules that compose it. In the following sections, we will precisely quantify the amounts of energy and the ranges of properties required to vaporize and liquefy one fluid in particular: water.

5.1.4 Industrial use of water and liquids/vapors

When using a fluid to convert work and heat, it can be advantageous to exploit phase change phenomena.

In vapor form, a fluid behaves like a gas and spontaneously occupies all the volume made available to it. It is often used in this form to move mechanical parts (pistons in cylinders or blades in a turbine).

In liquid form, the fluid has a significantly higher density. It is often used in this form to transfer heat (heating or cooling) because much smaller conduits can be used. For example, in order to achieve the same power, a radiator filled with a gas should have a volume roughly a thousand times larger than if the fluid was liquefied.

Historically, water has been used in the very first engines in history for these reasons, and because changes in volume during phase changes allow for easier control of machines with low technology. Nowadays, liquids/vapors are mainly used in two major types of applications:

In power plants where liquids/vapors allow efficient heat extraction from external sources (waste combustion, nuclear reactions, geothermal energy). Water is used there, because it is abundant and easy to manipulate. Chapter 9 (*steam power cycles*) is entirely dedicated to these machines.

In refrigeration systems where liquids/vapors enable the use of compact components, especially pumps. The use of liquids/vapors also allows to drop the temperature of the fluid without having to use moving parts, using a simple valve, which is not possible with an ideal gas (see §4.3.2). A variety of fluids (then called “refrigerants,” although they are nothing extraordinary) are used for these purposes, selected according to their range of physical properties, cost, impact on the ozone layer, and contribution to global warming.

In this book, we focus on water, but the phenomena and calculation methods apply equally well to other liquids/vapors.

5.2 Qualitative description of water properties

5.2.1 Limits of the ideal gas

> « Gases exhibit in their deportment, particularly as regards the relations of volume, temperature and pressure, expressed by the laws of Mariotte and Gay-Lussac, so much regularity as to lead us to the notion that the mutual attraction of the particles which takes place in solid and fluid bodies is in their case annulled; so that while with solids and fluids the heat necessary to effect an expansion has to contend with both an inner and an outer resistance, the latter only is effective in the case of gases. »
>
> Rudolf Clausius, 1850 [10, 11, 21]

As we slow down and bring the molecules of a gas closer together, the ideal gas model describes its properties less and less accurately. We observe a threshold below which liquefaction and evaporation occur, in other words, where the two liquid and gaseous phases coexist; this threshold is described in terms of a temperature and a pressure which are named *critical*. The critical temperatures and pressures of some common fluids are indicated in table 5.1. It should be noted that air, a mixture of several gases, will see different substances in its composition condense at different temperatures.

When a fluid is maintained at a temperature and pressure significantly higher than its critical values, it behaves like an ideal gas. All fluids that we traditionally consider as liquids (for example, mercury) or gases (for example, CO_2) can transition from one state to the other.

		$T_{\text{cr.}}$ (K)	$p_{\text{cr.}}$ (MPa)
Air	–	132	3.8
Chlorine	Cl_2	417	7.71
Carbon dioxide	CO_2	304.2	7.39
Water	H_2O	647.1	22.06
Helium	He	5.3	0.23
Oxygen	O_2	154.8	5.08
R-134a	CF_3CH_2F	374.2	4.059
Xenon	Xe	289.8	5.88

Table 5.1: Critical temperatures and pressures of some substances. In practice, in the industry, engineers will mainly use the properties of two substances: water (in steam engines) and the refrigerant R-134a (in heat pumps and refrigerators). In this chapter, we will only use water, but the principles remain the same for all substances.

5.2.2 The temperature-volume (T-v) diagram

Let’s observe the temperature and volume of a mass of a liquid, here water, that is steadily heated while it is placed in a container at constant pressure (figure 5.4). Then we measure the temperature of the water as a function of its volume (figure 5.5).

> « Steam can be considered at the very moment of its formation in the boiler, still in contact with the liquid from which it emanates, or else separated from that same liquid; and in each of these cases, its properties are different. »
>
> François-Marie Guyonneau de Pambour, 1839
> *Théorie de la machine à vapeur* [7]

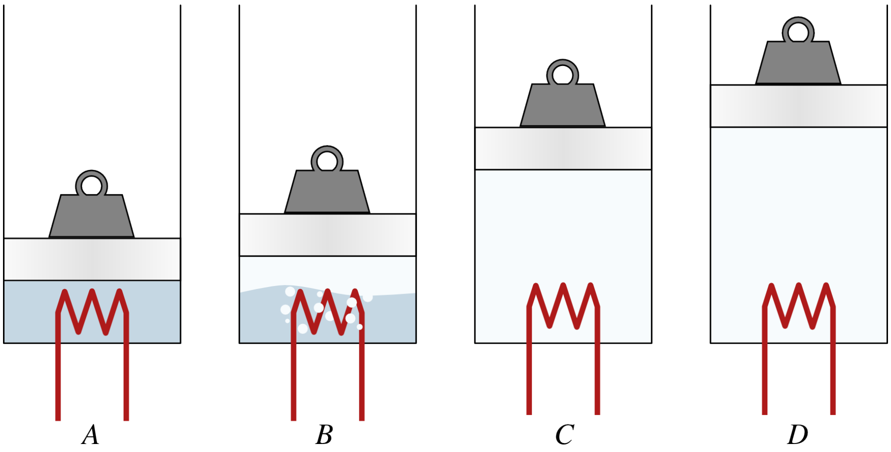

Figure 5.4: Heating of a fixed quantity of water at constant pressure.
A: subcooled liquid; B: liquid-vapor mixture; C: saturated vapor; D: superheated vapor.

Diagram CC-0 *Olivier Cleynen*

Initially, when the water is liquid, the temperature increases linearly with the volume, with a steep gradient. This state is referred to as *subcooled liquid* (sometimes alternatively called *compressed* or *unsaturated liquid*).

Then, suddenly, while the volume continues to increase, the temperature stops rising. The mixture in the cylinder is now two-phase: part liquid, and part vapor. Adding heat does not cause any increase in temperature (unlike an ideal gas), but only the transformation of more liquid into vapor: this is called evaporation or boiling. In this state, the substance is called *liquid-vapor mixture*.[1]

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[1] Strictly speaking, the mixture is called *saturated liquid-vapor mixture*, since it consists of *saturated liquid* and *saturated vapor*. It can also be named *wet vapor*.

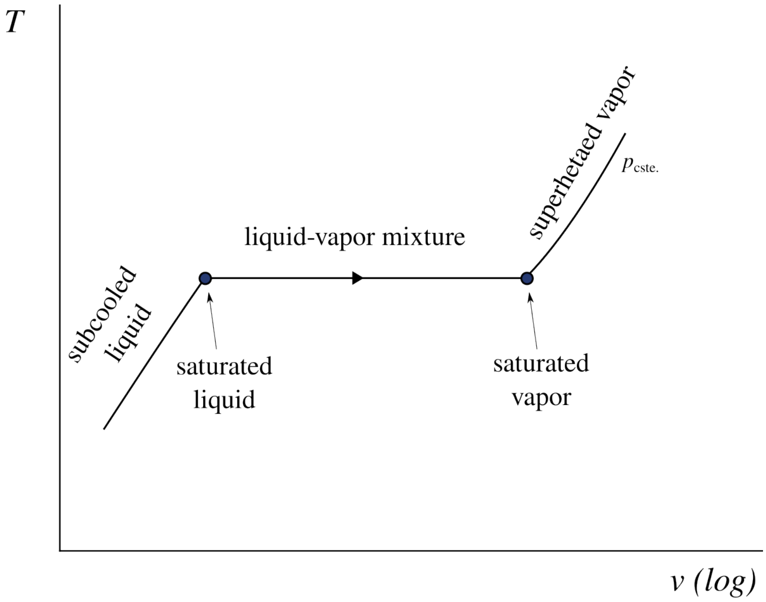


Figure 5.5: Vocabulary: states of the water during a process at constant pressure.

Diagram CC-0 *Olivier Cleynen*

Finally, once the last drop of liquid has been transformed into vapor, the temperature resumes its increase as more heat is added. The fluid is then in a state called *dry* or *superheated vapor*.

> « Water being unable to vaporize under high pressure except by virtue of a higher temperature, we have reason to believe that, all other circumstances being equal, the machine must be capable of vaporizing less water under a more considerable pressure. »
>
> François-Marie Guyonneau de Pambour, 1835
> *Traité théorique et pratique des machines locomotives* [6]

The experiment can be repeated at different pressures (figure 5.6). When the imposed pressure increases, two important facts are observed:

- The temperature of the phase change increases;
- The change of volume during the phase change is reduced.

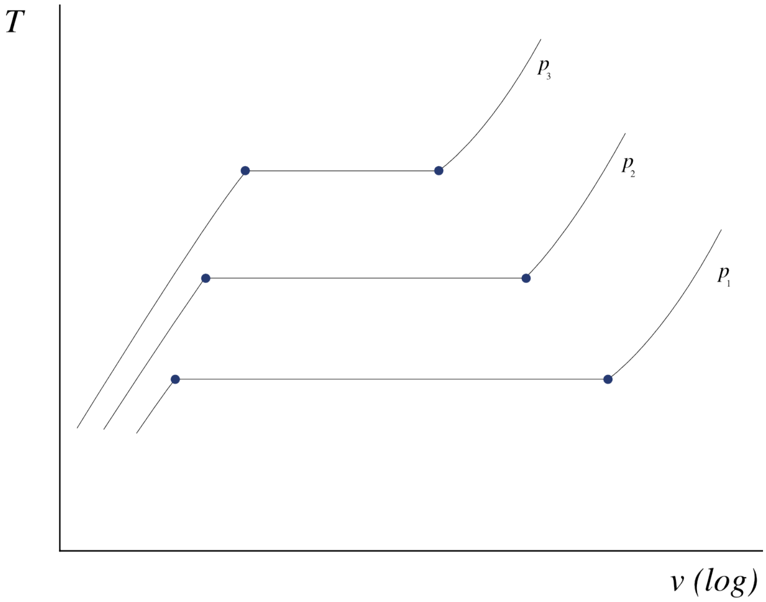


Figure 5.6: Properties of water plotted on a temperature-volume diagram, when conducting the experiment described in figure 5.4 at different pressures. It can be observed that the higher the pressure, the smaller the boiling range becomes.

Diagram CC-0 *Olivier Cleynen*

Above a certain pressure called *critical pressure* $p_{\text{cr.}}$, the phase change occurs indistinctly and there is no longer a range of constant temperature. The liquid turns into vapor without boiling!

At the end, we can connect all the phase change points, at all different pressures: we obtain a curve called *saturation curve*. All this information can be gathered on a temperature-volume (T-v) diagram represented in figure 5.7, which well describes the properties of liquid-vapor mixtures. The student is encouraged to practice reproducing it.

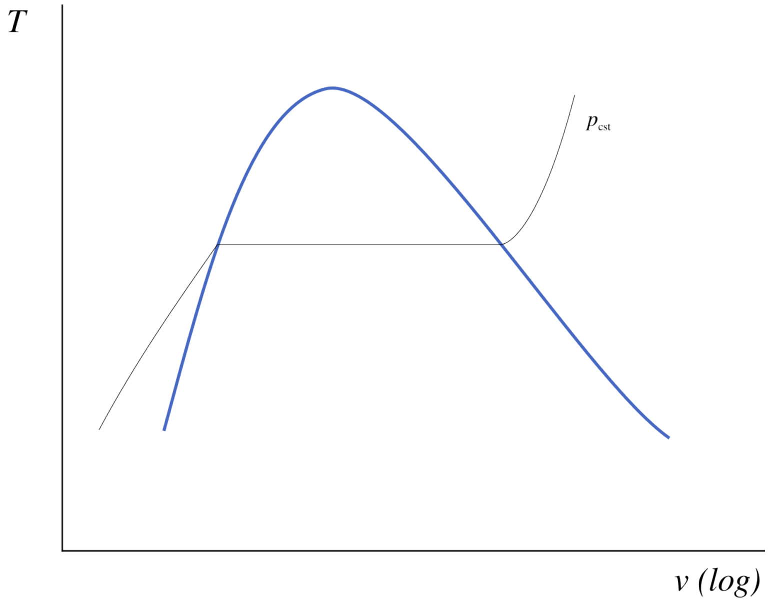


Figure 5.7: Temperature-volume diagram of water, represented with a constant pressure (isobaric) process. The saturation curve is represented in blue.

Diagram CC-0 *Olivier Cleynen*

5.2.3 The pressure-volume diagram (p-v)

In order to fully understand the phase change phenomenon, let's now imagine a slightly different experiment.

We propose to vary the volume of a given mass of fluid, again here water, while keeping its temperature constant (for example, by submerging the container in a lukewarm water bath). We then observe the pressure inside the container (figure 5.8).

As long as the water is liquid, we observe that the pressure drops sharply as we increase its volume. Then, suddenly, the pressure stops decreasing and remains perfectly constant, while the volume continues to increase: inside the cylinder, the water starts to boil and we have a liquid-vapor mixture. Finally, once the last drop of liquid water has evaporated in the cylinder, the pressure again starts to decrease.

If we replicate the experiment at different temperatures, we observe that the higher the temperature, and the shorter the phase change range becomes. Above a certain temperature, which we call *critical temperature* ($T_{\text{cr.}}$), the range disappears completely.

The behavior of a liquid-vapor in this experiment can be described on a pressure-volume ($p - v$) diagram as shown in figure 5.9. The student is also encouraged to reconstruct this diagram.

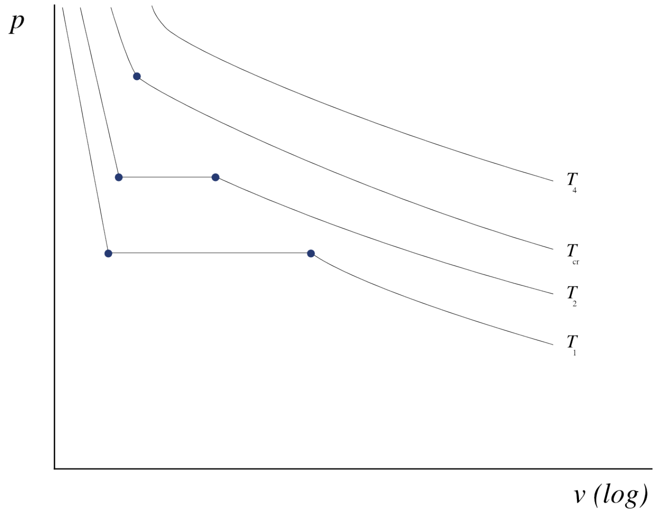


Figure 5.8: Properties of water plotted on a pressure-volume diagram, when maintaining constant temperature by varying the volume.

Diagram CC-0 *Olivier Cleynen*

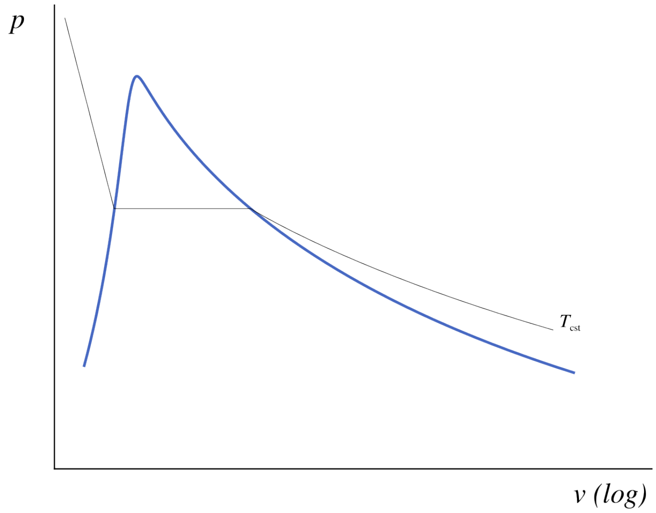


Figure 5.9: Pressure-volume diagram of water, represented with a constant temperature (isothermal) process. The saturation curve is represented in blue.

Diagram CC-0 *Olivier Cleynen*

5.2.4 A student’s false friend

The most important notion to remember from the behavior of liquid-vapors is that in contrast to ideal gases, *their temperature is completely deregulated.* It no longer simply dictates the other properties.

Let’s emphasize this. For a fluid close to a phase change:

$$pv \not\propto T \tag{5/1}$$

$$u \not\propto T \tag{5/2}$$

$$h \not\propto T \tag{5/3}$$

Almost everything that was covered in chapter 4 (*the ideal gas*) must be forgotten when dealing with a liquid/vapor. Fortunately, the first three chapters have not lost any of their utility.

5.2.5 Water in everyday life

The phenomena we describe here are easily observable and reproducible with water in everyday life. However, it should be noted that:

- Water vapor is transparent and almost invisible. What is observed above a boiling pot of water or in the form of clouds is *liquid* water suspended in the air (figure 5.10). These fine liquid droplets can gather to form drops (as a droplet grows, the surface area offering frictional resistance increases less rapidly than its weight, and its falling velocity increases) or evaporate again and become invisible once more.

Figure 5.10: The visible water above a container of hot liquid, sometimes called "steam", is in the liquid state and not gaseous. These droplets are observable to the naked eye.

Photo by Jorge Barrios (public domain, cropped)

- Air is partially composed of water vapor (and its ability to carry water increases with temperature). When boiling liquid water in open air, it must not be forgotten that it is the air that hosts the water vapor; thus, the evaporation unfolds quite differently from the experiment described in figure 5.4. For example, the temperature of liquid water drops significantly during evaporation at constant pressure in the air. Another particularityis that condensation is catalyzed by the presence of dust particles in the air.

5.3 Quantifying the properties of water

For a liquid/vapor, there is no simple way to quantify the internal energy u and enthalpy h that interest us so much. Indeed, from p and v, we cannot calculate the temperature ($pv \not\propto T$) and from T, we cannot calculate u and h ($u \not\propto T$ and $h \not\propto T$).

- The bad news is that we will have to use tables of previously-measured properties, called *steam tables*, which can be tedious at times;
- The good news is that these tables spare us from using the dreadful mathematical relations (such as $(T_1/T_2)^{1/\gamma-1} = ...$) that described the properties of fluids in chapter 4 (*the ideal gas*).

5.3.1 Subcooled liquid and superheated vapor

Let's start by heating a fixed amount of liquid water while maintaining its pressure constant, as we did in figure 5.4. For each temperature, we measure

v, u, and h (as well as s, but that's a surprise we keep for chapter 8). The experiment is then repeated at a different pressure.

The set of measurements is tabulated in Steam Table 1 (see pp. 308-311), of which an excerpt is presented in table 5.2.

> « Hence we see that very distinguished mathematicians have proposed, regarding the motion of the piston in steam engines, analytical formulas which would be very true if, indeed, things occurred in the machine as they suppose; but which, lacking a true starting point in their calculations, collapse by themselves in the face of the facts. Thus it also follows that, in practice, the proportions of these machines have only been determined through multiple trials, and that the art of constructing them still proceeds by trial and by imitation. »
>
> François-Marie Guyonneau de Pambour, 1835
> *Theoretical and Practical Treatise on Locomotive Engines* [6]

°C T	$\frac{\text{m}^3}{\text{kg}}$ v	$\frac{\text{kJ}}{\text{kg}}$ u	$\frac{\text{kJ}}{\text{kg}}$ h	$\frac{\text{kJ}}{\text{K kg}}$ s
	$p = 1.6\,\text{MPa}$ ($T_{\text{sat.}} = 201.37\,^\circ\text{C}$)			
10	0.001	42	43.6	0.1509
20	0.001001	83.8	85.4	0.2962
50	0.001011	209.1	210.7	0.7031
100	0.001043	418.6	420.3	1.306
200	0.001156	850.4	852.3	2.3305
300	0.15866	2 781.5	3 035.4	6.8863
500	0.22029	3 120.1	3 472.6	7.5409
600	0.24999	3 293.9	3 693.9	7.81
700	0.2794	3 473.5	3 920.5	8.0557
800	0.30865	3 659.5	4 153.3	8.2834
900	0.3378	3 852.1	4 392.6	8.4965
1 000	0.36687	4 051.2	4 638.2	8.6974
1 100	0.39589	4 256.6	4 890	8.8878
1 200	0.42487	4 467.9	5 147.7	9.0689
1 500	0.51169	5 133.7	5 952.4	9.5656
2 000	0.65615	6 326.8	7 376.6	10.272

Table 5.2: Excerpt from Steam Table 1 (see in Appendix A1 pp. 308-311). Here the measurements are made at 1.6 MPa, in other words, 16 bar (232 psi). A discontinuity is observed between 200 °C and 300 °C: this is the state change that occurred at $T_{\text{sat.}} = 201.37\,^\circ\text{C}$, the saturation temperature for this pressure.

This steam table allows us to answer many questions. Here are a few examples:

Example 5.1

At 16 bar and 600 °C, what is the volume occupied by 2 kg of water?

The pressure is 1.6 MPa. In Steam Table 1, at this pressure, at 600 °C, we can read its specific volume as $v = 0.249\,99\,\text{m}^3\,\text{kg}^{-1}$. The total volume will thus be $V = m\,v = 0.499\,98\,\text{m}^3$, which we confidently round to $0.5\,\text{m}^3$.

☞ Note that the temperature is higher than the saturation temperature (201.37 °C), indicating that the water is in the superheated vapor state.

☞ With an ideal gas, we could simply *calculate* the result ($v = \frac{RT}{p}$); but this method does not work for liquid/vapor mixtures.

Example 5.2

How much energy does this water lose as it undergoes a process from 600 °C and 16 bar to 20 °C and 6 bar?

From Steam Table 1, at 1.6 MPa and 600 °C, we read $u_1 = 3293.9\,\mathrm{kJ\,kg^{-1}}$.
For a pressure of 0.6 MPa at 20 °C, we read $u_2 = 83.9\,\mathrm{kJ\,kg^{-1}}$.
We can then quantify the change in energy as $\Delta U = m(u_2 - u_1) = -6420\,\mathrm{kJ}$ (hence a loss by the water).

> ☞ We were able to quantify ΔU, but we cannot determine the proportions of heat ($Q_{1\to 2}$) and work ($W_{1\to 2}$) in this change. The less reversible the process, and the smaller the work $W_{1\to 2}$ will be compared to $Q_{1\to 2}$. After chapter 8 (*entropy*), we will be able to use *entropy* to quantify the maximum amount of work that can be obtained between 1 and 2.

Example 5.3

A medium-sized turbine operates with a steam flow rate of $3\,\mathrm{kg\,s^{-1}}$ (6.61 lb/s) and a heat loss of 200 kW. At the inlet, the steam is at 600 °C and 16 bar; at the outlet, the steam is at 1 bar and 300 °C. What is the power delivered in the form of work?

At the inlet (1.6 MPa and 600 °C), we read $h_1 = 3693.9\,\mathrm{kJ\,kg^{-1}}$.
At the outlet (0.1 MPa and 300 °C), we read $h_2 = 3074.5\,\mathrm{kJ\,kg^{-1}}$.
Now, in an open system operating in steady state, neglecting changes in mechanical energy, we have $q_{1\to 2} + w_{1\to 2} = \Delta h$ (3/15). Therefore, $\dot{W}_{1\to 2} = \dot{m}\,\Delta h - \dot{Q}_{1\to 2} = 3 \times (3074.5 \times 10^3 - 3693.9 \times 10^3) - (-200 \times 10^3) = -1.6582 \times 10^6\,\mathrm{W} = -1658.2\,\mathrm{kW}$.

Example 5.4

What is the specific internal energy of water at 16 bar and 585 °C?

We interpolate between two lines of Steam Table 1. We have $u_{500\,^\circ\mathrm{C}} = 3120.1\,\mathrm{kJ\,kg^{-1}}$ and $u_{600\,^\circ\mathrm{C}} = 3293.9\,\mathrm{kJ\,kg^{-1}}$. We have progressed by a factor $y = \frac{585-500}{600-500} = 0.85$ between the two lines.
We obtain by interpolation $u_{585\,^\circ\mathrm{C}} = u_{500\,^\circ\mathrm{C}} + y \times (u_{600\,^\circ\mathrm{C}} - u_{580\,^\circ\mathrm{C}}) = 3267.83\,\mathrm{kJ\,kg^{-1}}$.

> ☞ After interpolating, always quickly check the order of magnitude of the results. Here $u_{585\,^\circ\mathrm{C}}$ is indeed between $u_{500\,^\circ\mathrm{C}}$ and $u_{600\,^\circ\mathrm{C}}$, and closer to $u_{600\,^\circ\mathrm{C}}$.

5.3.2 Saturation points

In order to precisely quantify the properties of water when it changes phase, we use Steam Tables 2 and 3. The properties of water in the form of saturated liquid (subscript L) and saturated vapor (subscript V) are tabulated for each temperature.

In Steam Table 2 (see pp. 312-313), the data is sorted by pressure (with each pressure corresponding to a single saturation temperature). Steam Table 3 (see pp. 314-315) presents exactly the same data, but sorted by temperature (with each temperature corresponding to one saturation pressure). Excerpts from these steam tables are presented in Tables 5.3 and 5.4.

°C	MPa	$\mathrm{kJ\,kg^{-1}}$			$\mathrm{kJ\,kg^{-1}}$			$\mathrm{m^3\,kg^{-1}}$	
$T_{\text{sat.}}$	$p_{\text{sat.}}$	u_L	u_V	$\Delta u_{L\to V}$	h_L	h_V	$\Delta h_{L\to V}$	v_L	v_V
...	...	...	...	...	...	...	...	...	...
115	0.16918	482.4	2 523.4	2 041	482.6	2 698.6	2 216	0.001056	1.0358
120	0.19867	503.6	2 528.8	2 025.2	503.8	2 705.9	2 202.1	0.00106	0.89121
125	0.23224	524.8	2 534.3	2 009.4	525.1	2 713.1	2 188	0.001065	0.77003
130	0.27028	546.1	2 539.6	1 993.5	546.4	2 720.1	2 173.7	0.00107	0.668
...	...	...	...	...	...	...	...	...	...

Table 5.3: Excerpt from Steam Table 2 (see in Appendix A1 pp. 312-313). Subscript L corresponds to saturated liquid, and subscript V corresponds to saturated vapor. The difference between these values is sometimes noted with an index LV: for example $u_{LV} \equiv \Delta u_{L\to V} \equiv u_V - u_L$.

MPa	°C	$\mathrm{kJ\,kg^{-1}}$			$\mathrm{kJ\,kg^{-1}}$			$\mathrm{m^3\,kg^{-1}}$	
$p_{\text{sat.}}$	$T_{\text{sat.}}$	u_L	u_V	$\Delta u_{L\to V}$	h_L	h_V	$\Delta h_{L\to V}$	v_L	v_V
...	...	...	...	...	...	...	...	...	...
0.2	120.21	504.5	2 529.1	2 024.6	504.7	2 706.2	2 201.5	0.001061	0.88568
0.25	127.41	535.1	2 536.8	2 001.8	535.3	2 716.5	2 181.1	0.001067	0.71866
0.3	133.52	561.1	2 543.2	1 982.1	561.4	2 724.9	2 163.5	0.001073	0.60576
0.35	138.86	583.9	2 548.5	1 964.7	584.3	2 732	2 147.7	0.001079	0.52418
...	...	...	...	...	...	...	...	...	...

Table 5.4: Excerpt from Steam Table 3 (see in Appendix A1 pp. 314-315). These are the same data as in Steam Table 2; they are merely sorted by pressure instead of temperature.

We can already answer simple questions using these tables:

Example 5.5

What is the boiling temperature of water when the pressure is 3 bar?

Water is boiling, so it is at saturation (liquid-vapor mixture). We refer to Steam Table 3 (excerpt in table 5.4) where the data is sorted by pressure. At 0.3 MPa, the saturation temperature is 133.52 °C.

☞ As long as the water continues to boil or condense, it will remain at 133.52 °C. In order to achieve boiling at a different temperature, the pressure must be adjusted.

Example 5.6

What is the increase in volume when water is vaporized at 130 °C?

Water goes from a volume v_L (a liquid just about to boil) to a volume v_V (when the last drop has evaporated).
We refer to Steam Table 2 (excerpt in table 5.3) where the data is sorted by temperature. At 130 °C, the specific volume increases by $v_{LV} \equiv \Delta v_{L \to V} \equiv v_V - v_L = 0.668 - 0.001\,07 = 0.666\,93\,\mathrm{m^3\,kg^{-1}}$ (it is multiplied by about 600).

☞ Here, evaporation occurs entirely at 130 °C (which is quite easy to achieve in practice, since it simply involves maintaining constant pressure, see figure 5.7). If the temperature and pressure were not held constant, the final volume would be different.

Example 5.7

How much heat is needed to fully (and slowly) vaporize 4 L of saturated liquid water at 3 bar?

The water will receive heat, but will also do work (by expanding at constant pressure of 3 bar). We will calculate $q_{\text{evap.}} = q_{1\to2} = (u_2 - u_1) - w_{1\to2}$ (2/2).
The water starts as a saturated liquid (state 1 = subscript L) and ends up as a saturated vapor (state 2 = subscript V).
Since the process is slow and at constant pressure, the work $w_{1\to2} = -\int_1^2 p\,\mathrm{d}v$ simply becomes $-p_{\text{cst.}}(v_2 - v_1)$.
Let's gather all of this information in one equation: $q_{\text{evap.}} = (u_V - u_L) + p_{\text{cst.}}(v_V - v_L) = h_V - h_L = h_{LV} \equiv \Delta h_{L\to V} = 2163.5\,\mathrm{kJ\,kg^{-1}}$.
At 3 bar, our 4 L of saturated liquid water corresponds to a mass $m = \frac{V}{v_L} = \frac{4\times10^{-3}}{0.001073} = 3.7279\,\mathrm{kg}$. So, in the end, $Q_{\text{evap.}} = m\,q_{\text{evap.}} = 8065.2\,\mathrm{kJ}$.

☞ If we had used the usual approximation of 1000 liters per cubic meter of liquid water ($v_L \approx 10^{-3}\,\mathrm{m^3\,kg^{-1}}$), we would have made an error of +7.3%.

It is worth mentioning that the term $h_{LV} \equiv \Delta h_{L\to V} \equiv h_L - h_V$ is sometimes called *heat of vaporization* or *latent heat*. Indeed, for evaporation in a closed system at a given temperature, $q_{\text{evap.}} = \Delta u - w_{\text{evap.}} = (u_V - u_L) + p_{\text{sat.}}(v_V - v_L) = h_{LV}$ (the same result is obtained in an open system).

5.3.3 The liquid-vapor mixture

Finally, we want to quantify the properties of water *between* the saturation points, in other words, when it is only partially liquid. Experiment shows that in this region, fluids behave linearly, and its properties can be easily quantified.

In order to “position” a liquid-vapor mixture between the two saturation points, we define the *quality* or *dryness fraction* as follows:

> The dryness fraction x is the mass fraction of saturated vapor contained in a liquid-vapor mixture.

For example, a mass of 1 kg of water with a dryness fraction of 0.2 contains 0.8 kg of saturated liquid and 0.2 kg of saturated vapor. This 0.2 kg does occupy the majority of the available volume. We could say that the dryness fraction quantifies the progression of a liquid-vapor mixture between its two saturation points (figure 5.11). The concept of dryness fraction applies only to two-phase mixtures, thus we always have $0 \leq x \leq 1$.

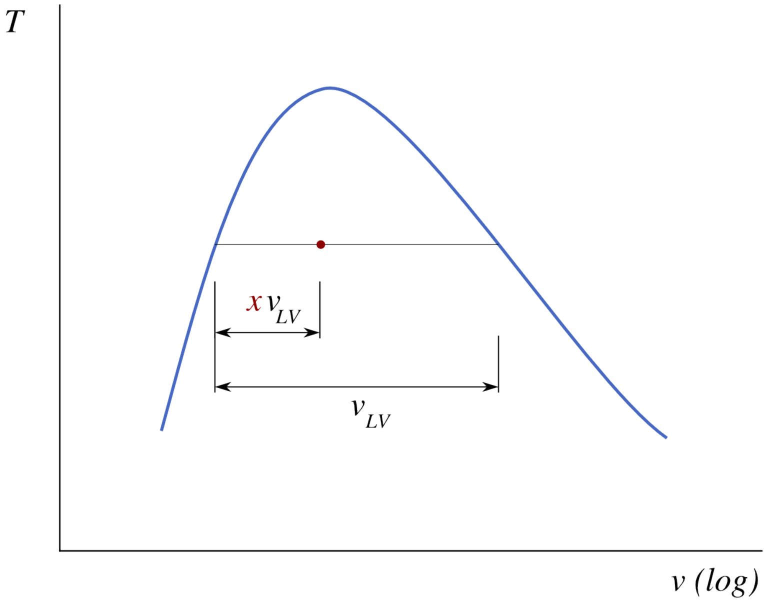


Figure 5.11: The vapor dryness fraction represented by the point position on a $T - v$ diagram.

Diagram CC-0 *Olivier Cleynen*

We can now express the properties u, h, and v in terms of the dryness fraction:

The enthalpy h of a liquid-vapor mixture is equal to the sum of the enthalpy of the liquid and that of the vapor. We have, as illustrated in figure 5.12:

$$\begin{aligned} h_x &= (1-x)h_L + x\,h_V \\ &= h_L + x(h_V - h_L) \\ h_x &= h_L + x\,h_{LV} \end{aligned} \tag{5/4}$$

where h_x is the specific enthalpy of the mixture at hand ($\mathrm{J\,kg^{-1}}$),
x is its dryness fraction (unitless),
and $h_{LV} \equiv \Delta h_{L\to V} \equiv h_V - h_L$ (tabulated value) is the specific enthalpy of vaporization at its temperature ($\mathrm{J\,kg^{-1}}$).

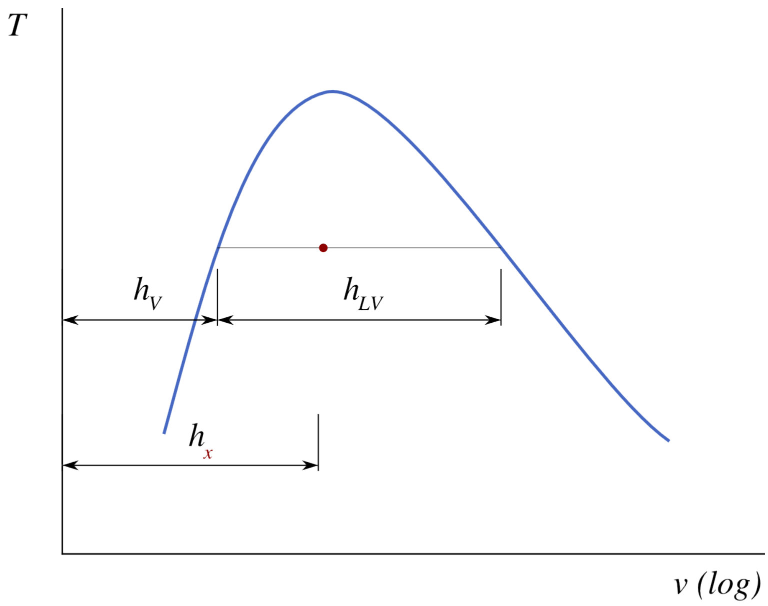


Figure 5.12: Enthalpy h_x of a mixture as a function of the enthalpies in the saturated state and of vaporization.

Diagram CC-0 *Olivier Cleynen*

The internal energy u of a liquid-vapor mixture is quantified in exactly the same way:

$$u_x = u_L + x\,u_{LV} \tag{5/5}$$

where u_x is the specific energy of the mixture under study ($\mathrm{J\,kg^{-1}}$),
x is its dryness fraction (unitless),
and $u_{LV} \equiv \Delta u_{L\to V} \equiv u_V - u_L$ (tabulated value) is the difference of specific internal energies at saturation, at its temperature ($\mathrm{J\,kg^{-1}}$).

The specific volume of a liquid-vapor mixture, finally, is quantified even more simply. The total volume of the mixture equals the volume of the gas plus the volume of the liquid, thus:

$$v_x = (1-x)v_L + x\,v_V$$

However, the specific volume v_L of the saturated liquid is usually small compared to that of the vapor. A brief examination of Steam Table 2 will reveal that this is approximately a factor of 10^3 (this factor is not very well highlighted by the $T-v$ and $p-v$ diagrams in this chapter, whose abscissa scales are logarithmic). We can therefore neglect v_L and simply write:

$$v_x \approx x\,v_V \tag{5/6}$$

where v_x is the specific volume of the mixture under study ($\mathrm{m^3\,kg^{-1}}$),
x is its dryness fraction (unitless),
and v_V (tabulated value) is the specific volume of the saturated vapor at its temperature ($\mathrm{m^3\,kg^{-1}}$).

This approximation is illustrated in figure 5.13.

We can now use the same Steam Tables 2 and 3 to quantify what happens between the saturation points.

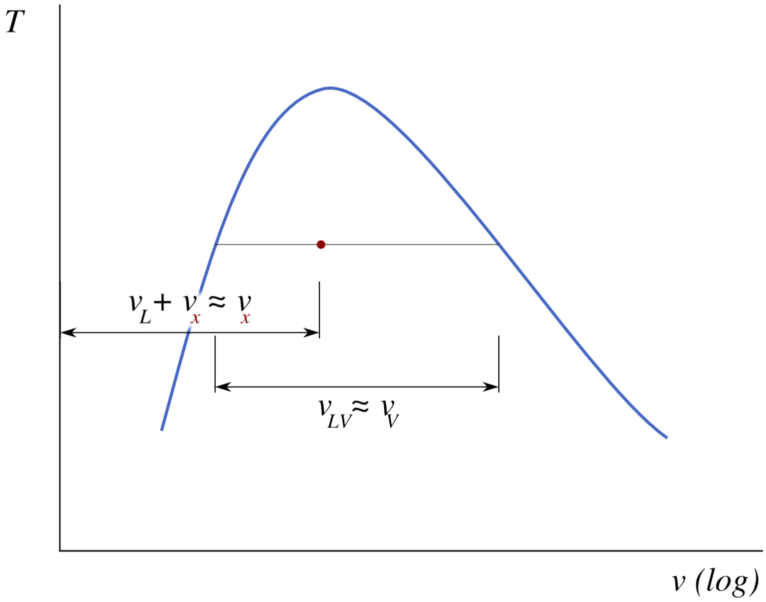


Figure 5.13: Approximations used in calculating the volume occupied by a liquid-vapor mixture. It should be noted that the abscissa scale is logarithmic: v_L is generally several hundred times smaller than v_V, and the approximation is not graphically emphasized. *Diagram* CC-0 *Olivier Cleynen*

Example 5.8

What is the internal energy and volume occupied by a mass of 4 kg of water, 75 % vaporized, at 115 °C (239 °F)?

We have a liquid-vapor mixture and the dryness is 0.75. We refer to Steam Table 2 (excerpt in table 5.3) to find the saturation temperature of 115 °C.

Then, we simply apply equation 5/5: $u_x = u_L + 0.75 \times u_{LV} = 482.4 + 0.75 \times 2041 = 2013.15\,\text{kJ}\,\text{kg}^{-1}$.

Similarly, with equation 5/6: $v_x = 0.75 \times v_V = 0.75 \times 1.0358 = 0.776\,85\,\text{m}^3\,\text{kg}^{-1}$.

Therefore, we have $U = m\,u = 8052.6\,\text{kJ}$ and $V = m\,v = 3.1074\,\text{m}^3 = 820.89\,\text{US gal}$.

Example 5.9

What is the dryness of water at 2.5 bar whose enthalpy is $1500\,\text{kJ}\,\text{kg}^{-1}$?

We have a liquid-vapor mixture; we look in Steam Table 3 (excerpt in table 5.4) for the line corresponding to $p_{\text{sat.}} = 0.25\,\text{MPa}$. We use equation 5/4.

From there, we get: $x = \frac{h_x - h_L}{h_{LV}} = \frac{1500 - 535.3}{2181.1} = 0.442$.

☞ A quick check: at $1500\,\text{kJ}\,\text{kg}^{-1}$ we are roughly halfway between $h_L \approx 500$ and $h_V \approx 2700\,\text{kJ}\,\text{kg}^{-1}$.

5.4 Elementary Reversible Processes

We are now able to quantify the terms pv, u, and h of a liquid/vapor in all cases. Here, we intend to proceed just as in the previous chapter (§4.4): we want to calculate the energy transfers involved when compressing or expanding a liquid/vapor under entirely arbitrary constraints of volume, pressure, or temperature.

5.4.1 What is this chapter section for?

The answer is the same as in chapter chapter 4 (*the ideal gas*) (§4.4.1). The liquid/vapor processes we study here are highly hypothetical but interesting for two reasons:

1. The behavior of a liquid/vapor is intrinsically complex. These elementary processes serve as exercises to learn how to describe it step by step;
2. These elementary processes are conceptual tools that we will later assemble: first to quantify the theoretical limits of machines (in chapter 7), and finally to describe the behavior of fluids in industrial machines (in chapter 9).

5.4.2 Processes at constant pressure

It is possible to heat or cool a liquid/vapor while maintaining its pressure constant (figure 5.14). A process at constant pressure is called *isobaric*. To generate one, we should:

- with a fixed amount of fluid (closed system), constrain it with a surface that exerts a constant force regardless of the volume;
- with fluid in steady flow (open system), simply transfer heat by letting it flow through a conduit without moving parts. This is what happens in a boiler or condenser, for example.

> « It is well known that when water is made to vaporize under atmospheric pressure, in vain is additional heat continuously supplied to it by means of the furnace, neither the temperature of the water nor that of the steam ever rises beyond 100° of the centigrade thermometer, or 212° of the Fahrenheit thermometer. »
>
> François-Marie Guyonneau de Pambour, 1839
> *Théorie de la machine à vapeur* [7]

In a closed system, we have $q_{1\to2} + w_{1\to2} = \Delta u$ (2/2). If the process is reversible, both heat and work can be quantified as follows:

$$w_{1\to2} = -\int_1^2 p \, \mathrm{d}v = -p_{\text{cst.}} \int_1^2 \mathrm{d}v$$

$$w_{1\to2} = -p_{\text{cst.}} \, \Delta v \tag{5/7}$$

in a reversible process at constant pressure $p_{\text{cst.}}$, in a closed system.

$$q_{1\to2} = \Delta u - w_{1\to2} = \Delta u + p_{\text{cst.}} \, \Delta v$$

$$q_{1\to2} = \Delta h \tag{5/8}$$

in a reversible process at constant pressure, in a closed system.

In an open system, we have $q_{1\to2} + w_{1\to2} = \Delta h$ (3/15). If the process is reversible, both heat and work can be quantified as follows:

$$w_{1\to2} = \int_1^2 v \, \mathrm{d}p$$

$$w_{1\to2} = 0 \tag{5/9}$$

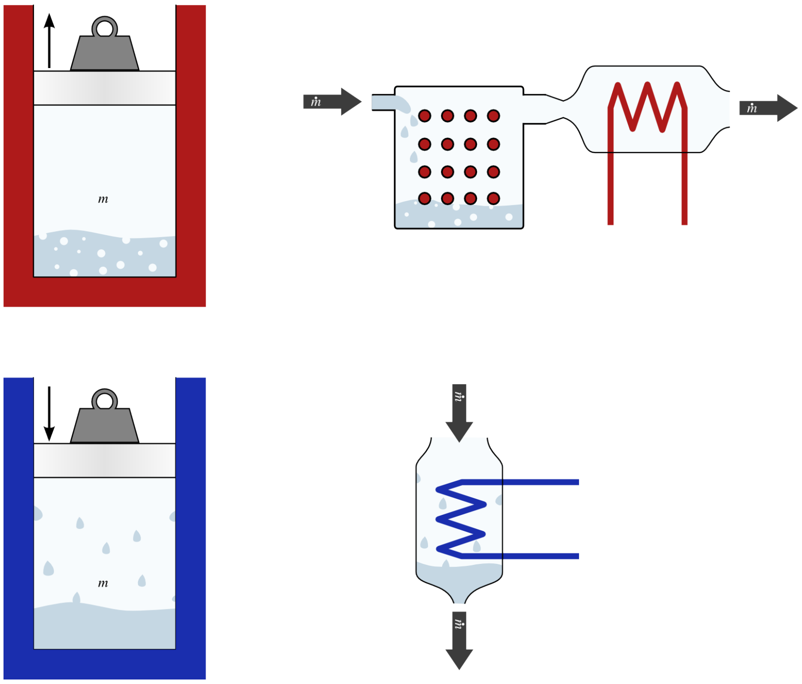


Figure 5.14: Constant-pressure (isobaric) process undergone by a liquid/vapor. In a closed system (on the left), the piston exerts a constant force throughout the process. In an open system (on the right), no work is done.

Diagram CC-BY-SA *Olivier Cleynen*

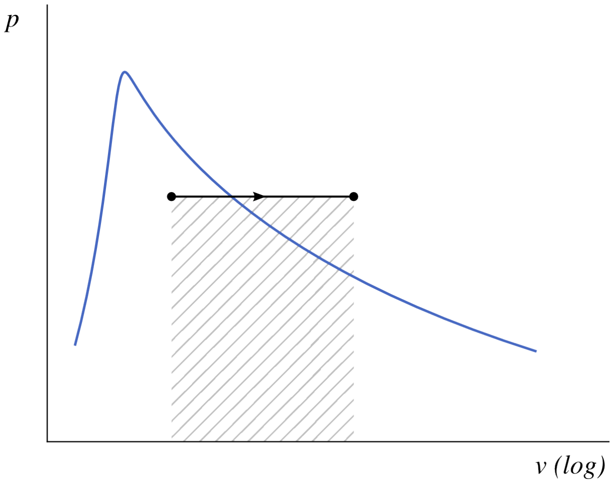


Figure 5.15: Heating at constant pressure of a liquid/vapor, represented on a pressure-volume diagram. *Diagram* CC-0 *Olivier Cleynen*

in a reversible process at constant pressure, in an open system.

$$q_{1\to2} = \Delta h - w_{1\to2}$$

$$q_{1\to2} = \Delta h \qquad (5/10)$$

in a reversible process at constant pressure, in an open system.

Example 5.10

How much work and heat are needed to slowly heat 2 kg of saturated liquid water at constant pressure (3 bar) until the volume reaches 1 m^3?

We start from the saturated liquid state, with $v_1 = v_L$ and $h_1 = h_L$. We need the final specific volume and enthalpy in order to quantify $W_{1\to 2}$ and $Q_{1\to 2}$. The final volume will be $v_2 = \frac{V_2}{m} = 0.5\,\text{m}^3\,\text{kg}^{-1}$.

☞ We notice that v_2 is less than v_V at our temperature. At the end of the heating process, the water will still be partially liquid, and we will need to calculate its dryness fraction.

☞ Liquid-vapor mixture? We are heading towards Steam Tables 2 and 3. We know the pressure (0.3 MPa), therefore we need Steam Table 3.

The final dryness fraction is $x_2 \approx \frac{v_x}{v_V} = \frac{0.5}{0.605\,76} = 0.825$ (5/6). Therefore, $h_2 = h_L + x_2\, h_{LV} = 561.4 + 0.825 \times 2163.5 = 2347.2\,\text{kJ}\,\text{kg}^{-1}$ (5/4).

The work is obtained using equation 5/9: $W_{1\to 2} = m\ w_{1\to 2} = -m\ p_{\text{cst.}}\ \Delta v = -2{\times}0.3{\times}10^6{\times}(0.5{-}0.001\,073) = -2.994{\times}10^5\,\text{J} = -299.4\,\text{kJ}$.

Finally, the heat transfer is calculated using equation 5/10: $Q_{1\to 2} = m\ q_{1\to 2} = m\ \Delta h = 2 \times (2347.2 \times 10^3 - 561.4 \times 10^3) = +3.5715 \times 10^6\,\text{J} = +3571.5\,\text{kJ}$.

☞ The heat transfer involved is ten times more significant than the work done. In this case, we are heating a lot, and the fluid, at low pressure, does little work.

☞ It is probably simpler and less risky to derive these equations 5/9 and 5/10 by hand rather than trying to memorize them.

One can notice that when heating water in a liquid/vapor mixture (below the saturation curve), the volume increase is significant. In practice, just a few milliliters of liquid water can lead to an expansion of several liters at constant pressure, with a very moderate and constant temperature. This is why all the early engines, in the 19th century, operated with water rather than air. The large volume amplification allowed for more compact engines with large strokes (simpler mechanisms), the constant pressure meant forces were easier to manage, and the modest temperatures allowed the use of simple materials. This combination made the liquid-vapor mixture of water appealing at a time when metallic technology was limited. We will see in chapters 7 (*the second law*) and 9 (*steam power cycles*) that these advantages unfortunately translate into staggeringly low efficiencies. In order to overcome this, the temperature must be increased: that would be for the 20th century.

5.4.3 Processes at constant volume

It is possible to transfer heat to or from a liquid/vapor while maintaining its volume constant (figure 5.16). A process at constant volume is called an *isochoric* process. To generate one, we should:

- With a fixed amount of fluid (closed system), simply transfer heat with a fixed and closed container;
- with a fluid in steady flow (open system), carry out a more complex process: the liquid-vapor must be compressed while heating it to prevent its volume from increasing; similarly, to prevent its volume from decreasing when cooling it, it must be expanded.

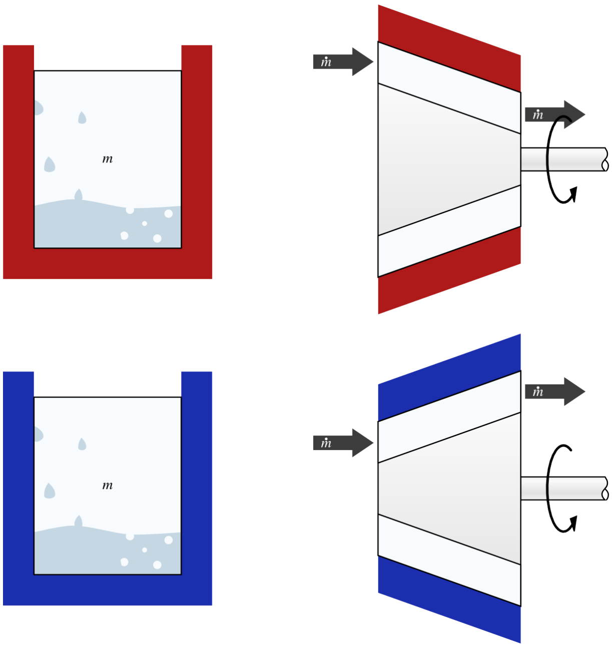


Figure 5.16: A constant-volume (isochoric) process undergone by a liquid-vapor. In a closed system (left), the volume is fixed and no work is done. In an open system (right), the fluid must be compressed while being heated and expanded while being cooled, in order to maintain constant specific volume.

Diagram CC-by-sa *Olivier Cleynen*

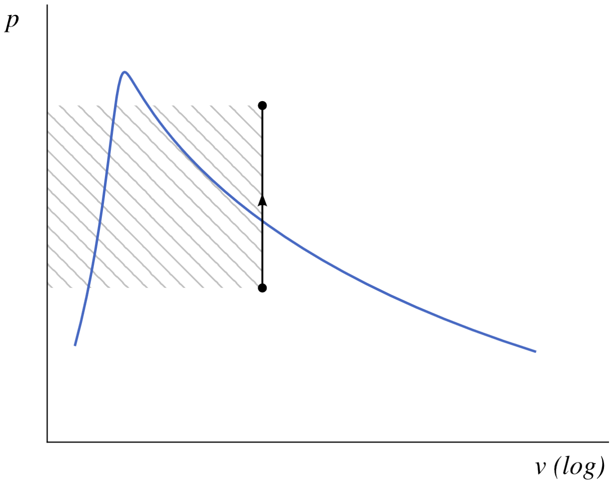


Figure 5.17: Heating at constant volume of a liquid-vapor, represented on a pressure-volume diagram.

Diagram CC-0 *Olivier Cleynen*

In a closed system, we have $q_{1\to 2} + w_{1\to 2} = \Delta u$. The heat and work can each be quantified as follows:

$$\begin{aligned} w_{1\to 2} &= -\int_1^2 p \, \mathrm{d}v \\ w_{1\to 2} &= 0 \end{aligned} \tag{5/11}$$

in a constant volume process, in a closed system.

$$\begin{aligned} q_{1\to 2} &= \Delta u - w_{1\to 2} \\ q_{1\to 2} &= \Delta u \end{aligned} \tag{5/12}$$

in a constant volume process, in a closed system.

When the process occurs in an open system, we have $q_{1\to 2} + w_{1\to 2} = \Delta h$. if the process is reversible, the heat and work can each be quantified as follows:

$$\begin{aligned} w_{1\to 2} &= \int_1^2 v \, \mathrm{d}p = v_{\text{cst.}} \int_1^2 \mathrm{d}p \\ w_{1\to 2} &= v_{\text{cst.}} \, \Delta p \end{aligned} \tag{5/13}$$

in a reversible constant volume process, in an open system.

$$\begin{aligned} q_{1\to 2} &= \Delta h - w_{1\to 2} = \Delta h - v_{\text{cst.}} \, \Delta p \\ q_{1\to 2} &= \Delta u \end{aligned} \tag{5/14}$$

in a reversible constant volume process, in an open system.

We note that depending on its dryness fraction at the beginning, a liquid-vapor mixture, when heated at constant volume, can become either entirely liquid, or entirely gaseous.

5.4.4 Processes at constant temperature

It is possible to heat or cool a liquid-vapor while maintaining its temperature constant (figure 5.18). A process at constant temperature is called an *isothermal* process.

When the fluid is in a mix of phases (inside of the saturation curve), the constant temperature process also occurs at constant pressure, as described in section §5.4.2 above. In order to quantify the energy transfers, we only need to refer to equations 5/9 and 5/10.

However, as soon as we cross the saturation curve, things change. Once the saturation is reached, the pressure starts to decrease, and we do not have an analytical way to describe this process.

The consequence is that for now, we cannot quantify the work and heat involved when evolving steam at constant temperature! We must wait until chapter 8, where we will use the concept of *entropy* to tackle the problem.

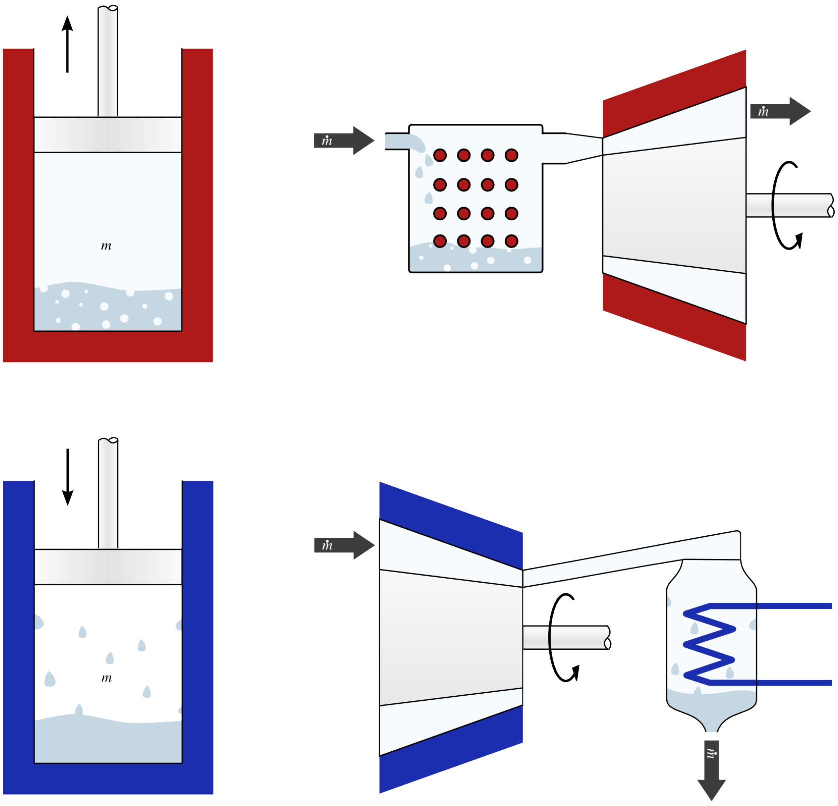


Figure 5.18: A constant-temperature (isothermal) process undergone by a liquid-vapor. In a closed system (left), the gas is allowed to do work on a piston while being heated, and conversely, work is done on it when cooling. In an open system (right), the same manipulations are carried out continuously.

Diagram CC-BY-SA *Olivier Cleynen*

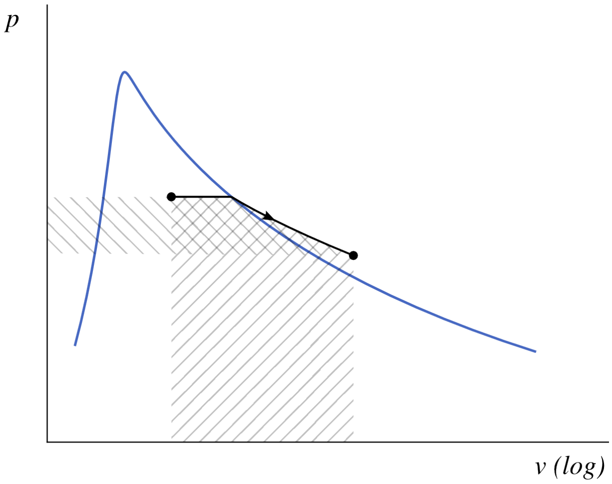


Figure 5.19: Expansion (heating) at constant temperature of a liquid-vapor, represented on a pressure-volume diagram.

Diagram CC-0 *Olivier Cleynen*

Example 5.11

How much work and heat are required to slowly heat 2 kg (4.41 lb) of saturated liquid water at constant temperature (130 °C = 266 °F), until its volume reaches 1 m^3 (220 imp gal)?

We first observe the final state. The final volume will be $v_2 = \frac{V_2}{m} = 0.5\,\text{m}^3\,\text{kg}^{-1}$, which is less than v_V at our temperature. Therefore, at the end of the heating process, the water will still be partially liquid.

The process will also occur at constant pressure (at the saturation pressure, $p_{\text{sat}130\,^\circ\text{C}} = 0.270\,28\,\text{MPa}$). The calculation is exactly the same as for example 5.10 p. 135. We obtain a dryness fraction of 0.749, the work (done) is $W_{1\to2} = -269.7\,\text{kJ}$, and a heat (received) amounts to $Q_{1\to2} = +3256.2\,\text{kJ}$.

☞ As long as we are in a liquid-vapor mixture (inside the saturation curve), constant temperature = constant pressure. No problem.

Example 5.12

We revisit the same question as in Example 5.11 above with a larger final volume: 2 m^3 (440 imp gal). How much heat and work are needed?

We cannot yet answer this question! The final specific volume exceeds v_V and towards the end of the process, the pressure decreases (figure 5.19).

We could quantify the energy u_2 by interpolating between the *rows and columns* of Steam Table 1 (by looking for a volume v_2 at 130 °C), which would be imprecise and cumbersome. However, even if we quantified $\Delta u = u_2 - u_1$, we would be unable to determine the share of work and heat within: both the pressure and volume change, and we are missing a relationship between them to carry out the integral $\int p\,\mathrm{d}v$.

☞ With the ideal gas model, we could write that $pv = \text{cst.}$ at constant temperature, and thus calculate the work during expansion. But with a liquid-vapor mixture, this no longer works.

☞ In chapter 8 (*entropy*), we will be able to use the brilliant concept of *entropy* to answer this question.

5.4.5 Reversible adiabatic processes

An *adiabatic* process is a process during which there is no heat transfer (figure 5.20). This can be achieved by covering the container or the duct with a thick layer of thermal insulator.

A *reversible adiabatic* process is carried out infinitely slowly. For this to happen, a piston in a cylinder will have to be moved infinitely slowly, and a steady-flow turbine will have to be infinitely long. Adiabatic processes serve as a reference, a theoretical goal, to quantify the performances of real turbines, which we will study in chapter 9.

« I may here be allowed to refer to a fact proved by Rankine and myself, that when a quantity of vapour, at is maximum density and enclosed by a surface impenetrable to heat, expands and thereby displaces a moveable part of the enclosing surface, e.g. a piston, with its full force of expansion, a part of the vapour must undergo condensation… »

Rudolf Clausius, 1856
On the Application of the Mechanical Theory of Heat to the Steam Engine
[16, 18, 20]

Just like for an ideal gas, the temperature necessarily varies in such a process, since the work is non-zero. It is also noted that the curves of reversible adiabatic processes plotted on a pressure-volume diagram always intersect the saturation curve. In other words, dry steam expanded slowly without heat transfer will, sooner or later, be led to condense. This fact will have significant consequences in chapter 9 (*steam power cycles*).

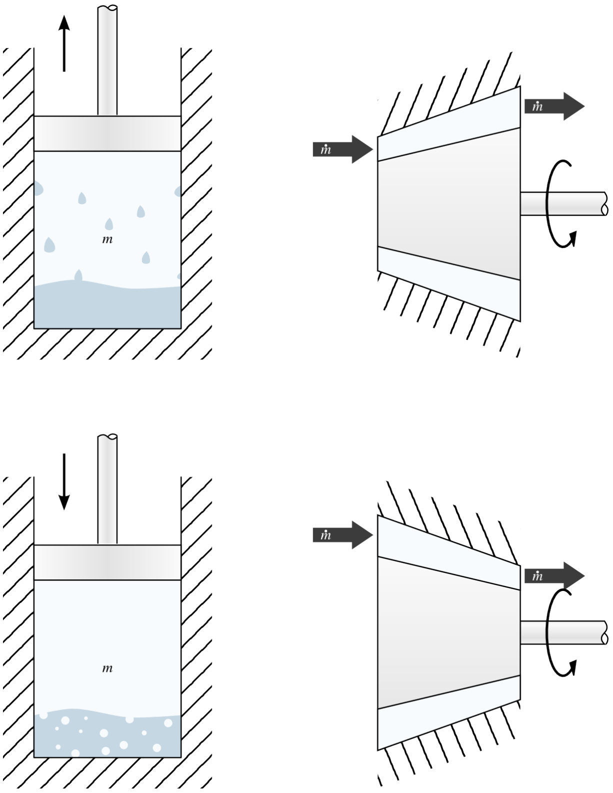


Figure 5.20: A reversible adiabatic (isentropic) process undergone by a liquid-vapor. In a closed system (left) as well as in an open system (right), the enclosure is perfectly insulated, so that there is no heat transfer, even if its temperature varies.

Diagram CC-BY-SA *Olivier Cleynen*

In any adiabatic process, the heat transfer is zero:

$$q_{1\to 2} = 0 \tag{5/15}$$

for any adiabatic process.

The work is therefore simply expressed as:

$$w_{1\to 2} = \Delta u \tag{5/16}$$

for any adiabatic process in a closed system;

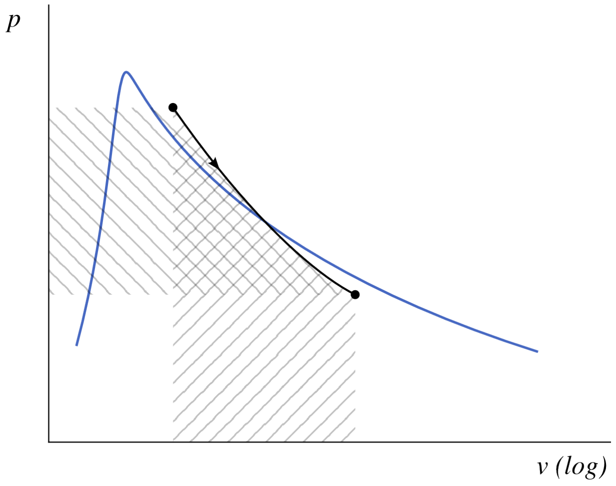


Figure 5.21: Reversible adiabatic expansion of a liquid-vapor, represented on a pressure-volume diagram.

Diagram CC-0 *Olivier Cleynen*

$$w_{1\to2} = \Delta h \tag{5/17}$$

for any adiabatic process in an open system.

How to quantify this Δu or Δh? Let's consider the example of an adiabatic expansion, starting from 40 bar and 500 °C. We try to extract the maximum work from the steam before releasing it at atmospheric pressure (1 bar).

- If the expansion is completely irreversible (very abrupt), then the work is zero. The steam is released with the same amount of energy (u, h) as at the inlet.
- The slower the expansion, the more work we receive.
- The best case – the maximum work – corresponds to a reversible adiabatic expansion (infinitely slow).

Unfortunately, we are still unable to quantify this maximum amount of work! To do this, we would need to be able to quantify the energy within the steam while it expands. We knew how to do this last chapter with an ideal gas (and the daunting relationships of the type $(T_1/T_2)^{1/\gamma-1} = ...$), but we do not have such a tool with liquids/vapors.

Later, in chapter 8 (*entropy*), we will see that a reversible adiabatic process occurs at constant *entropy* (which is why we will call these processes *isentropic*), and we will use this phenomenal tool to answer these questions.

5.4.6 Arbitrary processes

One must keep in mind that in practice, the properties of a liquid-vapor can be *arbitrarily changed* (figure 5.22), just like with a gas.

We have focused on four specific processes, because each plays an important role, for physicists and engineers, in the design of thermal machines. By cleverly controlling heat and work transfers, of course, we can change the fluid properties any way we would like.

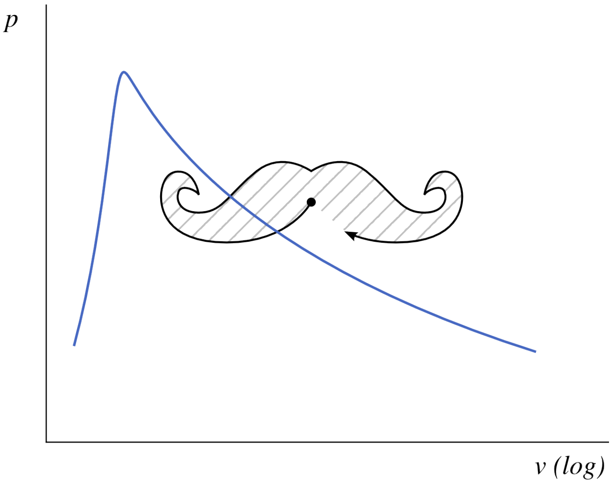


Figure 5.22: A completely arbitrary process undergone by a liquid-vapor represented on a pressure-volume diagram. In addition to a deplorable sense of humor, such a process requires an extremely complex combination of heat and work transfers, which the student is invited to imagine. *Diagram* CC-0 *Olivier Cleynen*

A Bit of History: the Horsepower

*

We traditionally associate the word *engine* with automobile propulsion: machines running on air and gasoline. However, the very first engines were quite different. Heavy, slow, incredibly large, running on coal and water, they were only used to pump water.

Let's go back to the beginning of the 19th century. At that time, Europe was heated by coal, which was extracted with great difficulty from constantly flooded mines. The water was removed by working horses through a primitive pumping mechanism. The first engines were installed to replace these horses – but they were hardly less expensive, and required just as much attention!

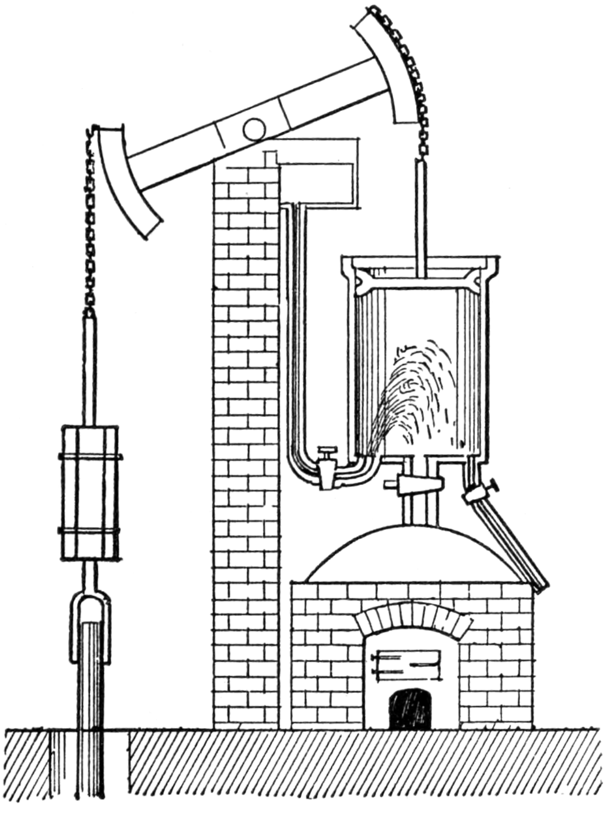


Figure 5.23: Cross-section diagram of one of the first steam engines (Newcomen engine, ~1720). The condensation caused by water injection into the cylinder led to a drop in internal pressure.

Engraving by Newton Henry Black & Harvey Nathaniel Davis, published in 1913 (public domain)

The level of development of metallurgy (cylinders were made of copper, by hand) and mechanical technology (valves had to be successively opened and closed by hand, one by one, to let the engine operate) meant that these engines could only operate with very low pressures.

For these engines, water is an excellent choice for a working fluid. When steam at moderate pressure is cooled (for example by mixing it with cold liquid water), it condenses and its pressure drops abruptly (figure 5.23). This is an opportunity to drive a piston that, subjected to atmospheric pressure on its other side, can supply work. Thus, one could almost speak of "implosion engines", since they make the atmosphere work on a cylinder of depressurized steam to produce work.

With this operating mode, the pressure difference obtained reaches a maximum of 1 bar, and the pace is lamentably slow. But these machines operated at reasonable temperatures and pressures, and the operators lacked neither coal nor water.

It is a young employee of the University of Manchester who first realized the tremendous development potential of the steam engine. By studying a scale model of an engine owned by the university, he made a series of modifications that doubled its efficiency.

The first and most important of these modifications was to separate in space the heating and cooling phases of the steam. Previously, condensation by injection of cold water also lowered the temperature

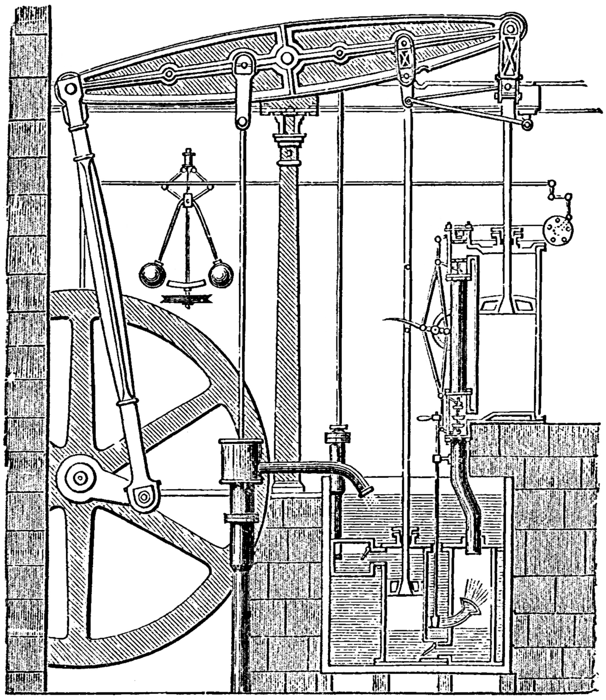


Figure 5.24: The *Boulton & Watt* engine with separate condensation and double-acting piston.

Engraving by Robert Henry Thurston (1878, public domain) of the metallic piston and cylinder, which had to be heated again at each cycle, with a significant energy cost. Now instead, the steam was cooled in a chamber maintained at low temperature by immersion in water (figure 5.24), while the engine cylinder was kept at a high temperature above the boiler.

The second modification consisted of exploiting the two faces of the piston. By using a system of pipes controlled by valves, it became possible to increase the pressure difference driving the piston movement. While the steam condensed at 0.2 bar (2.9 psi) on one side, the other face now met steam pressurized at 1.4 bar (20.3 psi). Not only was the work done during each piston movement greater, but also the speed (and thus the power) was doubled, since the piston was providing work in both the upward and downward movements.

Finally, a series of mechanical devices reduced the attention that had to be paid to the formidable machinery thus assembled. A large flywheel maintained speed, valve openings were mechanically linked to the engine's advancement, and the centrifugal ball governor, a technology imported from water mills, prevented engine runaway or stalling.

The young laboratory assistant, who went by the name of James Watt, found fortune by partnering with a cannon manufacturer and expert coppersmith, Matthew Boulton. The rest is history: *Boulton & Watt* captured the lion's share of the emerging market for heat engines.

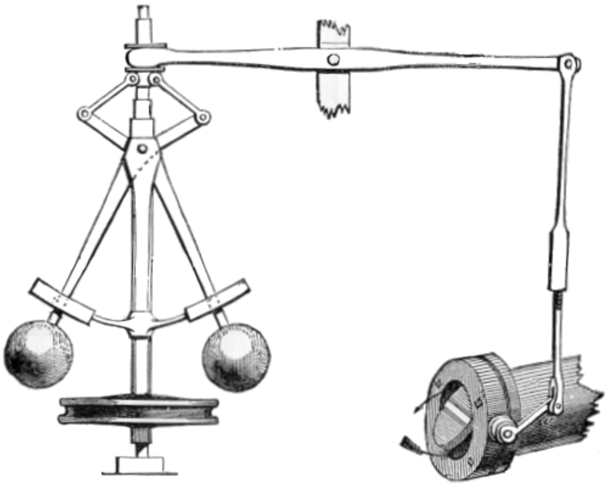


Figure 5.25: The ball governor, a mechanism originating from windmills and integrated into steam engines by James Watt.

Engraving by R. Routledge (1900, public domain)

Their success, unfortunately, would come much less from the technological innovations they brought than from the high-profile lawsuits they led to monetize them. Indeed, the two partners excelled in political connections and were at home in the peculiar world of patents and the royalties that result from them. The two Scotsmen in top hats, for example, received a percentage of the coal savings generated by the machines they sold across the country. And it would take nearly fifteen years before the legal possibility, in the United Kingdom, to use the "expansive power of steam" became finally open to everyone, a process deviously patented by the two partners!

Regardless, the *General Conference on Weights and Measures* assigned the unit watt to power in the SI system in 1960. It then dethroned the *horsepower* (hp)... a unit introduced by the very James Watt nearly a century earlier, while he was comparing his machines with the draft horses they were to replace.

$$\begin{aligned} 1\,\mathrm{hp}_{\mathrm{imperial}} &\equiv 33\,000\,\mathrm{ft\,lb_f\,min^{-1}} \\ &= 745.6999\,\mathrm{W} \end{aligned}$$

Problems

The properties of water are tabulated in Steam Tables 1, 2, and 3 (see Appendix A1 p. 307)

5.1 Temperature and Boiling Pressure

A student is traveling on a commercial flight and is served a hot drink by the cabin crew (figure 5.26); the drink is almost boiling. S/he measures the temperature to be 190.8 °F (88.2 °C).

1. What is the pressure in the cabin?

The aircraft undergoes rapid depressurization and the cabin pressure equalizes with the local atmospheric pressure (0.175 $\text{kg}_\text{f}/\text{cm}^2$ or 17.2 kPa). The student puts on their oxygen mask and unpleasantly notices that the drink, which is cooling down, has started to boil.

2. At what temperature will the boiling stop?

Figure 5.26: An aerial hot drink with an unidentified taste

Photo CC-BY *by Flickr User:notbrucelee*

5.2 Evaporation of Water

1. How much heat is needed to completely evaporate a pot of water (figure 5.27)? The container contains 2.5 L (0.66 US gal) of water at 50 °F (10 °C), and the ambient atmospheric pressure is 1 bar (14.5 psi).
2. Draw the process qualitatively (that is, without showing numerical values) on a temperature-volume diagram, showing the saturation curve.
3. The water is heated with an electric heating plate of 1500 W. How long does it take to vaporize the water, and what is the cost incurred by the experiment? The operator charges 0.15€ per kW h and the losses from the plate to the room are around 10 %.

Figure 5.27: An ordinary physics experiment

Photo CC-BY *by Indi Samarajiva*

5.3 Simple Recap Exercise

Describe very briefly an experiment where a fixed mass of subcooled liquid water is heated at constant temperature. Draw the process qualitatively on a pressure-volume diagram, showing the saturation curve.

5.4 High-Pressure Steam Generation

An industrial chemical process requires a steam flow rate of 2 $\text{kg}\,\text{s}^{-1}$ (4.41 lb/s) at 6 bar and 875 °C (87 psi and 1607 °F). The machine responsible for providing this steam is fed by a pressurized liquid water pipeline at 10 °C and 6 bar (50 °F and 87 psi).

1. What power in the form of work and heat is required to generate this steam flow?
2. Draw the process undergone by the water on a pressure-volume diagram, qualitatively (that is, without showing numerical values), showing the saturation curve.

5.5 Everything Depends On the Valve

A student decides to maintain a balanced diet, and to do so, cooks food in a pressure cooker (figure 5.28).

The valve of the pressure cooker is a small metal cylinder that is partly hollowed out, and sits freely on top of a small vertical exhaust pipe on the lid. The valve weighs 0.476 lb (216 g); it is placed on an exhaust pipe with a diameter of 5 mm (0.197 in). The ambient atmospheric pressure is 1.122 $\text{kg}_\text{f}/\text{cm}^2$ (1.1 bar).

1. At which temperature does the pressure cooker allow the student to food their cook?
2. What temperature and pressure could a person generate inside the pressure cooker by pressing on the valve? How could an accident be then prevented?

Figure 5.28: A pressure cooker, in which the increased pressure results in higher boiling temperature and thus faster cooking. It is affectionately known as a « *cocotte minute* » in France.

Photo CC-BY-SA *by Commons User:rama*

5.6 A First Steam Engine

An engineer conducts an experiment with water vapor, aiming to develop a very simple, small steam engine (figure 5.29).

S/he puts 2 L (0.44 imp gal) of liquid water at 20 °C into a large cylinder. The water is compressed to 2 bar by a piston.

S/he then heats the water, and the piston moves while maintaining constant pressure, until the volume reaches 300 L (66 imp gal).

1. Draw the process qualitatively (that is, without showing numerical values) on a pressure-volume diagram, showing the saturation curve.
2. What was the amount of work done?
3. How much heat had to be supplied?
4. What would be the transfers of work and heat if the expansion were continued until 4500 L (990 imp gal)?

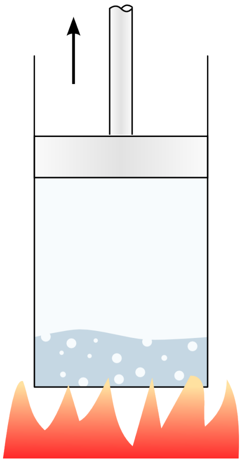


Figure 5.29: A very basic concept of a steam engine

Diagram CC-0 *by Olivier Cleynen*

5.7 Pumping Water

A pump is installed to draw liquid water at 5 °C located up from a lower tank (figure 5.30).

1. Up to what height Δz can the pumping be done?
2. How could we change the setup to pump water to a greater height?

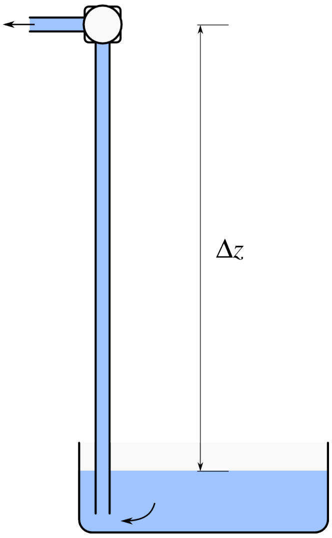


Figure 5.30: Water pumping from a reservoir located below. The first observation of the height limit calculated in this problem was made in 1630 by Giovanni Battista Baliani.

Sketch CC-0 *by Olivier Cleynen*

5.8 Steam Turbine On a Portable Power Plant

A company is developing a small steam power plant that can be carried in a standard-sized shipping container. Once connected to an external boiler, it is capable of converting heat from unrefined fuels (such as wood, paper, or coal) into electricity with a relatively high efficiency.

Within this power plant, the turbine is adiabatic and receives 5 t/h (11 023 lb/h) of steam at 90 bar and 510 °C from the boiler. The outlet pressure is barely above atmospheric pressure (we will take 1 bar). An engineer predicts, as we will also be able to do after chapter 8 (*entropy*), that the outlet specific internal energy of the steam will then be $2676.6\,\mathrm{kJ\,kg^{-1}}$.

The turbine is mechanically connected to a power generator with an efficiency of 85 %.

1. What is the power output of the generator?

At the other end of the container, an electric pump (the only other mechanical element in the plant) collects the condensed water in saturated liquid state (1 bar) and increases its pressure back to 90 bar to feed the boiler. It is assumed that during pumping, the specific volume of water varies negligibly, and that the compression is reversible.

2. What is the power required to power the pump?

5.9 The Crushed Barrel

In order to perform a physics demonstration, a group of students bring water to boil at ambient pressure in a steel drum. The drum's capacity is 55 US gal (208 L) and it is 34.6 in (88 cm) high.

The barrel is removed from the heat source and hermetically sealed. The purpose of the operation is to observe the barrel being crushed by the atmosphere due to the change of internal pressure as the contained water condenses.

1. Which pressure can be generated inside the barrel by letting it cool down?
2. What would be the vertical force exerted on the upper wall of the barrel at that point?

Some more challenging questions:

3. There are 5 L (1.32 US gal) of liquid left at the bottom of the barrel when the lid is closed. What is the steam dryness fraction?
4. How much steam has condensed during cooling?
5. How much heat had to be removed in order to reach the final pressure?

5.10 Newcomen Engine

In their time, around 1720, Newcomen engines (figure 5.23) were at the forefront of technology. Slightly overheated steam (1 bar, 250 °C) was injected into a large cylinder (height 1 m, diameter 1.5 m).

This steam was then cooled (by allowing a small amount of liquid water at atmospheric pressure and temperature to enter), maintaining the internal pressure at 0.1 bar. The piston would then descend, providing work.

The water available to the engine was at 1 bar, 10 °C.

1. Plot the process on a pressure-volume or temperature-volume diagram, showing the saturation curve.
2. How much heat must be supplied to fill the cylinder with steam before the descent can be performed?
3. How much work is supplied by the engine during the piston's descent?
   Hint: Consider the work done by the atmosphere on the outer face of the piston.
4. What is the efficiency of the engine, if friction and all other heat losses are neglected?

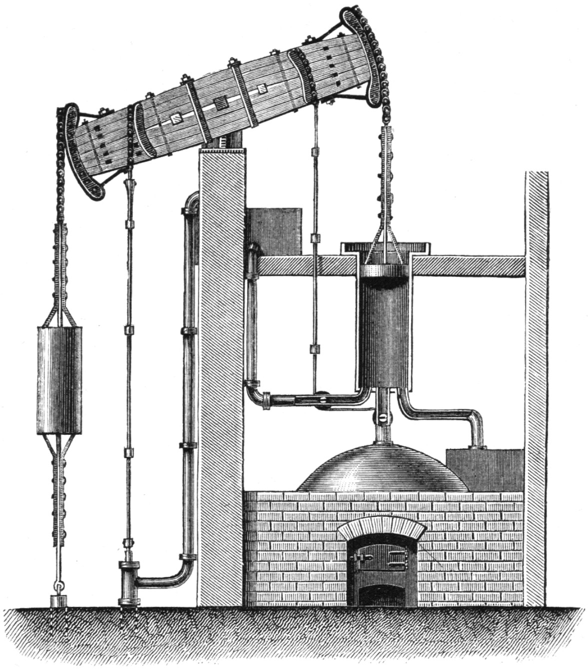


Figure 5.31: The ingenious atmospheric engine by Newcomen, the first true success in steam power.

Engraving C. L. Moll (1873, public domain)

5.11 Condenser of a Steam Power Plant

In a high-power electric power plant, the condenser is responsible for recovering water at the output of the turbines and removing energy from it, so that it may return to the liquid state and re-enter the pumps → boilers → turbines circuit. The water from the system (180 t/h) arrives at 0.5 bar with a specific volume of $3.1247\ \mathrm{m^3\ kg^{-1}}$; it must leave at the same pressure, in a saturated liquid state.

In order to extract heat from the water in the power plant, the condensers use a secondary water circuit directly from a river, where water is drawn at 10 °C.

In order to reduce the ecological impact of the power plant, the goal is to discharge the secondary water into the river at a temperature no higher than 35 °C.

1. What flow rate of secondary water should be drawn from the river?
2. In order to limit heat discharge into the river, where (and how) is the condenser heat also discharged in practice?

5.12 Aircraft Catapult On an Aircraft Carrier

An aircraft catapult is mounted on a military ship (figures 5.32 and 5.33). It consists of a steam reservoir connected to a long cylinder, in which a piston slides to propel the aircraft during takeoff.

At the beginning of the catapult launch, the steam is at 140 bar and 700 °C. After a brief run of 50 m, the aircraft has left the deck and the steam is at 4 bar and 410 °C.

1. How much energy did the catapult provide to the aircraft per kilogram of steam?
2. What must be the diameter of the piston and the total mass of steam, for the thrust provided to the aircraft to always exceed 2.5 $\mathrm{t_f}$ (5512 lbf)?

(And a question which we can not yet answer: what is the *maximum* amount of energy that the catapult could have supplied to the aircraft by allowing the steam to expand?)

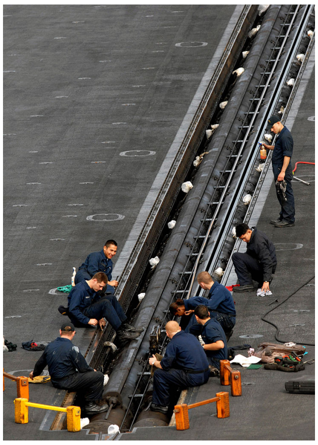


Figure 5.32: Cylinder of a steam catapult from the *USS Abraham Lincoln*
Photo by Geoffrey Lewis, U.S. Navy (public domain)

Figure 5.33: Piston of a steam catapult from the aircraft carrier *Charles de Gaulle*.
Photo CC-BY-SA *by Jean-Michel Roche, Netmarine.net*

5.13 Turbine of a Nuclear Power Plant

In a nuclear power plant, the electricity generator is driven by a steam turbine (figure 5.34).
Most of the steam (heated by the nuclear reactor) passes through the entire turbine. However, in the middle of the turbine, a steam bleed is carried out. It allows, on one hand, to heat the water in another part of the circuit (§9.4.5), and on the other hand, to precisely control the mass flow rate in circulation. The total flow rate at the inlet is 317 t/h of steam.

We measure the following steam properties:

Inlet:	120 bar; 565 °C
Extraction:	10 bar; 250 °C; 1.2 $\mathrm{kg\,s^{-1}}$
Outlet:	1 bar; 115 °C

What is the shaft power of the turbine?

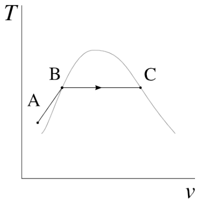
Figure 5.34: One of the turbines at the Russian Balakovo nuclear power station (~1 GW plant power), in maintenance (top) and during installation (bottom).

Photos 1 and 2 CC-BY-SA *The Centre of the Public Information Balakovo NPP*

Answers

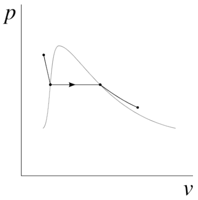
5.1 1) Interpolating between $T_{\text{sat.}} = 85\,^\circ\mathrm{C}$ and $T_{\text{sat.}} = 90\,^\circ\mathrm{C}$ in Steam Table 2, we obtain $p_{\text{sat.}} = 0.67\,\mathrm{bar}$
2) Interpolating between $p_{\text{sat.}} = 0.016\,\mathrm{MPa}$ and $p_{\text{sat.}} = 0.018\,\mathrm{MPa}$ in Steam Table 3, we obtain $T_{\text{sat.}} = 56.8\,^\circ\mathrm{C} = 134.2\,^\circ\mathrm{F}$ (yuck!)

5.2



1) To heat up the water until the boiling point (B) then complete evaporation (C): $Q_{\text{A}\to\text{C}} = m(q_{\text{A}\to\text{B}} + q_{\text{B}\to\text{C}}) = \frac{V_\text{A}}{v_\text{A}}(h_{L0.1\,\mathrm{MPa}} - h_\text{A} + h_{LV0.1\,\mathrm{MPa}}) = +6582\,\mathrm{kJ}$.
3) $\Delta t = \frac{Q_{\text{A}\to\text{C}}}{\dot{Q}_{\text{average}}} = 1\,\mathrm{h}\ 21\,\mathrm{min}$ with a cost of 0.29€.

5.3

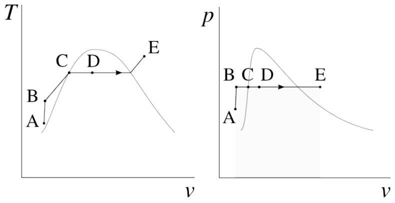


see §5.2.3 p. 123 & figure 5.9.

5.4 1) Reading in Steam Table 1 @ $p_\text{A} = 0.6\,\mathrm{MPa}$, $h_\text{A} = 42.6\,\mathrm{kJ\,kg^{-1}}$. Interpolating between 800 and 900 °C in this table, we obtain $h_\text{B} = 4336.6\,\mathrm{kJ\,kg^{-1}}$. Thus with equation 5/10, $\dot{Q}_{\text{A}\to\text{B}} = \dot{m}\Delta h = +8.59\,\mathrm{MW}$. $\dot{W}_{\text{A}\to\text{B}} = 0\,\mathrm{W}$ (5/9).

5.5 1) $p_{\text{inner}} = p_{\text{valve}} + p_{\text{atm.}} = \frac{F_{\text{weight}}}{S_{\text{outlet}}} + 1\,\mathrm{bar} = 2.0797\,\mathrm{bar}$. Interpolating in Steam Table 2, $T_{\text{sat.}p=2.0797\,\mathrm{bar}} = 121.37\,^\circ\mathrm{C} = 250.47\,^\circ\mathrm{F}$.

5.6



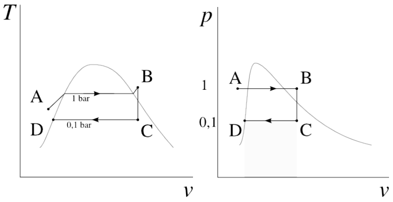
2) $W_{\text{A}\to\text{B}} \approx 0$ (5/11); with $m = \frac{V_\text{B}}{v_\text{B}}$ and $v_\text{D} = \frac{V_\text{D}}{m}$, we calculate $W_{\text{B}\to\text{D}} = -mp_{\text{cst.}}(v_\text{D} - v_\text{B}) = -59.6\,\mathrm{kJ}$ (5/7).
3) With v_D we calculate the dryness fraction $x_\text{D} \approx \frac{v_\text{D}}{v_{V0.2\,\mathrm{MPa}}} = 0.1697$. Thus, $Q_{\text{B}\to\text{D}} = m(h_\text{D} - h_\text{B}) = m(h_L + x_\text{D} h_{LV} - h_\text{B}) = +1585.2\,\mathrm{kJ}$ (5/8), so twenty-five times more…
4) The relationships are identical and yield $W_{\text{B}\to\text{E}} = -899.6\,\mathrm{kJ}$ and $Q_{\text{B}\to\text{E}} = +7694.3\,\mathrm{kJ}$ (the efficiency jumps from 3.8 to 11.7 %… there is a lead to follow here…)

5.7 1) Hydrostatic pressure in the pipe depends on height ($\Delta p = \rho g \Delta z$). When the pressure at the pump goes below $p_{\text{sat.}}$, the water starts boiling. For $\Delta p_{\text{boiling}} = 9.9127 \times 10^4\,\mathrm{Pa}$, $\Delta z_{\text{boiling}} = 10.1\,\mathrm{m}$.
2) One can do better than heating up the tank…

5.8 1) In A in Steam Table 1 we interpolate at 9 MPa between 500 and 600 °C to obtain $h_{510\,^\circ\mathrm{C}\&90\,\mathrm{bar}} = 3412\,\mathrm{kJ\,kg^{-1}}$. The same is done in B where $u_\text{B} > u_{V1\,\mathrm{bar}}$ ($h_\text{B} = 2899.4\,\mathrm{kJ\,kg^{-1}}$). Finally $\dot{W}_{\text{electrical}} = \eta_{\text{conversion}}\dot{m}\Delta h = -605.8\,\mathrm{kW}$.
2) $\dot{W}_{\text{pump}} = \dot{m}\int v\,\mathrm{d}p \approx \dot{m} v_L \Delta p = +12.9\,\mathrm{kW}$.

5.9 1) If we attain $T_\text{B} = 30\,^\circ\mathrm{C}$ with constant volume, then $p_{\text{inside min.}} = p_{\text{sat.}30\,^\circ\mathrm{C}} = 0.004\,247\,\mathrm{MPa}$. In that case, $\Delta p_{\max} = -9.575 \times 10^4\,\mathrm{Pa} = -13.89\,\mathrm{psi}$;
2) $F_{\max} = \Delta p_{\max} S_{\text{lid}} = -22.6\,\mathrm{kN} = 5081\,\mathrm{lb_f}$ (the barrel will of course be crushed before that);
3) $x_\text{A} = 0.024\,39$, $x_\text{B} = 1.288 \times 10^{-3}$;
4) $m_{\text{condensed}} = 0.113\,52\,\mathrm{kg} = 0.25\,\mathrm{lb}$;
5) $Q_{\text{A}\to\text{B}} = -1.669\,\mathrm{MJ}$.

5.10



2) Reading Steam Table 1 gives us $h_\text{A} = 42.1\,\mathrm{kJ\,kg^{-1}}$. Through interpolation we obtain $h_\text{B} = 2975\,\mathrm{kJ\,kg^{-1}}$ and $m = \frac{V_\text{B}}{v_\text{B}} = 0.7354\,\mathrm{kg}$. Thus we calculate $Q_{\text{A}\to\text{B}} = m\Delta h = +2154\,\mathrm{kJ}$ (5/8).
3) While calculating m we already obtained $v_\text{B} = 2.406\,\mathrm{m^3\,kg^{-1}}$. Thus $W_{\text{piston}\to\text{shaft}} = W_{\text{atm.}\to\text{piston}} + W_{\text{piston}\to\text{steam}} = m(p_\text{C} - p_{\text{ext.}})(v_\text{D} - v_\text{C}) = -159\,\mathrm{kJ}$.
4) $\eta \equiv \frac{Q_{\text{supplied}}}{W_{\text{useful}}} = 7.38\,\%$ (realistic value).

5.11

1) $x_\text{A} = 96.44\,\%$ & $x_\text{B} = 0$; thus $\dot{Q}_{\text{A}\to\text{B}} = -111.13\,\mathrm{MW}$, which is the reason why we need

$\dot{m}_{\text{secondary}} \geq 1062.4\,\text{kg}\,\text{s}^{-1}$ or $2342.2\,\text{lb/s}$.
2) Into the atmosphere, through the secondary water which is released through the large towers…

5.12

1) $\Delta u = -433.1\,\text{kJ}\,\text{kg}^{-1}$ ($= w_{\text{A}\to\text{B}}$ if we suppose that the process is adiabatic);
2) $D_{\text{min.}} = 32.26\,\text{cm}$ (care must be taken to take atmospheric pressure into account); $m = \frac{V_{\text{max}} - V_{\text{min}}}{v_{\text{max}} - v_{\text{min}}} = 5.243\,\text{kg}$.
Nota: the data given in this problem is hypothetical, for lack of reliable published data.

5.13

$\dot{W}_{\text{turbine}} = -71\,\text{MW}$.

CHAPTER 6

Thermodynamic Cycles

or

A Foray into the Sorcery of Heat Mechanics

Chapter 6 – Thermodynamic Cycles

Executive summary for chapter 6

When heating a compressed fluid, it provides more work upon expansion than was spent during compression. By expanding a fluid, its temperature drops and thus we can absorb heat from a relatively “cooler” body. With these two processes, we transform heat into work and vice versa.

Introduction

Thanks to chapters 4 and 5, we have learned to quantify energy in fluids based on their properties. We will now use these fluids in machines to convert heat into work, and work into heat. In this chapter 6 (*thermodynamic cycles*), we aim to formalize the concept of a cycle, addressing two questions:

- How do engines, refrigerators, and heat pumps work?
- How is their efficiency quantified?

6.1 Graphical Conventions

We start by agreeing on some graphical and notation conventions, which are summarized in figure 6.1.

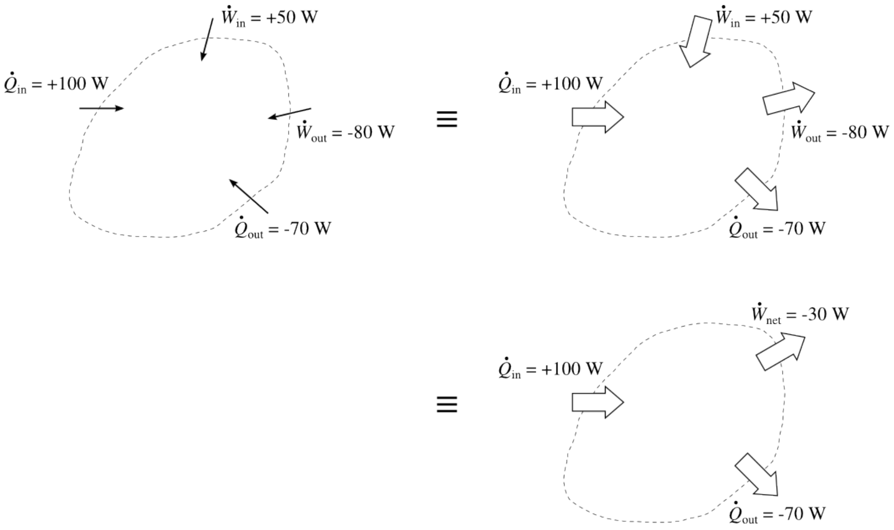


Figure 6.1: New graphical and notation conventions for energy transfers. The white arrows are oriented according to the physical direction of the transfers; The algebraic sum of all the work received and carried out is represented by a single transfer named *net work*.

Diagram CC-0 *Olivier Cleynen*

We use large white arrows to represent *the physical direction of transfers*. We do not change our sign convention (transfers are positive when directed toward the system and negative when coming from it), but only the graphical convention for their orientation, in order to make the visualization of transfers in machines more intuitive.

The algebraic sum of the work W_{in} received by a machine and W_{out} it supplies is named the *net work* W_{net}. The net work can be positive (done onto the the machine from external sources) or negative (done by the machine to an external receiver), depending on the application.

$$\begin{aligned} W_{net} &\equiv W_{in} + W_{out} \\ \dot{W}_{net} &\equiv \dot{W}_{in} + \dot{W}_{out} \\ w_{net} &\equiv w_{in} + w_{out} \end{aligned} \tag{6/1}$$

We define the *net heat* in the same way:

$$
\begin{aligned}
Q_{\text{net}} &\equiv Q_{\text{in}} + Q_{\text{out}} \\
\dot{Q}_{\text{net}} &\equiv \dot{Q}_{\text{in}} + \dot{Q}_{\text{out}} \\
q_{\text{net}} &\equiv q_{\text{in}} + q_{\text{out}} \qquad (6/2)
\end{aligned}
$$

Therefore, for example, a car's engine sees a positive net heat transfer (reception) and a negative net work (supplied to the gearbox).

6.2 Transforming Heat and Work

6.2.1 Building thermodynamic cycles

We want to compare different ways of transforming work and heat. For these comparisons to be valid, we must always take into account *all* the processes undergone by the fluid until it returns to its initial state.

For example, it is easy to cool a room with a compressed air bottle (simply make the fluid work during its expansion to lower its temperature); but if we want to continuously cool the room, then we also need to consider the energy required to *return* the air to the bottle, at its initial pressure and temperature, at the end of the process.

A second example is that of a car engine, which releases heat carried by the exhaust gases. In order to account for this lost energy, we count the heat that would need to be removed from the gases in order to bring them back to the engine's inlet temperature. This imaginary cooling takes place outside the engine in practice, but from a thermodynamic standpoint, it is an integral part of the energy transformation process.

Therefore, every time we analyze the operation of a thermodynamic machine, we will make sure to continue the fluid processes until it returns to its initial state (same temperature, same pressure, same internal energy, etc.). We then say that it has completed a *thermodynamic cycle* (§2.3).

6.2.2 Producing work with heat

Let us start by compressing a fluid: we increase its pressure and reduce its specific volume, which requires a certain amount of work. After that, we heat up this fluid: its pressure and volume tend to increase. By expanding the fluid back to its initial pressure, we will recover more work than what we initially invested. Finally, in order to bring the fluid back to its initial state, it needs to be cooled down.

In the end, the fluid has done more work when it was expended than was done onto it when it was compressed. Over a cycle, it will thus have *produced* work and *absorbed* heat (part of which it transformed). This is the operating principle of an engine.

There are infinitely many possible cycles to perform this process, but they all involve at least four energy transfers: compression, heating, expansion, and cooling. We can separate these processes in space, as shown in figure 6.3, or in time, as illustrated in figure 6.2. Depending on technological and practical constraints, some of these transfers may be performed simultaneously.

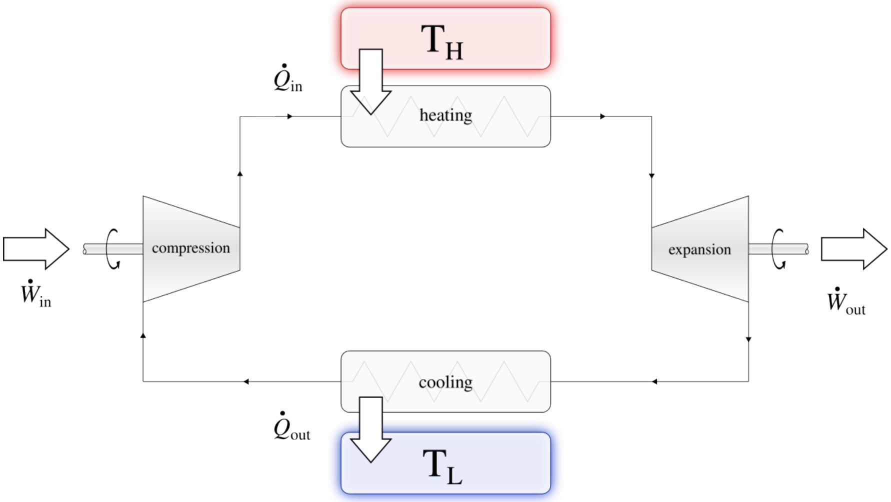


Figure 6.2: Engine thermodynamic cycle. The fluid absorbs heat supplied at high temperature T_H. The compression power is lower than the expansion power: the net power in the form of work $\dot{W}_{net} = \dot{W}_{in} + \dot{W}_{out}$ is negative.

Diagram CC-0 *Olivier Cleynen*

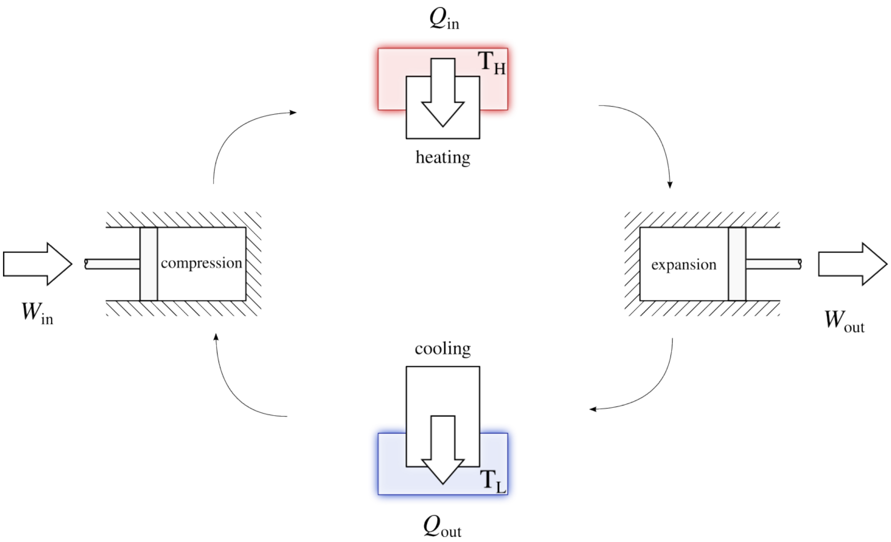


Figure 6.3: Engine thermodynamic cycle carried out by separating the stages in time (rather than in space as shown in figure 6.2). The fluid is heated by a high-temperature heat source T_H. The net work $W_{net} = W_{in} + W_{out}$ is negative.

Diagram CC-0 *Olivier Cleynen*

It is possible to mechanically link the sections that receive and supply energy in the form of work. In the case where the fluid circulates continuously, the compressor and the turbine can be connected by the same shaft, as shown in figure 6.4. In the case where the processes are separated in time, such as in an internal combustion engine, the processes can be linked by performing multiple offset cycles simultaneously (with multiple cylinders) or by storing energy in a flywheel. The engine then does not receive external work, and the resulting output is a power $\dot{W}_{net}$.

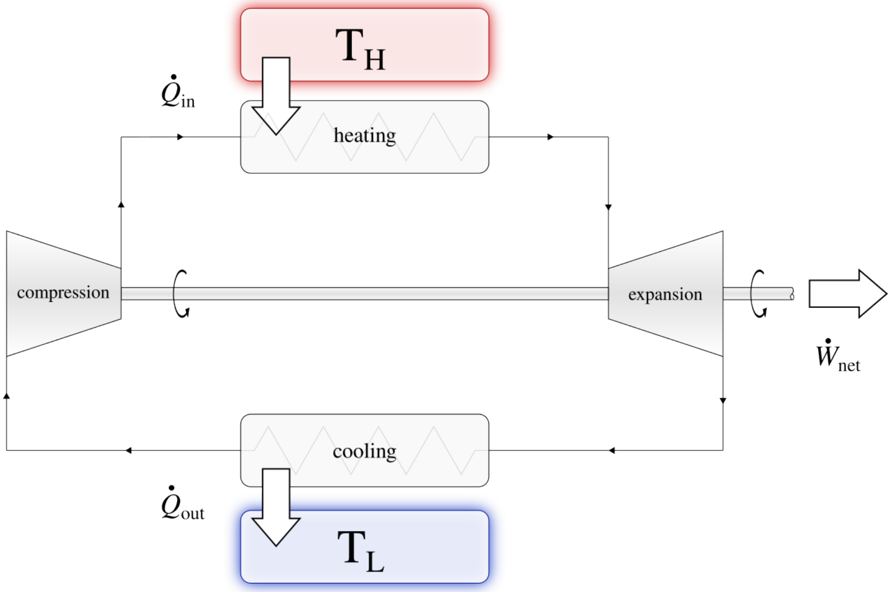


Figure 6.4: An engine thermodynamic cycle in which the compressor and the turbine are mechanically coupled. Since the turbine supplies a power $\dot{W}_{out}$ greater than that absorbed by the compressor ($\dot{W}_{in}$), it is able to not only drive the compressor but also to provide an excess $\dot{W}_{net}$ sent outside of the engine.

Diagram CC-0 *Olivier Cleynen*

6.2.3 Extracting heat with work

When work is done on a fluid, its temperature tends to rise (with a brief exception for liquid/vapors between their saturation points) and it can thus supply heat to a body that was initially at a higher temperature ("hotter") than itself.

Conversely, when a fluid is expanded, its temperature tends to decrease and it can thus absorb heat from a body that was initially "colder" than itself.

By performing these steps one after the other, we obtain a *refrigeration cycle*: a machine capable of extracting heat at low temperature and rejecting it at high temperature. Such a cycle is depicted in figures 6.6 (stages separated in space) and 6.5 (stages separated in time).

Careful examination of these two figures will reveal a major surprise: they are exactly the same arrangement as for an engine! The only difference lies in the operating temperatures. The temperature reached during compression must be **higher than the high temperature** T_H, and the temperature reached during expansion must be **lower than the low temperature** T_L. Unless these conditions are met, the heat transfers will occur in the wrong direction.

In a refrigeration cycle, the fluid has a larger volume when compressed (after having being heated) than when expanded (after having being cooled): this time, the compression requires more power than the expansion. The net power $\dot{W}_{net}$ in the form of work is therefore positive, meaning that the machine must be powered by an external source of work.

In practice in refrigeration systems, a trick is often used to lower the temperature: instead of a turbine, a simple valve (sometimes called a *throttling valve*) is used. In this component without moving parts, the fluid does not do work (therefore the power to be supplied to the machine is increased),

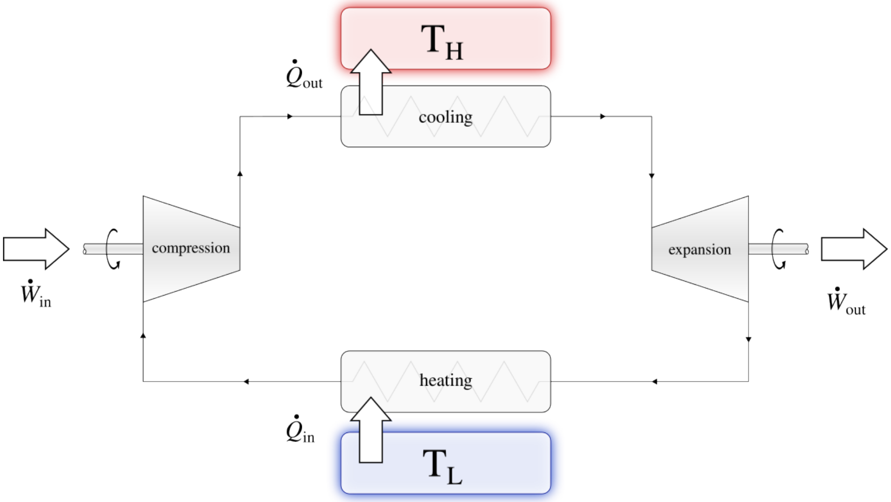


Figure 6.5: A refrigeration cycle, for use in refrigerators, air conditioners, and heat pumps.
A power $\dot{Q}_{in}$ in the form of heat is absorbed at low temperature (the fluid is then heated) while a power $\dot{Q}_{out}$ is rejected at high temperature (the fluid is then cooled).

Diagram CC-0 *Olivier Cleynen*

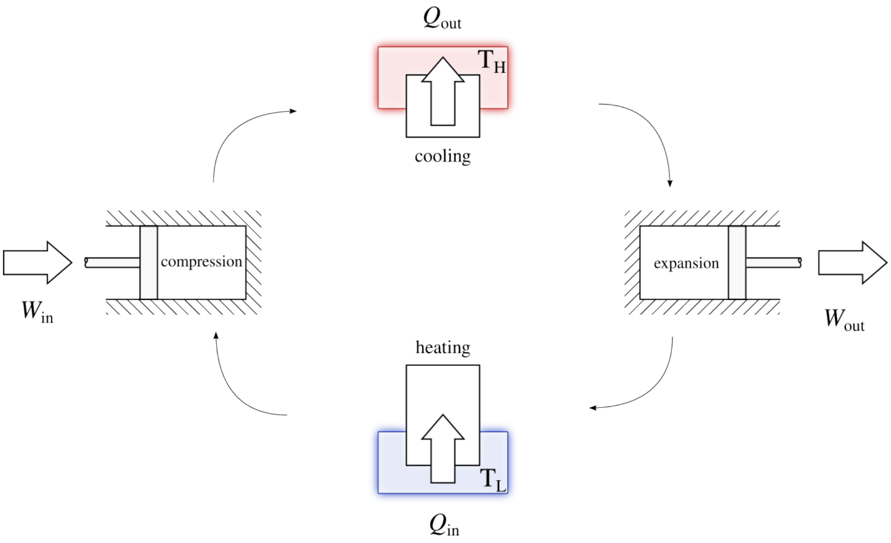


Figure 6.6: A refrigeration cycle carried out by separating the stages in time (rather than in space as shown in figure 6.5)

Diagram CC-0 *Olivier Cleynen*

but it is much simpler to manufacture and use. This modification is depicted in figure 6.7.

The throttle valve, in thermodynamic terms, allows for a completely irreversible expansion, increasing the volume and reducing the pressure without extracting work. If a perfect gas were used, this would have no effect on the temperature (as in the experiments of Joule and Gay-Lussac studied in §4.3.2 p. 90) and therefore no interest; but when liquids/vapors are used, the throttling expansion is a technologically simple way to lower the temperature.

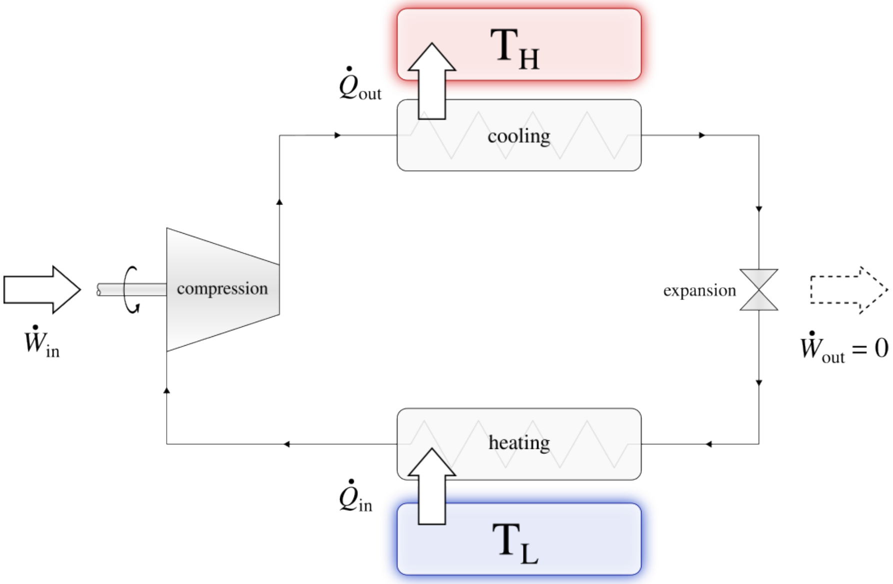


Figure 6.7: A modified refrigeration cycle using a throttling valve. When using liquids/vapors, it is possible to avoid extracting work during expansion. The use of a simple valve is sufficient to lower the temperature of the fluid.

Diagram CC-0 *Olivier Cleynen*

Refrigeration cycles have two main types of applications:

Heat pumps (figure 6.8) are arranged to reject heat to a high-temperature body, most often a building;

Refrigerators and air conditioners (figure 6.9) are arranged to extract heat from a low-temperature body (a cold enclosure).

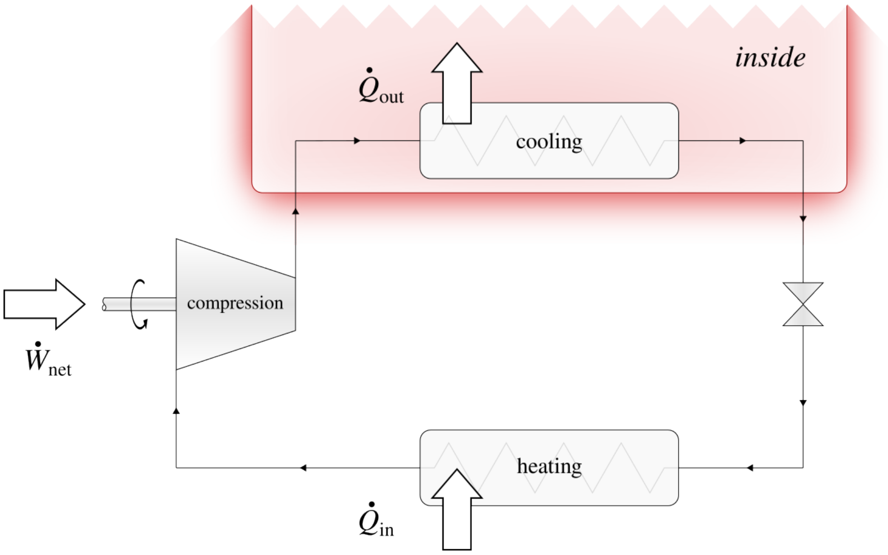


Figure 6.8: Arrangement of a heat pump. The machine is configured to reject heat inside (where the temperature is higher) which it extracts from outside (where the temperature is lower).

Diagram CC-0 *Olivier Cleynen*

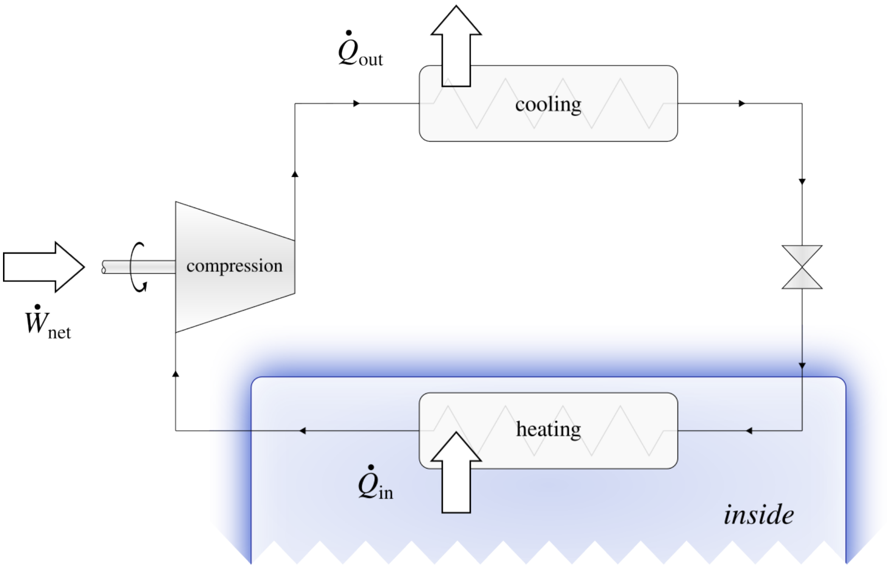


Figure 6.9: Arrangement of a refrigerator or air conditioner. The machine is configured to reject heat outside (where the temperature is higher) which it extracts from the inside (where the temperature is lower). This is exactly the same machine as in figure 6.8.

Diagram CC-0 *Olivier Cleynen*

In these two types of applications, it is exactly the same machine, operating with the same cycle. The only difference concerns the internal/external arrangement of the components: a heat pump is nothing more than a refrigerator positioned to "cool the outside".

The similarity between an air conditioner and a heat pump allows these two functions to be performed by a single machine, which is then referred to as *reversible* in the industry. In thermodynamics, the word "reversible" has a different meaning, as we will see in chapter 7 (*the second law*), and so we will call this kind of machine *bidirectional* here. Depending on the needs, the direction of fluid flow is reversed, which causes the inversion of heat transfers. This type of machine is shown in figure 6.10.

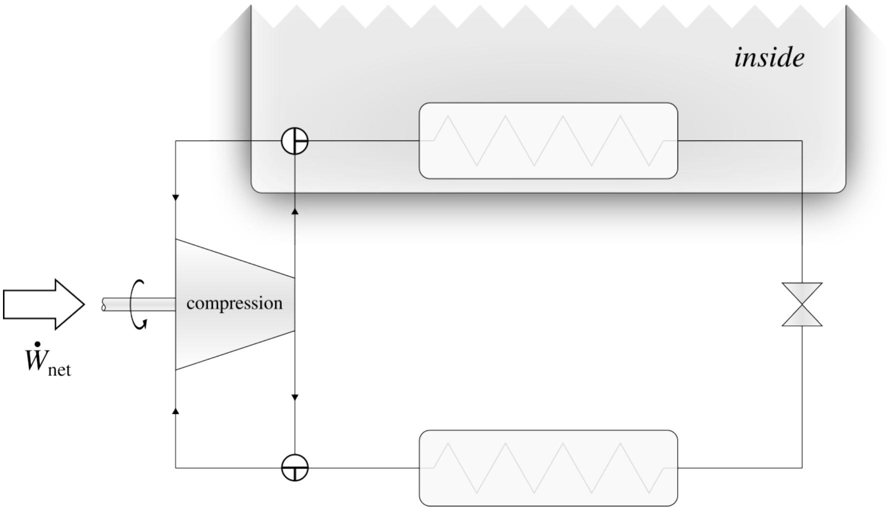


Figure 6.10: Layout of a bidirectional (commonly called "reversible") air conditioner. By rotating both valves 90 degrees counterclockwise, the function changes from a heat pump to an air conditioner.

Diagram CC-0 *Olivier Cleynen*

6.3 Cycle Efficiency

6.3.1 A basic formula

The *efficiency*[1] η of a thermodynamic machine compares the useful transfer or transformation it performs with the energy cost it incurs. We will adopt the following principle definition:

$$\eta \equiv \left|\frac{\text{useful transfer}}{\text{energy expenditure}}\right| \tag{6/3}$$

By convention, the efficiency is always expressed as a positive number; thus we use an absolute value in equation 6/3. For each of the three types of thermodynamic machine, we will define and quantify this “useful transfer” and this “energy expenditure”.

6.3.2 Efficiency of an engine

The function of a thermal engine, like those found on board road vehicles or in power plants, is to supply work, meaning a negative quantity $\dot{W}_{\text{net}}$ (figure 6.11). The expense incurred to generate this work is the heat it receives, namely the quantity $\dot{Q}_{\text{in}}$ (usually originating from the combustion of fuel or the fission of atomic nuclei).

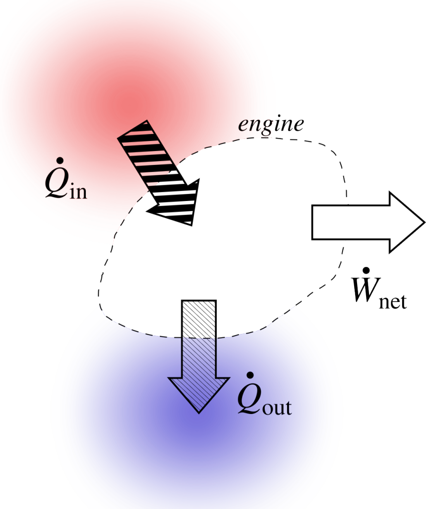


Figure 6.11: Energy transfers associated with an engine. We aim to obtain a large transfer $\dot{W}_{\text{net}}$ (result) from the transfer $\dot{Q}_{\text{in}}$ (cost). The rejection $\dot{Q}_{\text{out}}$ is undesirable.

Diagram CC-0 *Olivier Cleynen*

According to definition 6/3, the efficiency η_{engine} of the thermal engine is therefore:

$$\eta_{\text{engine}} \equiv \left|\frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}}\right| \tag{6/4}$$

---

[1] Some authors make a distinction between the efficiency η defined in 6/3 and a *relative efficiency* or *effectiveness* $\Phi \equiv \frac{\eta_{\text{real}}}{\eta_{\text{theoretical}}}$ comparing the efficiency achieved in practice with the maximum achievable efficiency by the machine in theory. It is then necessary to carefully define the assumptions associated with the calculation of the maximum efficiency.

Example 6.1

A car engine receives 100 kW in the form of heat from gasoline combustion; it supplies 55 kW as work at the transmission shaft. What is its efficiency?

The efficiency is $\eta_{\text{engine}} = \left|\frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}}\right| = \left|\frac{-55\times10^3}{+100\times10^3}\right| = 0.55 = 55\,\%$.

☞ This engine rejects $\dot{Q}_{\text{out}} = -\dot{W}_{\text{net}} - \dot{Q}_{\text{in}} = -(-55\times10^3) - 100\times10^3 = -45\,\text{kW}$ as heat. Most of this energy is carried away with the exhaust gases.

☞ Since one must always supply at least as much heat $\dot{Q}_{\text{in}}$ as the engine outputs in work $\dot{W}_{\text{net}}$, the efficiency of an engine will always necessarily be less than 1.

The net power $\dot{W}_{\text{net}}$ in the form of work can be expressed in terms of other energy transfers, as follows:

$$\dot{W}_{\text{net}} = \dot{W}_{\text{in}} + \dot{W}_{\text{out}} = -\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}$$

$$\eta_{\text{engine}} = 1 - \left|\frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}}\right| \tag{6/5}$$

This equation 6/5 will be very useful in the next chapter (§7.5.1 p. 197), where we will want to relate the heat transfers $\dot{Q}_{\text{in}}$ and $\dot{Q}_{\text{out}}$ to the temperatures at which they occur.

6.3.3 Efficiency of a refrigerator or air conditioner

The function of a refrigerator or an air conditioner is to extract heat, and thus to generate a positive power $\dot{Q}_{\text{in}}$ (heat extracted every second from the compartment to be cooled). This transfer (figure 6.12) is made possible by supplying work to the refrigerator, $\dot{W}_{\text{net}}$, an “expense” that must be positive, too.

According to the definition 6/3, the efficiency of a refrigerator or an air conditioner, also called the *coefficient of performance* $\text{COP}_{\text{refrigeration}}$, is therefore:

$$\eta_{\text{refrigerator}} = \eta_{\text{air conditioner}} \equiv \left|\frac{\dot{Q}_{\text{in}}}{\dot{W}_{\text{net}}}\right| \tag{6/6}$$

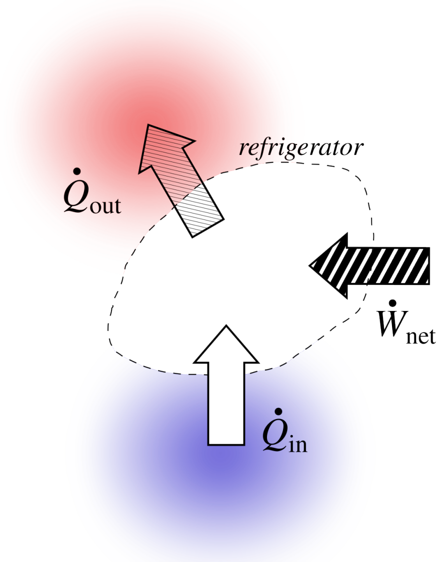


Figure 6.12: Energy transfers associated with a refrigerator or an air conditioner. We aim to obtain a large transfer $\dot{Q}_\text{in}$ (result) from the transfer $\dot{W}_\text{net}$ (cost).

Diagram CC-0 *Olivier Cleynen*

Example 6.2

A refrigerator receives an electrical power of 100 W; it extracts heat from the cold chamber with a power of 120 W. What is its efficiency?

The efficiency (or COP) is $\eta_\text{refrigerator} = \left|\frac{\dot{Q}_\text{in}}{\dot{W}_\text{net}}\right| = \left|\frac{+120}{+100}\right| = 1.2 = 120\,\%$.

☞ This refrigerator rejects $\dot{Q}_\text{out} = -\dot{W}_\text{net} - \dot{Q}_\text{in} = -100 - 120 = -220\,\text{W}$ outside of the cold chamber (usually, within the building itself).

☞ Household refrigerators and air conditioners often have efficiencies greater than 1, but depending on the desired temperatures, the efficiency can indeed be lower.

In order to prepare for the next chapter (§7.5.2 p. 198), and while paying attention to the pitfalls associated with the use of absolute values, we can express this efficiency in terms of heat transfers only:

$$\eta_\text{refrigerator} = \eta_\text{air conditioner} = \frac{1}{\left|\frac{\dot{Q}_\text{out}}{\dot{Q}_\text{in}}\right| - 1} \tag{6/7}$$

6.3.4 Efficiency of a heat pump

A heat pump operates exactly the same way as an air conditioner. Its function is to generate a transfer $\dot{Q}_\text{out}$ to the "hot" section (usually inside a building). This transfer, represented in figure 6.13, is made possible by supplying work to the heat pump, $\dot{W}_\text{net}$, an "expense" that is necessarily positive.

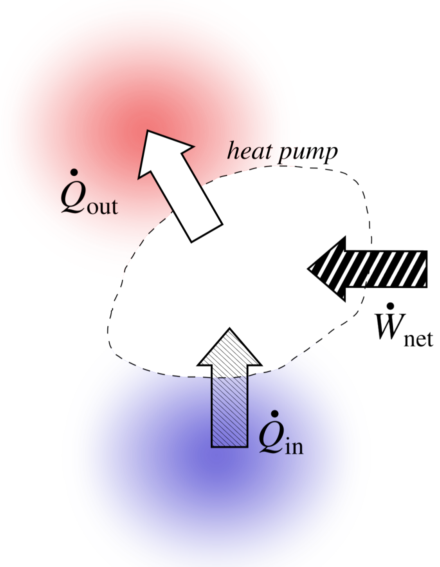


Figure 6.13: Energy transfers associated with a heat pump. We aim to obtain a large transfer $\dot{Q}_{\text{out}}$ (result) from the transfer $\dot{W}_{\text{net}}$ (cost).

Diagram CC-0 *Olivier Cleynen*

The efficiency $\eta_{\text{heat pump}}$ of the heat pump, also known as the *coefficient of performance* $\text{COP}_{\text{heat pump}}$, is thus defined as:

$$\eta_{\text{heat pump}} \equiv \left| \frac{\dot{Q}_{\text{out}}}{\dot{W}_{\text{net}}} \right| \tag{6/8}$$

Example 6.3

A heat pump receives an electrical power of 100 W; it heats the inside of a room with a power of 350 W. What is its efficiency?

The efficiency is $\eta_{\text{heat pump}} = \left|\frac{\dot{Q}_{\text{out}}}{\dot{W}_{\text{net}}}\right| = \left|\frac{-350}{+100}\right| = 3.5$.

☞ The heat pump rejects more energy as heat than it receives as work – that is its whole purpose. If the COP were equal to or less than 1, it would be more economical and much simpler to use an electric heater instead.

Just as we did for the previous sections, we can express this efficiency in terms of heat flows only:

$$\eta_{\text{heat pump}} = \frac{1}{1 - \left|\frac{\dot{Q}_{\text{in}}}{\dot{Q}_{\text{out}}}\right|} \tag{6/9}$$

6.3.5 On the low performance of machines

« The question has often been agitated as to whether the motory power of heat is limited, or whether it is boundless; whether the possible improvements of fire engines have an assignable limit, a limit which the nature of things prevents from being surpassed by any means whatsoever, or whether on the contrary these improvements are capable of indefinite extension… »

Sadi Carnot, 1824 [4]

In all the cases we have studied above, for each cycle, we have included an undesirable transfer. In the engine cycle, some of the energy is wasted in the form of heat rejection ($\dot{Q}_{\text{out}}$). In refrigeration cycles, work must be supplied ($\dot{W}_{\text{in}}$) to carry out a heat transfer that *a priori* could have seemed "free" ($\dot{Q}_{\text{out}}$ then being equal to $\dot{Q}_{\text{in}}$).

Engineering students will certainly be indignant about the role played by these losses in this chapter – and about the modest efficiencies achieved by the machines described in the examples. Why are the efficiencies calculated in the examples and in the following problems so low, and more importantly, how can we design cycles with greater efficiency? We take these worries to our heart, and will address them in chapter 7 (*the second law*).

A Bit of History: the Number of Strokes

*

Historically, we have learned to transform heat into work by manipulating fixed quantities of fluid trapped in enclaves. These enclaves have always been of cylindrical geometry, which greatly facilitates the manufacture of pistons that exploit the changes in volume of the fluid in order to extract work from it.

When the fluid is water, heat is supplied in a boiler and the steam is then transferred to the cylinder or cylinders in order to be expanded (§9.4). However, at the end of the 19th century, air began to be used, a fluid inside which heat could be directly generated by combustion. The complex process of conducting a separate combustion to heat a metallic boiler which then finally heats the water of the engine was eliminated, along with all the heat and temperature losses it entailed. This was the birth of the *internal combustion engine*, the development of which was notably driven by the German engineers Nicolaus Otto and Rudolf Diesel (§10.3). Combustion and expansion were now carried out in the same place, directly in the cylinder.

However, in order to achieve internal combustion, a new problem had to be solved: since the oxygen in the air is used, the combustion can only be carried out once. After each combustion, it is therefore essential to expel the reaction products (CO_2 and H_2O primarily) from the cylinder and to reintroduce “fresh” air containing the oxygen O_2 necessary for breaking the hydrocarbon molecules C_xH_y that produce heat. Two different solutions were then adopted.

The most common method dedicates one piston movement to each step: the first one for compression, the second one for expansion (after or during combustion), the third one for exhaust gases expulsion (exhaust), and the last one for admitting fresh air (intake). Engines following this process are called *four-stroke* (figure 6.14), and they have always been the most widely used.

The second method is bound to offend purists: it performs these four operations in *two strokes* only. In these engines, part of the gas expansion is used to perform the exhaust, and it is carried out simultaneously with the air intake (figures 6.15 and 6.16). Certainly, none of the four steps can be optimally performed: the compression and expansion phases are only carried out over a portion of the stroke, and the scavenging is necessarily incomplete due to the mixing of fresh and exhaust gases. On the other hand, the combustions are twice as frequent, since there is no need for intake and exhaust strokes during which no thermodynamic operation takes place. Thus, for the same displacement and speed of rotation, two-stroke engines are much more powerful than their four-stroke counterparts, even though they are also significantly less efficient.

Two- and four-stroke engines were developed at the same time, but two-stroke engine development really took off after the Second World War. The refinement by the German engineer Walter Kaaden of an ingenious exhaust system, whose geometry alone increases the airflow escaping during expan-

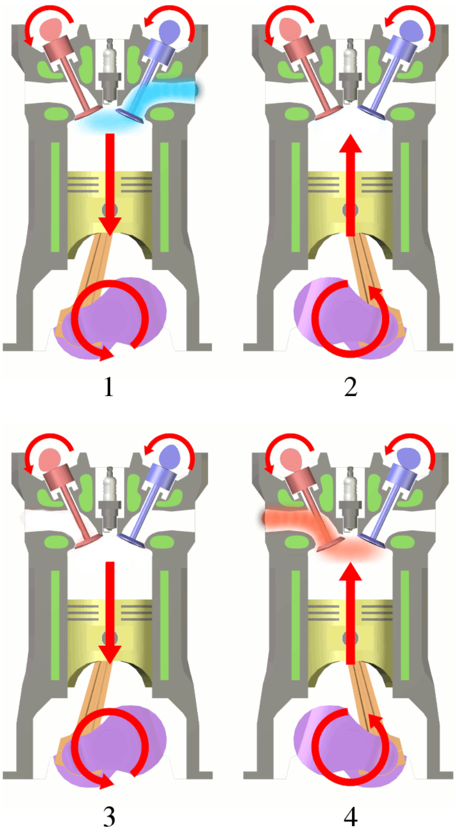


Figure 6.14: The four steps of a four-stroke engine. The piston descends to admit fresh air coming from the right (*intake* stroke, 1); it then rises to increase the air temperature (*compression* stroke, 2); useful work production occurs during a descent (*power* stroke, 3); finally, the air is expelled to the outside during a fourth and last movement (*exhaust* stroke, 4) before starting the cycle again. Most current piston-cylinder engines follow this process.

Diagrams 1 2 3 4 CC-by *by Eric Pierce*

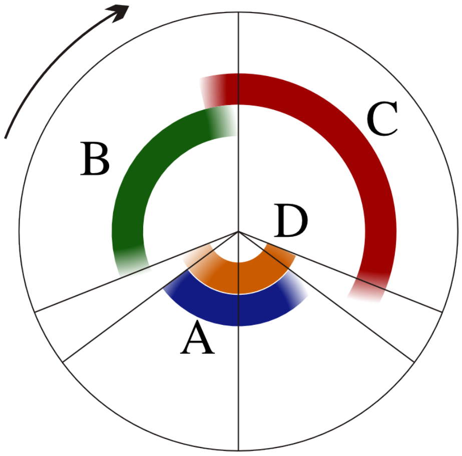


Figure 6.15: Cycle of a two-stroke engine. The four necessary steps for operation are carried out in a single revolution of the crankshaft, that is, two piston movements. Intake (A) occurs during the passage at bottom dead center, compression (B) starts late, and expansion (C) is interrupted to allow scavenging (D) when the piston approaches bottom dead center again.

Diagram derived from a diagram CC-BY-SA *by Commons User:A7N8X*

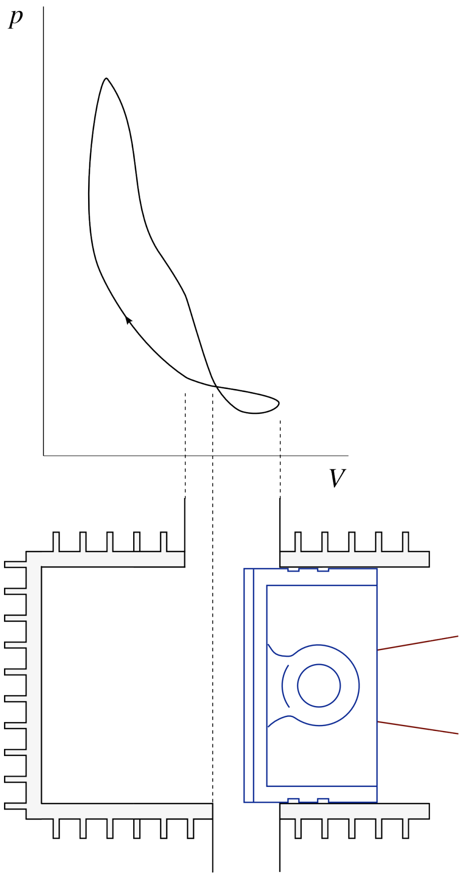


Figure 6.16: Schematic pressure-volume diagram of the cylinder of a two-stroke engine with crankcase intake. It is left to the student to determine which of the two ports (high or low) corresponds to intake and exhaust in the cylinder.

Diagram CC-BY-SA *by Commons User:Terraflorin & Olivier Cleynen*

sions and reduces it during compressions, made the engine very competitive in racing motorcycles; this *tuned expansion chamber* (figure 6.17) was adopted in many production models.

In parallel, the ideas formulated by the English entrepreneur Joseph Day at the end of the 19th century on the mechanism controlling intake spread widely. With his ingenious *crankcase intake*, it is the piston itself that serves as a valve (figure 6.18). The intake air first passes through the crankcase where the crankshaft rotates, then it is slightly compressed by the piston in its downward movement before entering the cylinder. The engine thus operates without any moving valve; lubrication can even be provided simply by injecting oil directly into the intake air.

With these two advantages, the engine found its application wherever constraints of weight, volume, acquisition cost, and maintenance took precedence over efficiency. After powering three million *Trabant* cars in East Germany, it was adopted on nearly all outdoor portable tools (chainsaws, lawnmowers, etc.). The engine can easily be miniaturized, leaving room for legs on a scooter motorcycle, allowing snowmobiles to start easily, in short, until the 90s, nothing —not even homeowner associations!— seemed to be able to halt its progress.

However, at the beginning of the 21st century, it becomes necessary to give up on these advantages. One can wearily accept the irritating sound emitted by the two-stroke engine, but its polluting emissions are staggering. Lubrication by oil injection into the intake air causes the atmospheric discharge of smoke, odors, and harmful particles. In addition, the always incomplete scavenging of the cylinder

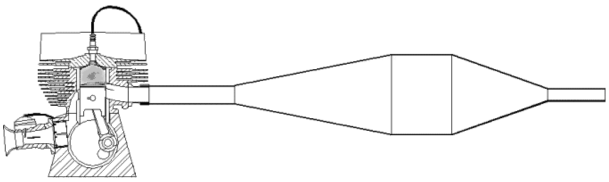


Figure 6.17: Tuned (sometimes called "harmonic") expansion chamber mounted on a two-stroke engine. Since the flow is unsteady, it is possible to manipulate the pressure exerted by fixed amounts of exhaust gases on the exhaust port when they pass through the chamber. Passing through the expansive part reduces the pressure (thus facilitating scavenging during the piston descent), while passage through the contraction, on the contrary, increases this pressure (thus reducing gas losses during the piston ascent).

Diagram CC-BY-SA *by Achim Agster*

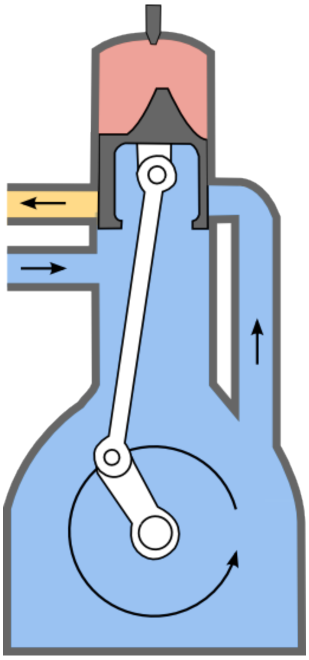


Figure 6.18: Crankcase intake system. The intake air, laden with fuel for combustion and oil for lubricating mechanical parts, first enters the crankcase. It is compressed and then inserted into the cylinder with the sole downward movement of the piston. There is no need for any valve or flap.

Diagram public domain by Commons User:Tomeq183

greatly limits the efficiency of combustion and the thermal efficiency. The tightening of regulations controlling emissions gradually forces the replacement of these engines with four-stroke ones or electric systems — whose batteries are often charged with energy from power plants... powered by steam engines. We see that seemingly minor technological decisions can sometimes have consequences on a global scale!

Problems

The properties of water are tabulated in Steam Tables 1, 2, and 3 (see Appendix A1 p. 307)

Air is considered an ideal gas.

$c_{v\,(\text{air})} = 718\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$ $R_{\text{air}} = 287\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$

$c_{p\,(\text{air})} = 1005\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$ $\gamma_{\text{air}} = 1.4$

We assume that for a reversible adiabatic process (without heat transfer and infinitely slow), the properties of air are linked according to the following three relationships:

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{v_2}{v_1}\right)^{\gamma-1} \tag{4/36}$$

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \tag{4/37}$$

$$\left(\frac{p_1}{p_2}\right) = \left(\frac{v_2}{v_1}\right)^{\gamma} \tag{4/38}$$

6.1 Engine Efficiency

The Diesel engine of an excavator has an efficiency of 40 % and delivers a continuous power of 60 kW (approximately 80 hp). It is powered by fuel with a calorific value of 35 MJ kg^{-1}.

1. What is the hourly fuel consumption of the machine?
2. What is the power rejected as heat in the exhaust pipe?

6.2 Refrigerator Efficiency

A refrigerator with a COP of 1.2 must extract 100 kJ from food placed in the cold chamber. How much electrical energy must be provided for this? How much heat will it have rejected at the end of the cooling process?

6.3 Heat Pump Efficiency

A heat pump with a COP of 3.1 provides a power of 4000 W to an apartment. What is the electrical power required? What is the power absorbed from the atmosphere?

6.4 Thermodynamics Party

A group of students, made thirsty by an endless thermodynamics class, prepares for the weekend by placing ten six-packs of bottles containing a beverage containing mostly mineral water in the refrigerator (figure 6.19).

An experiment conducted on a bottle shows that it is made of 172 g of glass with a specific thermal capacity of 0.75 kJ kg^{-1} K^{-1}, and that it contains 25 cL (8.45 fl oz) of liquid with a specific thermal capacity of 4.2 kJ kg^{-1} K^{-1}.

When they are inserted in the refrigerator, the packs are at room temperature (19 °C or 66.2 °F). Four hours later, they have reached a temperature of 5 °C (41 °F).

Figure 6.19: A pack of six bottles containing a liquid used to drown the exasperation resulting from the study of thermodynamics

Photo CC-BY *Moritz Barcelona*

The refrigerator has an efficiency of 95 %. The imperfectly insulated walls of the refrigerator absorb heat from the room at a rate of 10 W.

1. How much electrical energy did the refrigerator receive during these four hours?

The local electricity grid operator applies a tariff of 0.15 €/(kW h).

2. What is the financial cost of the cooling performed?
3. Has the room where the refrigerator is stored cooled down or warmed up?
4. Will the room cool down if the refrigerator door is left open?

6.5 Operation of a Heat Pump

Describe the path followed by the fluid inside a heat pump, indicating the direction of heat flows and the location (inside/outside) of the different components.

Why do we let the fluid expand in a valve instead of using a turbine that could supply work?

(One can also practice by focusing on the cycles and configurations of an air conditioner, a refrigerator, or an engine: which part is heated and where?)

6.6 Algebra

Show, starting from the definition of the efficiency of an air conditioner, that it can be expressed as the relation:

$$\eta_{\text{air conditioner}} = \frac{1}{\left|\frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}}\right| - 1} \tag{6/7}$$

(This demonstration will be useful in the next chapter (§7.5). One can also practice by attacking in the same way equations eq. 6/5 p. 161 and eq. 6/9 p. 163. Pay attention to absolute values!)

6.7 Cooling of a Wind Tunnel

The cryogenic wind tunnel ETW (for *European Transonic Windtunnel*, figure 6.20) allows the circulation of nitrogen in a closed circuit to observe flows around models. It allows reaching Mach 0.8 at 4 bar and −200 °C (58 psi and −328 °F), using a 50 MW fan.

The walls of the wind tunnel are highly insulated, so that its thermal transfers with the outside are negligible compared to other energy transfers. The nitrogen cooling system has a COP of 0.8.

When the wind tunnel is operating at full capacity, what power as work is received by the cooling system? What power is rejected as heat into the atmosphere?

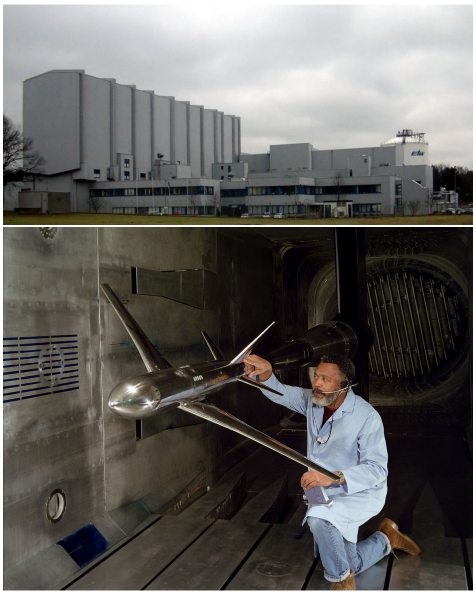


Figure 6.20: Buildings of the ETW (European Transonic Windtunnel) in Cologne and test section of the National Transonic Facility of NASA, of similar size and capabilities.

Photo 1 CC-BY-SA *by Commons User:Dantor*
Photo 2 by Fred Jones (NASA, public domain)

6.8 Electricity Generation With a Gas Turbine

A gas turbine (in English the term *gas turbine* can be used to describe the engine as a whole) is set up to operate an electric generator (figure 6.21); it operates with a flow rate of 0.5 kg s^{-1} of atmospheric air.

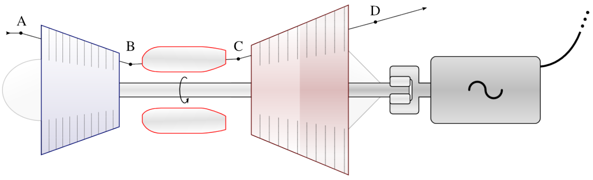


Figure 6.21: A static gas turbine engine, in this configuration named *turboshaft*, powering an electric generator. The gas is typically expanded (in the turbine, between C and D) to atmospheric pressure.

Diagram CC-BY-SA *Olivier Cleynen*

- The air enters the machine at 20 °C (68 °F) and 1 bar; it is compressed (A → B) to 30 bar in the compressor.
- The air then receives heat through combustion, at constant pressure (B → C), until its temperature reaches 1000 °C.
- Finally, the air is expanded in a turbine (C → D) until it reaches atmospheric pressure and is discharged outside.

The compressor is mechanically powered by the turbine, and the shaft connecting them also drives the electric current generator.

In order to quantify the maximum efficiency that could be achieved by the machine, we consider that the compressor and the turbine are reversible adiabatic (meaning that compression and expansion occur very slowly and without heat transfer).

1. Draw the process undergone by the air during one cycle on a pressure-volume diagram, qualitatively (that is, without showing numerical values).
2. At what temperature does the air exit the compressor?
3. What is the power of the compressor?
4. At what temperature is the air rejected into the atmosphere? What power is rejected as heat into the atmosphere?
5. What is the efficiency of the machine?
6. How do the four energy transfers of this theoretical machine compare to those of a real machine, where the compressor and the turbine cannot be reversible?

6.9 Cycle of a Steam Engine

In a steam power plant, water circulates continuously through four components:

- A quasi-adiabatic pump where water enters as saturated liquid and its pressure is raised from 0.5 bar to 40 bar;
- A boiler where its temperature is raised to 650 °C at constant pressure;
- A quasi-adiabatic turbine that allows the water to return to 0.5 bar while expending energy as work;
- A condenser that cools the water at constant pressure (0.5 bar) until its return to the pump.

We accept the following assumptions:

- At the turbine outlet, the steam is at a temperature of 110 °C (after chapter 8, we will be able to predict this outlet temperature);
- The compression in the pump is reversible, and the density of water does not vary as it passes through it.

1. Draw the process undergone by the water on a pressure-volume diagram, qualitatively (that is, without showing numerical values) and showing the saturation curve.
2. What is the efficiency of the engine?
3. What would happen if, in order to eliminate heat rejection, we removed the condenser by connecting the pump inlet directly to the turbine outlet?

6.10 Industrial Refrigeration

A supermarket chain is seeking your expertise to assess the profitability of an ambitious project to renew a fleet of refrigerators.

All supermarkets in the company use the same model of refrigerator. Its efficiency is 100 %.

You travel to a representative supermarket, which allows you to take measurements and quantify the thermal transfers of the building. You highlight that:

- The power absorbed as heat by the cold chambers of the refrigerators, averaged over the year, is 80 kW.
- In the winter, the building loses heat with an average power of 400 kW. It is heated with a heat pump unit of COP 4.
- In the summer, the building absorbs heat with an average power of 160 kW. It is cooled with an air conditioning unit of COP 0.9.
- During autumn and spring, the heating/cooling needs are almost negligible.

The company is considering replacing its entire fleet of refrigerators with a model of 220 % efficiency, which requires a significant investment. It has a total of 100 supermarkets and pays 0.15 € per kW h on average for electricity.

What would be the annual financial savings generated by changing the refrigerator model?

6.11 Operation of an AC Unit

An air conditioner operates according to the circuit schematized in figure 6.22. The fluid used in the circuit is air (in practice, fluids that liquefy and evaporate within the machine are often used instead, but the operating principle remains the same).

The air inside the circuit circulates in a steady flow. Both the compressor and the turbine are adiabatic and we consider them to be reversible. Heat transfers occur at constant pressure.

When the air conditioner is started, the outside temperature and the inside temperature are both 86 °F (30 °C).

The temperatures of the air inside the circuit are T_A = 68 °F = 20 °C, T_B = 140 °F = 60 °C, and T_C = 104 °F = 40 °C.

1. Draw the process qualitatively (that is, without showing numerical values) on a pressure-volume diagram; and indicate the work and heat transfers.
2. What is the pressure ratio between A and B?
3. What is the temperature of the air at point D?
4. Calculate the specific powers for each of the four energy transfers, and thus calculate the efficiency of the air conditioner.
5. The owner wants to obtain a flow of cool air at 53.6 °F (12 °C) with a flow rate of 529.7 CFM (0.25 $m^3 s^{-1}$). What electrical power must be supplied to the air conditioner for this purpose?

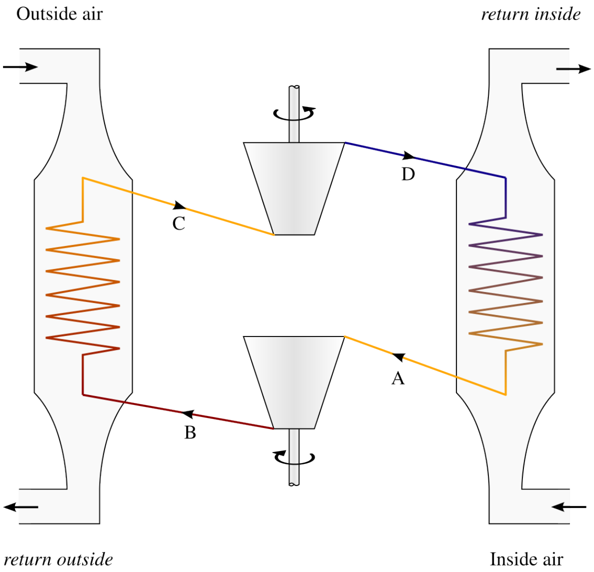


Figure 6.22: Schematic diagram of an air conditioner. The air in the air conditioner circuit circulates in a steady flow (A → B → C → D → A), without ever leaving the machine.

Diagram CC-BY-SA *Olivier Cleynen*

6. What will be the minimum flow rate of outside air to be circulated in the outdoor section of the air conditioner?
7. During the winter, the owner wants to modify the air conditioner to turn it into a heat pump. Describe (qualitatively) a modification of the circuit for this purpose, and draw the process undergone by the air in the circuit on a new pressure-volume diagram, indicating the energy transfers.

6.12 Conditioning Pack of an Aircraft

A “conditioning pack” or simply “air pack” is a thermodynamic machine used in commercial aircraft to pressurize the fuselage and to maintain the temperature in the cockpit, cabin, and cargo holds at a comfortable level regardless of external conditions.

The packs (often called ECS or ACM, for Environment Control System and Air Cycle Machine) are often placed around the wing box in the unpressurized area of aircraft (figure 6.27). An interesting feature of their operation is that it is the air from the thermodynamic circuit itself that is inserted into the cabin for the passengers.

Here we study the heating and air conditioning functions of a conditioning pack: for this purpose, we model its operation in a simplified manner.

The air intended for the cabin begins its journey at the inlet of the jet engines of the airplane (except when the airplane, on the ground, is connected to a source of conditioned or pressurized air). In the compressor of one of these engines, its pressure is multiplied by 5 during an approximately adiabatic and reversible process; then it is led to the pack.

As it enters the pack, this air passes through a heat exchanger where it loses heat (figure 6.24). This heat is extracted by a separate air flow, called RAM: it is air from outside at atmospheric conditions, extracted and discharged under the fuselage. The cabin air and RAM air circuits are at very different pressures and are never mixed.

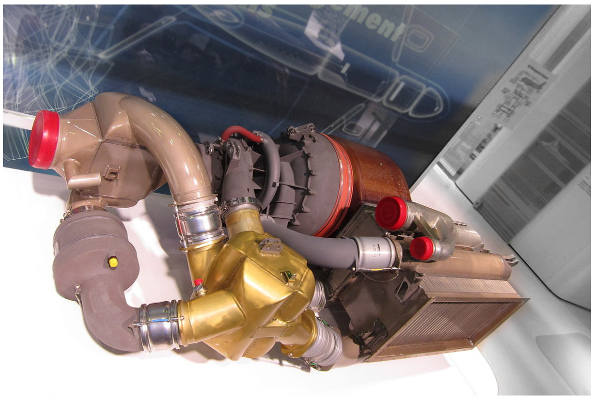


Figure 6.23: A ECS intended for a Comac C919, approximately 1.5 m in length.

Photo CC-BY-SA *Olivier Cleynen*

After passing through the heat exchanger, the air can follow three distinct circuits in the conditioning pack before reaching the cabin:

Circuit A is used in cold weather when one wants to raise or maintain the cabin at a higher temperature than the outside temperature;

Circuit B is used in moderate weather when the cabin needs to be kept at a temperature close to the outside temperature;

Circuit C is used in hot weather when there is a high demand for cabin air cooling.

The pack automatically controls the flow of outside air (RAM) and selects the circuit for the air intended for the cabin to follow, in order to bring its temperature to the value requested by the crew in the cockpit (figure 6.28).

Circuit A: heating in cold weather

In circuit A, the air intended for the cabin is simply expanded in a valve (figure 6.25) before being inserted into the cabin.

In the valve, the pressure drops sharply and the specific volume increases; however, no work or heat transfer occurs. This is a process known as a “Joule & Gay-

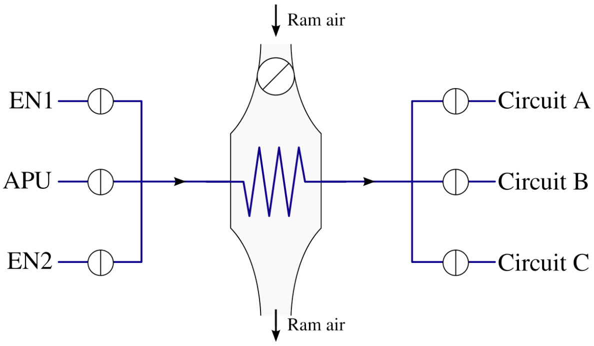


Figure 6.24: Diagram representing the air arriving in the conditioning pack from the engines (EN1 and EN2) or the auxiliary power unit (APU) on the left. This air exits into one of the three circuits A, B, or C after losing heat to the RAM air.

Diagram CC-0 *Olivier Cleynen*

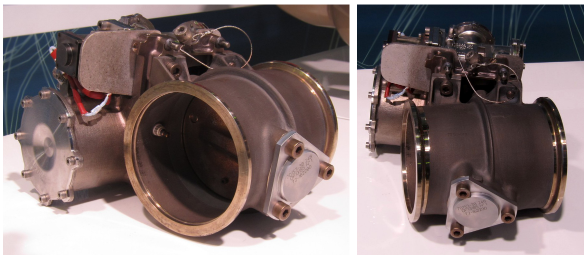


Figure 6.25: Airflow regulation valve of an ECS intended for a Comac C919.

Photos 1 & 2 CC-0 *Olivier Cleynen*

Lussac expansion" (see §4.3.2 p. 90). The process is entirely irreversible.

When the airplane is on the ground in cold weather conditions (−35 °C, 1 bar or −31 °F and 14.5 psi):

1. What is the *maximum* temperature of the air that the pack can supply to the cabin?
(for this, we will fully close the outside air RAM circuit).
2. Draw the process qualitatively on a pressure-volume diagram.
3. What is the minimum flow rate of outside air required to circulate in the RAM circuit in order to bring 0.5 kg/s of conditioned air at 24 °C (75 °F) into the cabin?
4. Draw the process that the conditioned air would undergo on the pressure-volume diagram above.

Circuit B: cabin air conditioning in moderate weather

In practice, it is possible to lower the temperature of the air intended for the cabin with a much lower RAM air flow. For this reason, when cooling needs are significant, the air intended for the cabin goes through circuit B. It is then expanded using a turbine to the cabin pressure (1 bar). We assume that the turbine is ideal (reversible adiabatic expansion).

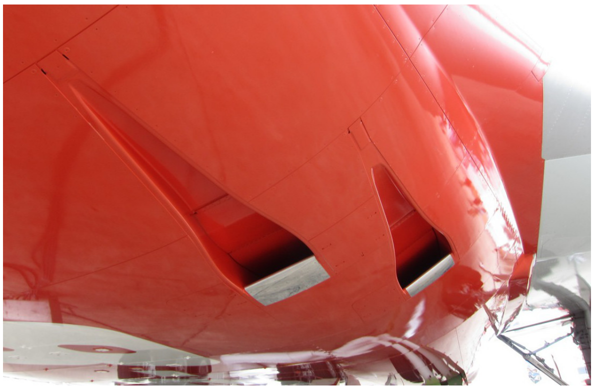


Figure 6.26: The air intakes of the RAM circuit at the wing box of a Boeing 747-8I.

Photo CC-BY-SA *Olivier Cleynen*

Figure 6.27: Air pack positioned at the wing root of a Sukhoi SuperJet SSJ100

Photo CC-BY-SA *A.Katranzhi*

When the external conditions are 20 °C, 1 bar (68 °F and 14.5 psi)):

5. At what temperature will the air enter the cabin if the RAM circuit is closed?
6. How much energy will be extracted from the air by the turbine?
7. Draw the process undergone by the air on a pressure-volume diagram, qualitatively.
8. What is the *minimum* temperature to which the circuit can bring the air intended for the cabin?
9. How much energy will be extracted from the air by the turbine in that case?
10. Draw the process on the pressure-volume diagram above.

Circuit C: cabin air conditioning in hot weather

When the aircraft is on the ground in very hot weather conditions (45 °C or 113 °F, 1 bar), the air intended for the cabin goes through circuit C.

Upon passing through the heat exchanger, its temperature only drops to 217 °C (422.6 °F).

It is then compressed in a compressor (reversible adiabatic process) to 20 bar (290.1 psi).

It then passes again through a heat exchanger crossed by the RAM air circuit.

Finally, it is expanded in a turbine (in practice, this is the turbine used in circuit B) down to atmospheric pressure, and then fed into the cabin. We model the expansion as a reversible adiabatic process.

11. Make a sketch of the path followed by the air through the conditioner and draw the process qualitatively on a pressure-volume diagram.
12. At what temperature should the air be brought to in the second heat exchanger, before expansion, to obtain an air flow at 5 °C (41 °F) in the cabin?
13. What is the work that the pack receives or supplies to operate in this case?

Conclusion

14. What is the COP of the heating system generated with circuit A in question 3?
15. What is the COP of the air conditioning performed with circuit C in question 12?

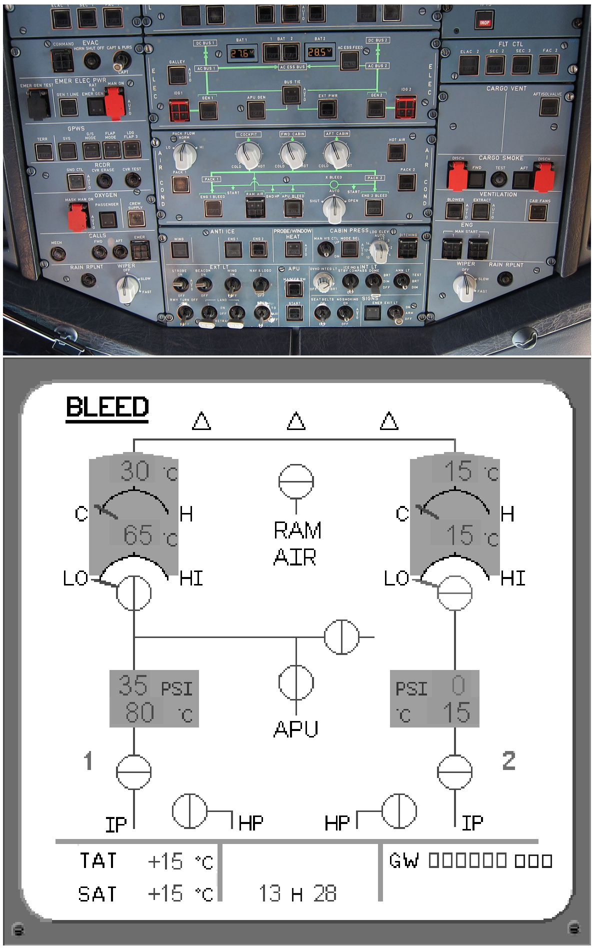


Figure 6.28: Control interface of the ECS in the center of the upper panel of the cockpit of an Airbus A320 and the corresponding EFIS display panel.

Photo CC-BY-SA *Olivier Cleynen*

Answers

6.1 1) $\dot{Q}_{\text{in}} = \frac{-\dot{W}_{\text{net}}}{\eta_{\text{engine}}} = \frac{-(60\times10^3)}{0.4} = +150\,\text{kW}$; thus $\dot{m} = \frac{\dot{Q}_{\text{in}}}{c_{\text{fuel}}} = 15.4\,\text{kg}\,\text{h}^{-1}$ (about 12 liters or 3 US gallons per hour);
2) $\dot{Q}_{\text{out}} = -\dot{Q}_{\text{in}} - \dot{W}_{\text{net}} = -90\,\text{kW}$.

6.2 $W_{\text{net}} = \frac{Q_{\text{in}}}{\eta_{\text{refrigeration}}} = +83.3\,\text{kJ}$; $Q_{\text{out}} = -Q_{\text{in}} - W_{\text{net}} = -183.3\,\text{kJ}$.

6.3 $\dot{W}_{\text{net}} = \frac{-\dot{Q}_{\text{out}}}{\eta_{\text{heat pump}}} = \frac{-(-4\times10^3)}{3.1} = +1.29\,\text{kW}$; $\dot{Q}_{\text{in}} = -\dot{W}_{\text{net}} - \dot{Q}_{\text{out}} = +2.71\,\text{kW}$.

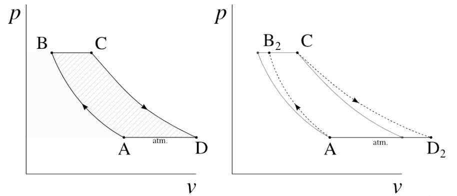
6.4 1) If we assume that the density of the liquid is equal to that of liquid water ($\rho_{\text{liquid}} = 10^3\,\text{kg}\,\text{m}^{-3}$), the heat Q_{in} absorbed by the refrigerator is $Q_{\text{in}} = -Q_{\text{glass}} - Q_{\text{liquid}} - Q_{\text{walls}} = -n_{\text{bottles}}(m_{\text{glass}}c_{\text{glass}} + m_{\text{liquid}}c_{\text{liquid}})(\Delta T)_{\text{packs}} - \dot{Q}_{\text{walls}}\Delta t = -60(0.172 \times 0.75\times10^3 + 0.25\times4.2\times10^3)\times(5-19) - 10\times4\times3600 = +1134.4\,\text{kJ}$. Thus, $W_{\text{net}} = \frac{Q_{\text{in}}}{\eta_{\text{refrigeration}}} = +1194.1\,\text{kJ}$.
2) $W_{\text{net}} = +1194.1\,\text{kJ} = +1194.1\,\text{kW}\,\text{s} = +0.332\,\text{kW}\,\text{h}$. Thus the cost adds up to 0.05€ (!).
3) The room will be heated up by the heat rejection $Q_{\text{out}} = -Q_{\text{in}} - W_{\text{net}} = -2.329\,\text{MJ}$.
4) Opening the door only increases the heat Q_{in} that needs to be extracted from the cold chamber, which will consequently increase Q_{out} and the warming of the room (with a net power $\dot{Q}_{\text{net}} = \dot{Q}_{\text{in}} + \dot{Q}_{\text{out}} = -\dot{W}_{\text{net}}$).

6.5 See §6.2.3 p. 156, and in particular figures 6.6, 6.7 and 6.8.

6.6 $\eta_{\text{conditioner}} \equiv \left|\frac{\dot{Q}_{\text{in}}}{\dot{W}_{\text{net}}}\right| = \frac{\dot{Q}_{\text{in}}}{\dot{W}_{\text{net}}} = \frac{\dot{Q}_{\text{in}}}{-\dot{Q}_{\text{in}}-\dot{Q}_{\text{out}}} = \frac{1}{-1-\frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}}}$. Now, by definition $\dot{Q}_{\text{out}} < 0$ and $\dot{Q}_{\text{in}} > 0$; thus $\frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} = -\left|\frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}}\right|$. We thus have $\eta_{\text{conditioner}} = \frac{1}{-1+\left|\frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}}\right|}$.
It is now your turn, with equations 6/5 et 6/9!

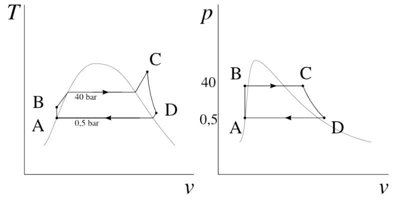
6.7 The 50 MW expended by the fan are entirely dissipated as friction in the wind tunnel, and thus converted into heat that must be removed if we want to maintain a constant temperature. Therefore, $\dot{W}_{\text{net}} = \frac{\dot{Q}_{\text{in}}}{\eta_{\text{refrigeration}}} = +62.5\,\text{MW}$ (quite a refrigerator…). It follows that $\dot{Q}_{\text{out}} = -\dot{Q}_{\text{in}} - \dot{W}_{\text{net}} = -112.5\,\text{MW}$.

6.8



2) With equation 4/37, $T_{\text{B}} = T_{\text{A}}\left(\frac{p_{\text{B}}}{p_{\text{B}}}\right)^{\frac{\gamma-1}{\gamma}} = 774.7\,\text{K}$;
3) With equations 3/14 and 4/14, $\dot{W}_{\text{A}\to\text{B}} = \dot{m}c_p(T_{\text{B}} - T_{\text{A}}) = +242\,\text{kW}$;
4) With equation 4/37, $T_{\text{D}} = T_{\text{C}}\left(\frac{p_{\text{D}}}{p_{\text{C}}}\right)^{\frac{\gamma-1}{\gamma}} = T_{\text{C}}\left(\frac{p_{\text{B}}}{p_{\text{A}}}\right)^{-\frac{\gamma-1}{\gamma}} = 481.8\,\text{K}$. Therefore, the rejected air must lose $\dot{Q}_{\text{D}\to\text{A}} = c_p\Delta T = -94.8\,\text{kW}$ in order to return to its initial state (§6.2.1);
5) $\eta_{\text{engine}} \equiv \left|\frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}}\right| = \left|\frac{\dot{W}_{\text{A}\to\text{B}}+\dot{W}_{\text{C}\to\text{D}}}{\dot{Q}_{\text{B}\to\text{C}}}\right| = 62.3\,\%$ (quite honorable, only attainable with perfect turbine and compressor);
6) With a real compressor $\dot{W}_{\text{A}\to\text{B}_2} > \dot{W}_{\text{A}\to\text{B}}$ and $T_{\text{B}_2} > T_{\text{B}}$. It follows that if T_{C} is kept constant, $\dot{Q}_{\text{B}\to\text{C}_2} < \dot{Q}_{\text{B}\to\text{C}}$. Nevertheless, we still have $\dot{W}_{\text{C}\to\text{D}_2} < \dot{W}_{\text{C}\to\text{D}}$ and $T_{\text{D}_2} > T_{\text{D}}$ in the turbine. The power $\dot{W}_{\text{net}}$ decreases, the rejection $\dot{Q}_{\text{out}}$ increases. We will see in chapter 7 (*the second law*) that the efficiency also decreases.
Note: We had already studied this engine in problems 1.9 p. 29 and especially 3.2 p. 75. Our ability to analyze and quantify performance improves each time…

6.9



1) To assist in constructing these diagrams, one can review figures 5.7 p. 123 and 5.9 p. 124, as well as section §5.4 p. 133.
2) $h_{\text{A}} = h_{L0.5\,\text{bar}}$; $h_{\text{B}} = h_{\text{A}} + \int_{\text{A}}^{\text{B}} v\,\text{d}p = h_{\text{A}} + v_L\Delta p$ (5/13 & 5/14 with $q_{\text{A}\to\text{B}} = 0$); $h_{\text{C}} = h_{650\,^\circ\text{C}\&4\,\text{MPa}}$; $h_{\text{C}} = h_{110\,^\circ\text{C}\&0.05\,\text{MPa}}$; With these data, we easily calculate $\eta_{\text{engine}} \equiv \left|\frac{w_{\text{net}}}{q_{\text{in}}}\right| = -\frac{w_{\text{net}}}{q_{\text{in}}} = -\frac{w_{\text{turbine}}+w_{\text{pump}}}{q_{\text{boiler}}} = -\frac{(h_{\text{D}}-h_{\text{C}})+(h_{\text{B}}-h_{\text{A}})}{h_{\text{C}}-h_{\text{B}}} = 31.47\,\%$ (interesting insofar as any type of fuel can be used).
3) In this case, the pump would (almost) force the vapor to follow the path from D to C, bringing it back to a temperature of 650 °C and making it impossible to add heat in the boiler. We would

then effectively have a machine made up of a turbine and a pump exchanging water: it would be impossible to generate work this way…

6.10

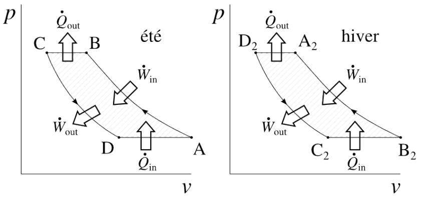
In a supermarket, both in spring and autumn, the energy savings provided by the new refrigerators amount to 43.6 kW.

In summer, the heat to be absorbed by the air conditioners is reduced. Therefore, an additional saving of 48.5 kW in electricity can be added at the level of the air conditioners.

In winter, the heat required by the heat pumps is increased. Therefore, an additional expense of 10.9 kW must be added at the level of the heat pumps.

In the end, this represents an annual savings of 1.672×10^{12} J, or 69.7 k€ per supermarket, which must be compared to the required investment and the costs incurred (including the call for your expertise…).

6.11



2) With equation 4/37, $\frac{p_B}{p_A} = \left(\frac{T_B}{T_A}\right)^{\frac{\gamma}{\gamma-1}} = 1.565$;

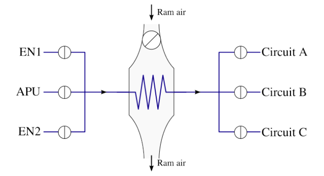
3) Idem, expansion C → D is reversible adiabatic, $T_D = T_C \left(\frac{p_D}{p_C}\right)^{\frac{\gamma-1}{\gamma}} = T_C \left(\frac{p_A}{p_B}\right)^{\frac{\gamma-1}{\gamma}} = T_C \frac{T_A}{T_B} = 275.6\,\text{K}$ or 2.4 °C or 36.3 °F;

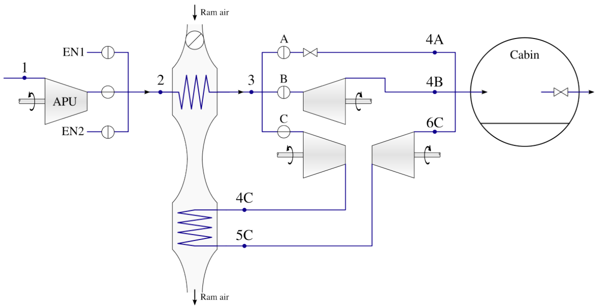
4) With equations 3/15 and 4/14, $w_{in} = +40.2\,\text{kJ}\,\text{kg}^{-1}$; $q_{out} = -20.1\,\text{kJ}\,\text{kg}^{-1}$; $w_{out} = +37.76\,\text{kJ}\,\text{kg}^{-1}$; $q_{in} = +17.6\,\text{kJ}\,\text{kg}^{-1}$; Thus with equation 6/6, $\eta_{conditioner} = 7.213$;

5) We want to obtain in 2 (return inside) $\dot{m}_{internal\ air} = \frac{\dot{V}_2}{v_2} = \frac{\dot{V}_2 p_2}{RT_2} = 0.305\,\text{kg}\,\text{s}^{-1}$. We must therefore remove a power $\dot{Q}_{internal\ air} = \dot{m}_{internal\ air} c_p (T_2 - T_1) = -5.51\,\text{kW}$ (3/14 & 4/14) from the internal air. The air conditioning unit will therefore require $\dot{W}_{net} = \frac{-\dot{Q}_{internal\ air}}{\eta_{conditioner}} = 765\,\text{W}$.

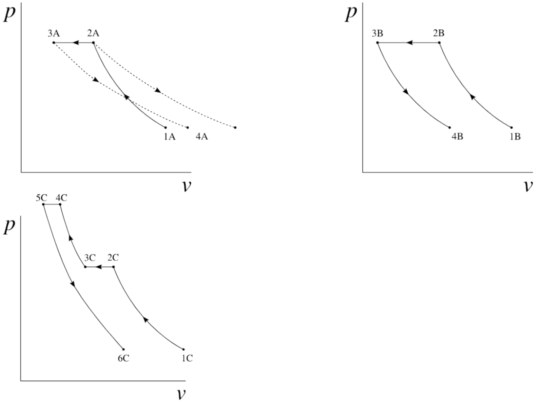
6) To minimize $\dot{m}_{external\ air}$, we need to maximize its outlet temperature T_4. However, we necessarily have $T_4 \leq T_B$, otherwise the heat transfer would occur in the wrong direction. Thus, $\dot{m}_{external\ air\ min.} = \frac{-\dot{Q}_{out\ air\ conditioner}}{c_p(T_{4max.} - T_3)} = -\frac{-\dot{Q}_{in} - \dot{W}_{net}}{c_p(T_{4max.} - T_3)} = 0.208\,\text{kg}\,\text{s}^{-1}$ (theoretical minimum).

7) In principle, it is sufficient to reverse the positions of the compressor and the turbine. In practice, the temperature ranges will also need to be shifted to allow heat absorption in cold weather.





6.12



1) If the RAM circuit is closed, then $T_{3A} = T_{2A}$; and since $T_{4A} = T_{3A}$ (§4.3.2) we obtain $T_{4A} = T_{1A}\left(\frac{p_{2B}}{p_{1B}}\right)^{\frac{\gamma-1}{\gamma}} = 377.19\,\text{K} = 104.1\,^{\circ}\text{C} = {}^{\circ}\text{F}$.

3) In the RAM exchanger we have $\dot{Q}_{cabin\ air} = -\dot{Q}_{ram\ air}$ so $\dot{m}_{cabin\ air} q_{cabin\ air} = -\dot{m}_{ram\ air} q_{ram\ air}$ or $\dot{m}_{cabin\ air} c_p (T_{3A} - T_{2A}) = -\dot{m}_{ram\ air} c_p (T_{RAM\ outlet} - T_{RAM\ inlet})$. Thus, $m_{ram\ air}$ is minimized when $T_{RAM\ outlet}$ is maximized; now we necessarily have $T_{RAM\ outlet} \leq T_{2C}$. Thus, $\dot{m}_{ram\ air\ min.} = -\dot{m}_{cabin\ air} \frac{T_{3A} - T_{2A}}{T_{2C} - T_{RAM\ inlet}} = 0.29\,\text{kg}\,\text{s}^{-1}$ (so about $200\,\text{L}\,\text{s}^{-1}$ or 424 CFM at the inlet).

5) If the RAM circuit is closed, then $T_{3B} = T_{2B}$; we have $T_{4B} = T_{3B}\left(\frac{p_{4B}}{p_{3B}}\right)^{\frac{\gamma-1}{\gamma}} = T_{2B}\left(\frac{p_{1B}}{p_{2B}}\right)^{\frac{\gamma-1}{\gamma}} = T_{1B} = 293.15\,\text{K} = 20\,^{\circ}\text{C} = 68\,^{\circ}\text{F}$.

6) $w_{turbine\ B} = c_p(T_{4B} - T_{3B}) = -w_{compression\ B} = -172\,\text{kJ}\,\text{kg}^{-1}$

8) We obtain $T_{4Bmin.}$ when $T_{3B} = T_{3Bmin.} = T_{external}$. Then $T_{4Bmin.} = T_{3Bmin.}\left(\frac{p_{4B}}{p_{3B}}\right)^{\frac{\gamma-1}{\gamma}} = 185.1\,\text{K} = -88.1\,^{\circ}\text{C} = -127\,^{\circ}\text{F}$ (a purely theoretical result,

of course);

9) It decreases : $w_{\text{turbine B2}} = c_p(T_{4\text{Bmin.}} - T_{3\text{Bmin.}}) = -108.6\,\text{kJ}\,\text{kg}^{-1}$.

12) $T_{5\text{C}} = T_{6\text{C}} \left(\frac{p_{5\text{B}}}{p_{6\text{B}}}\right)^{\frac{\gamma-1}{\gamma}} = 654.6\,\text{K} = 381.5\,^{\circ}\text{C} = 718.7\,^{\circ}\text{F}$.

13) We first calculate $T_{4\text{C}} = T_{3\text{C}} \left(\frac{p_{4\text{C}}}{p_{3\text{C}}}\right)^{\frac{\gamma-1}{\gamma}} = 728.4\,\text{K}$. In the pack, the work transfers are $w_{\text{pack}} = w_{\text{pack compressor}} + w_{\text{pack turbine}} = c_p(T_{4\text{C}} - T_{3\text{C}}) + c_p(T_{6\text{C}} - T_{5\text{C}}) = -138.9\,\text{kJ}\,\text{kg}^{-1}$; Thus, the air supplies net work in the pack, which in this case has a net surplus of shaft power.

14) In order to calculate these COP, the cycles must be completed by returning the air from the cabin condition back to the inlet condition (§6.2.1). In question 3 we have $\eta_{\text{heat pump}} = \left|\frac{q_{\text{out}}}{w_{\text{in}}}\right| = -\frac{c_p(T_{4\text{A}} - T_{1\text{A}})}{c_p(T_{2\text{A}} - T_{1\text{A}})} = 0.424$ (a rare application where a COP less than 100 % is acceptable).

15) In question 12 we have $\eta_{\text{conditioner}} = \left|\frac{q_{\text{in}}}{w_{\text{in}}}\right| = \frac{c_p(T_{1\text{C}} - T_{6\text{C}})}{c_p(T_{2\text{C}} - T_{1\text{C}} + T_{4\text{C}} - T_{3\text{C}} + T_{6\text{C}} - T_{5\text{C}})} = 0.842$.
In practice, however, the net work done by the air in the pack is not recovered: it is dissipated through friction into the RAM air.We thus have $w_{\text{net}} = c_p(T_{2\text{C}} - T_{1\text{C}})$ and the COP is decreased.
A few final comments : 1) In reality, adiabatic processes are not reversible, which further reduces the efficiencies calculated here. 2) The low values of these efficiencies result from compromises made to reduce the size, complexity, and especially the weight of the onboard systems. In this application, the available power as work is high, pneumatic energy is widely available, and any excess in volume or weight has disproportionate consequences. 3) In more recent aircraft (referred to as *more electric*), such as the B787 and the A350, the packs are now powered by electricity rather than using the same air intended for the cabin.

CHAPTER 7
The Second Law

or

Engineer Carnot's Troubling Discovery

Chapter 7 – The Second Law

Executive summary for chapter 7

Heat only moves towards a body at a lower temperature. This poses a fundamental limit on the efficiency of work-heat transformations. This limit depends on the maximum and minimum temperatures at which a machine operates, and is usually very low. Carnot described an engine of maximum efficiency.

Introduction

In chapter 6 (*thermodynamic cycles*), we studied the nature of different cycles for converting heat and work. We now aim to study, explain, and quantify their very limits. This chapter 7 (*the second law*) aims to answer two questions:

- Why do all heat engines always have an efficiency less than 100 %?
- How can the efficiency of a heat machine be maximized?

7.1 The Second Law

7.1.1 Statement

The *second law of thermodynamics* is expressed as follows:

Heat spontaneously moves
only towards a lower temperature.

This statement can be made more specific as follows:

> Heat transfer to a higher temperature
> can only occur with the input of energy.

We will see that this simple observation has multiple, profound consequences for engineers. In particular, it determines the maximum efficiency of all engines and refrigeration machines!

7.1.2 The evidence of the second law

The statement above seems so obvious that it is almost offending. Two remarks are necessary here.

- The second law can be stated in multiple ways. It is more striking to talk about "increase in entropy" than the spontaneous behavior of heat; yet these different statements, which we will gradually address, are all equivalent.
- The apparent manifest evidence of the postulate – of course no one has ever seen, at room temperature, a cup of hot tea spontaneously heat up, or a cold drink spontaneously cool down – crumbles as soon as one studies phenomena at the microscopic level.

  Indeed, if temperature is only the level of agitation of particles, then there is *a priori* nothing preventing it from locally increasing even if the ambient temperature is lower. It took half a century of arduous work by thermodynamicists to provide a satisfactory answer to this (curious students will learn of *Maxwell's demon* to their delight). The application of probabilities to thermodynamics, and in particular the work of Ludwig

> Boltzmann (§8.5.3), would reconcile it with the Newtonian mechanistic view of the world during the 20th century. This is not a trivial problem.

In our study and from our engineering point of view, we will accept the above postulate as obvious, without seeking to justify it or explain it.

7.1.3 Zeroth law and third law

The *zeroth law* of thermodynamics states that if two bodies are in thermal equilibrium with a third, then all three are in equilibrium with each other. The *third law* states that the entropy (a property that we will study in the next chapter) of a crystal at zero temperature is zero. Neither of these laws is of any importance to the engineer.

7.2 The Second Law and Thermal Machines

7.2.1 All engines reject heat

Let's imagine that we want to generate work by taking heat from a "hot" object, that is, at a high temperature: for example, 100 °C (212 °F), as shown in figure 7.1. We connect a cylinder filled with a fluid to this object, and let the fluid push on a piston as it receives heat.

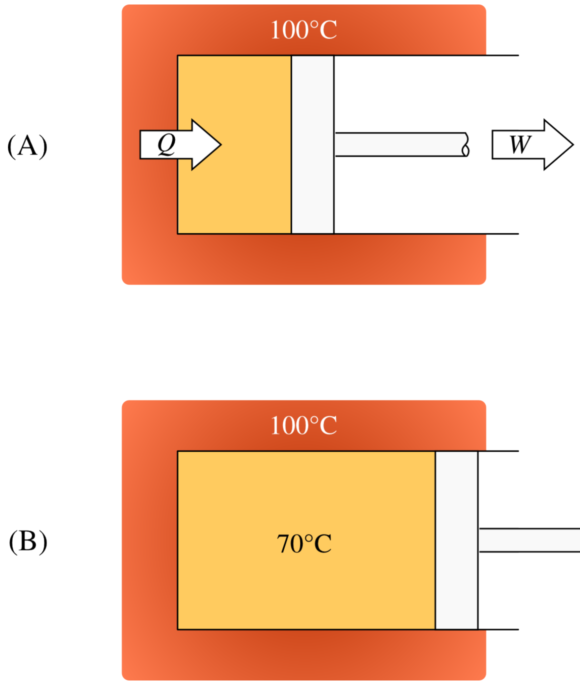


Figure 7.1: Production of work with heat from a body at 100 °C. The heat transfer allows work to be done, but also causes an increase in the fluid's temperature.

Diagram CC-0 *Olivier Cleynen*

Once it has done some work (at B in figure 7.1), the fluid has increased in volume. If we want to continue converting heat into work and do not want the engine to "inflate" indefinitely, we need to cool this gas to return it to its initial volume.

Unfortunately, *the only way* to extract heat from the gas is to put it in contact with a “cooler” body, as shown in figure 7.2. In particular, it is impossible to return the accumulated heat in the gas to the “hot” body – for that to happen, the gas temperature would need to be higher than this body. This accumulated energy is therefore irretrievably lost.

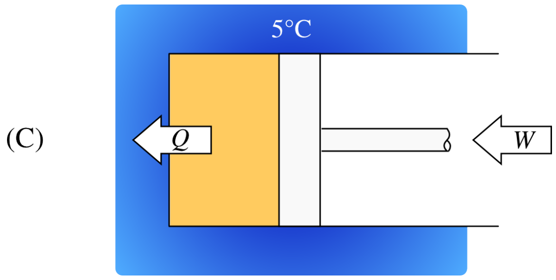


Figure 7.2: The inevitable cooling of the engine. The only way to return the fluid to its initial state (A) in the experiment in figure 7.1 is by extracting heat from it, which can only be done with a “heat sink” at a lower temperature. The heat and work transfers here are smaller than on the outward journey, but both are non-zero.

Diagram CC-0 *Olivier Cleynen*

> « According to this principle, it is not sufficient, in order to give rise to motive power, to produce heat: one must also procure cold; without it, heat would be useless.
> And indeed, if one were to find only bodies around them as hot as our furnaces, how would one succeed in condensing the steam? Where would one place it once it had come into existence? One must not believe that it could be simply discharged into the atmosphere, as is done in certain machines: the atmosphere would not receive it. It receives it, in the present state of things, only because it serves the function of a vast condenser, because it is at a colder temperature: otherwise it would soon be filled with it, or rather, it would already be saturated with it. »
>
> Sadi Carnot, 1824 [4]

Thus, for an engine to operate continuously, it must, in addition to a high-temperature source from which to capture heat, have a “sink” at low temperature to reject the heat that it can no longer make use of.

This reasoning applies in the same way to refrigerators, air conditioners, and heat pumps, which are designed to absorb heat at a *low* temperature. Once the heat has been absorbed into the low-temperature fluid, the only way to reject it at a higher temperature is to increase the temperature of the fluid. This requires a non-zero compression work. Thus, for a refrigerator to operate continuously, it must receive energy in the form of work.

7.2.2 Limits of thermal machines

In order to study heat and work transformations more rigorously, we will use the following notation to describe thermal machines:

- T_H and T_L will represent the high and low temperatures respectively;
- We will always call $\dot{Q}_{TH}$ the power in the form of heat transferred at high temperature, and $\dot{Q}_{TL}$ its equivalent at low temperature (each can be positive or negative).

Thus, the thermal machine in its most general representation looks like figure 7.3.

Regardless of the operation mode and efficiency of the machine, it cannot create or destroy energy (§1.1.2); and we will always have:

$$\dot{Q}_{TH} + \dot{Q}_{TL} + \dot{W}_{\text{net}} = 0 \qquad (7/1)$$

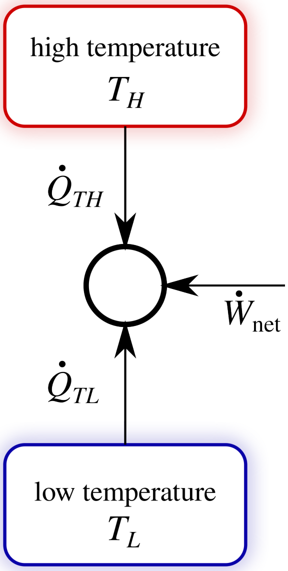


Figure 7.3: A thermal machine transforming work and heat in its most general representation.

Diagram CC-0 *Olivier Cleynen*

The second law has particular consequences for each of the two main types of thermal machines:

An engine takes heat from a high-temperature source ($\dot{Q}_{TH} > 0$) and produces work ($\dot{W}_{\text{net}} < 0$, figure 7.4). We have just seen that if we want to carry out this transformation continuously, we have no choice but to reject heat into a low-temperature reservoir ($\dot{Q}_{TL} < 0$).

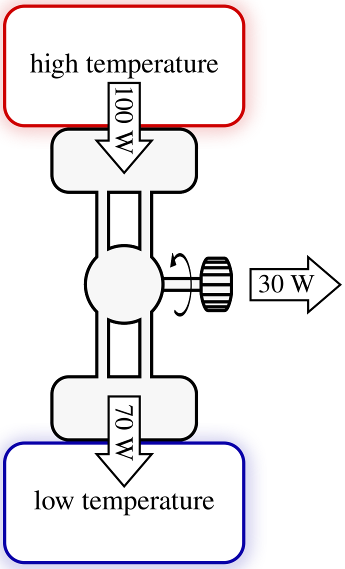


Figure 7.4: An example of energy transfers to a thermal engine.

Diagram CC-0 *Olivier Cleynen*

In power plants, the two temperature zones are easily identifiable: the steam takes heat from the core of the plant (nuclear reactor, gas boiler, or coal boiler) and releases heat through the large cooling chimneys.

Automotive and aeronautical engines, on the other hand, must discharge the air that serves as their working fluid because of the combustion products that prevent its reuse. For this reason, cooling takes place in the atmosphere, outside the engine casing. Their "cooling zone" is not easily distinguishable.

Applied to the engine, the second law can be expressed as follows:

> No engine can continuously convert heat into work from a single heat source. All engines reject heat at a lower temperature.

> The continuous operation of an engine requires two heat reservoirs, each at a different temperature.

Purists will express this corollary, called the *Kelvin-Planck corollary*, with the following inequality:

$$\dot{Q}_{TH} > -\dot{W}_{\text{net}} \tag{7/2}$$

for any thermal engine.

A refrigerator, an air conditioner, or a heat pump operates in the opposite way to engines. These machines extract heat from a low-temperature source ($\dot{Q}_{TL} > 0$) to reject it into a reservoir at a higher temperature ($\dot{Q}_{TH} < 0$, figure 7.5). An inevitable consequence is that they must receive work for this ($\dot{W}_{\text{net}} > 0$).

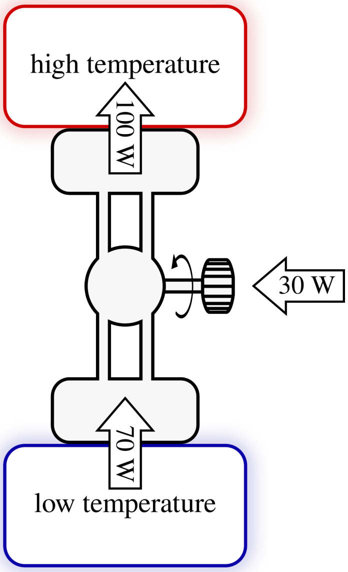


Figure 7.5: An example of energy transfers in a refrigerator, air conditioner, or heat pump in operation.

Diagram CC-0 *Olivier Cleynen*

Applied to a refrigerator, the second law can be expressed as follows:

> Any machine transferring heat from a body to another at a higher temperature receives work.

Purists will enjoy translating this corollary, called the *Clausius corollary*, as follows:

$$\dot{W}_{\text{net}} > 0 \tag{7/3}$$

for any refrigerator, air conditioner, or heat pump.

7.3 The Carnot Cycle

7.3.1 Some context

« To consider in all its generality the principle of the production of motion by heat, one must conceive of it independently of any mechanism, of any particular agent; one must establish reasoning applicable not only to steam engines, but to any imaginable heat engine, regardless of the substance employed and regardless of the manner in which it is acted upon. »

Sadi Carnot, 1824 [4]

At the beginning of the 19th century, a young Parisian *Polytechnique* engineer named Sadi Carnot became interested in the operation of thermal engines, which were booming at the time. Carnot sought to predict *the maximum amount of work* that can be generated from a given amount of coal.

Carnot's approach is interesting in that he completely abstracted the technological aspect to investigate the underlying principles of engine operation. This is even more challenging because at that time, engines operated by vaporizing and condensing steam, and the concepts of a cycle or of energy conservation were not yet established. This abstraction and the clarity of his writing established his only work, *Reflections on the Motive Power of Fire and on Machines Fitted to Develop that Power*, 1824 [4], in the history of physics.

Carnot died shortly after his publication and before his work could be recognized; his conception of heat was fundamentally incorrect;[1] yet the theoretical engine he described, an essential passage for engineering students, serves as a reference in the design offices of all engine manufacturers today.

7.3.2 Concept of reversible machine

Carnot sought the theoretical engine whose efficiency would be the maximum possible. He imagined a unique way to transform heat into work and work into heat. His machine can operate in both directions: as an engine or as a refrigerator.

In thermodynamic terms, the machine he conceptualized is not only *bidirectional*, meaning that the direction of fluid flow can be changed to change its function (like many domestic heat pump / AC systems on the market today), but it is also *reversible*: by reversing its operation, all heat flows become exactly opposite. In this way, if Carnot's refrigerator is powered by Carnot's engine, then the flows will be exactly compensated, as depicted in figure 7.6.

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[1] Carnot figured out the second law correctly, but not the first law: he still used Antoine Lavoisier's *caloric theory*, which was to be dismantled by James Prescott Joule twenty years later.

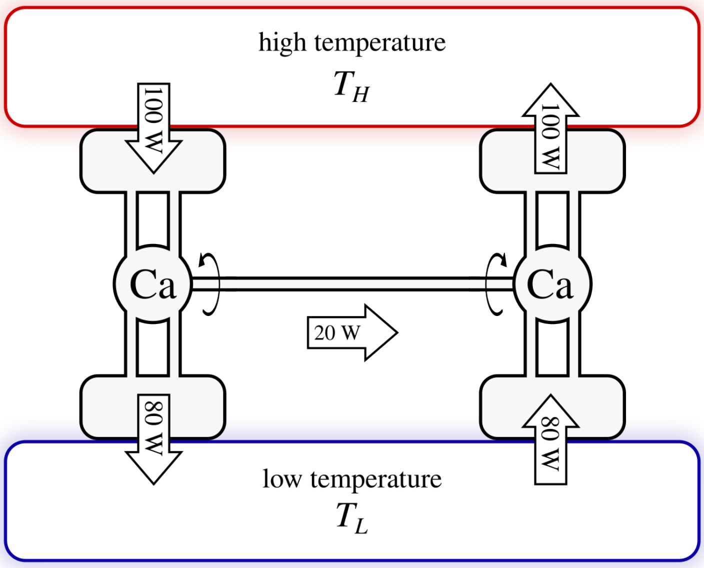


Figure 7.6: Two Carnot machines, an engine (left) and a refrigerator (right). The first powers the second, and since they are reversible (in the thermodynamic sense of the word), the heat flows are balanced.

Diagram CC-0 *Olivier Cleynen*

Why would such a machine be the most efficient one that can be designed? It can be proven by contradiction that an engine with *higher* efficiency than a reversible engine cannot exist (figure 7.7). The work supplied by this hypothetical machine could be used to power a reversible refrigerator. These two machines together would then receive no net work, but would still create a heat flow from the cold reservoir to the hot reservoir. According to Carnot, and based on the second law whose validity we have accepted, this is impossible: such a machine cannot exist.

> « This maximum [of work] has the property, that, by its *consumption*, a quantity of heat may be carried from the cold body B to the warm one A equal to that which passed from A to B during its *production*. »
>
> Rudolf Clausius, 1850 [10, 11, 21]

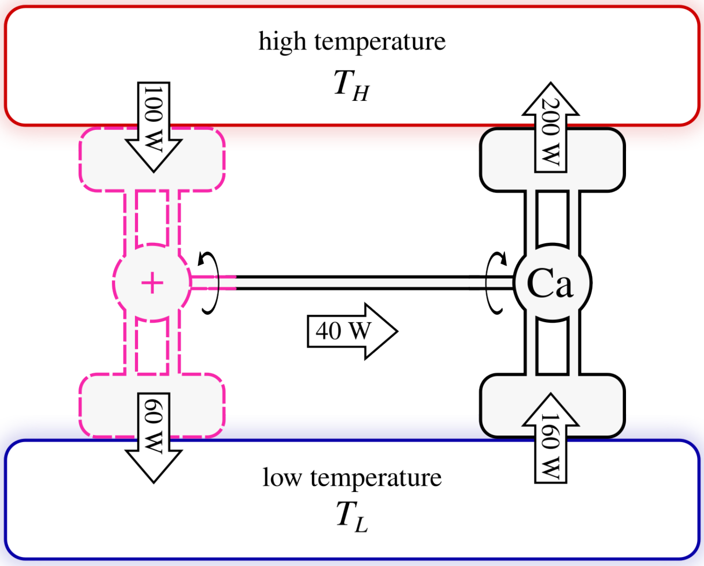


Figure 7.7: Proof by contradiction that the best possible engine is reversible. A hypothetical engine (on the left) with higher efficiency than a reversible refrigerator (on the right) could simply power the latter. Thus, we would obtain a spontaneous net heat flow (here of 100 W) from the cold source to the hot source, without a net input of work: according to the second law, this is impossible.

Diagram CC-0 *Olivier Cleynen*

This method of reasoning by combining hypothetical and theoretical machines, even if it can be initially confusing, is an excellent way to approach the theory of heat machines. The student is strongly encouraged to experiment in this way, for example by answering the following questions:

- Why does the best possible refrigerator operate reversibly?
- Why can't we improve the efficiency of an engine by redirecting its heat rejection towards the hot source by use of a reversible heat pump?

7.3.3 Development of the Carnot cycle

> « But in order to draw truly advantageous results from high-pressure machines, it is necessary that the fall of the caloric in them be utilized as effectively as possible. It is not enough for the steam to originate at a high temperature: it must also be that by the expansion of its volume it reach a sufficiently low temperature. The mark of a good steam engine must therefore be not only to employ steam under a high pressure, but *to employ it under successive pressures that are highly-variable, greatly-different from one another, and progressively decreasing.* »
>
> Sadi Carnot, 1824 [4]

We have now seen that the maximum efficiency of a machine is reached when its operation is reversible. From this observation, Carnot reasoned as follows:

1. All thermal machines operate by the expansion and contraction of a substance alternately subjected to two temperatures;
2. For them to be reversible, namely, to ensure they can be carried out in the reverse direction, all heat transfers must be done with infinitesimal temperature differences: these processes will then be *isothermal*;
3. For them to be reversible, the phases where the substance changes temperature (in order to move from one heat reservoir to another) must occur without heat transfer: these processes will then be *adiabatic*.
4. In order to allow each process to be run backwards, they must all be *reversible* (infinitely slow).

The essentials are here. Carnot outlined a theoretical thermodynamic cycle which consists of two isothermal and two adiabatic processes. He did not need to quantify any transfer; and did not yet concern himself with any technological detail. However, it is certain that the thermodynamic cycle that he described is the most efficient –the least inefficient!– that can be realized.

7.3.4 The four stages of the Carnot engine

We can describe the *Carnot cycle* with a fixed amount of mass maintained inside a cylinder undergoing four processes (figure 7.8). It commutes between temperatures T_H ("hot" source at high temperature) and T_L ("cold" source at low temperature), in order to to produce a net work:

Reversible adiabatic compression from 1 to 2

In this stage, we aim to raise the temperature of the fluid to a high level without adding any heat to it.

The cycle starts at 1, when the fluid is in the cylinder at the low temperature T_L. In order to raise it to a high temperature (thus enabling a reversible heat transfer in the following phase 2 → 3), the fluid is reversibly adiabatically compressed (§4.4.5 & §5.4.5). The fluid's temperature increases from T_L to T_H.

This phase requires work *input* ($W_{1\to2} > 0$).

T_H **Isothermal heating from 2 to 3** In this stage, we aim to absorb a quantity Q_{TH} of heat from the high-temperature source.

At 2, the fluid has been compressed in the piston at temperature T_H. The cylinder is brought into contact with the hot source (temperature T_H) and heat is supplied with an infinitesimal temperature difference: this is an isothermal expansion (§4.4.4 & §5.4.4). The fluid's temperature remains constant at T_H.

This phase generates work output ($W_{2\to3} < 0$).

▨ **Reversible adiabatic expansion from 3 to 4** In this stage, we aim to decrease the temperature of the fluid to that of the cold source (T_L).

At 3, the fluid is still at temperature T_H. The cylinder is then thermally isolated and the fluid is expanded in order to have the fluid do work and reduce its temperature without heat transfer: this is a reversible adiabatic expansion. The piston continues its slow retreat, and the fluid's temperature decreases down to T_L.

This phase generates work output ($W_{3\to4} < 0$).

T_B **Isothermal cooling from 4 to 1** In this final stage, we aim to reject a quantity Q_{TL} of heat into the low-temperature sink.

At 4, the fluid is at low temperature T_L. In order to bring it back to its initial volume, heat must be removed. We proceed with an isothermal cooling: the piston is gradually advanced, and the fluid's temperature is kept constant at T_L.

This phase requires work input ($W_{4\to1} > 0$).

Overall, the engine received a quantity of heat $|Q_{TH}|$ at high temperature and rejected a smaller quantity $|Q_{TL}|$ at low temperature. The difference between these two quantities is the produced work, $W_{\text{net}} = W_{1\to2} + W_{2\to3} + W_{3\to4} + W_{4\to1} = -Q_{TH} - Q_{TL}$ (6/1).

This quantity of work W_{net} represents the maximum that can be obtained from a quantity of heat Q_{TH} between two given temperatures T_L and T_H.

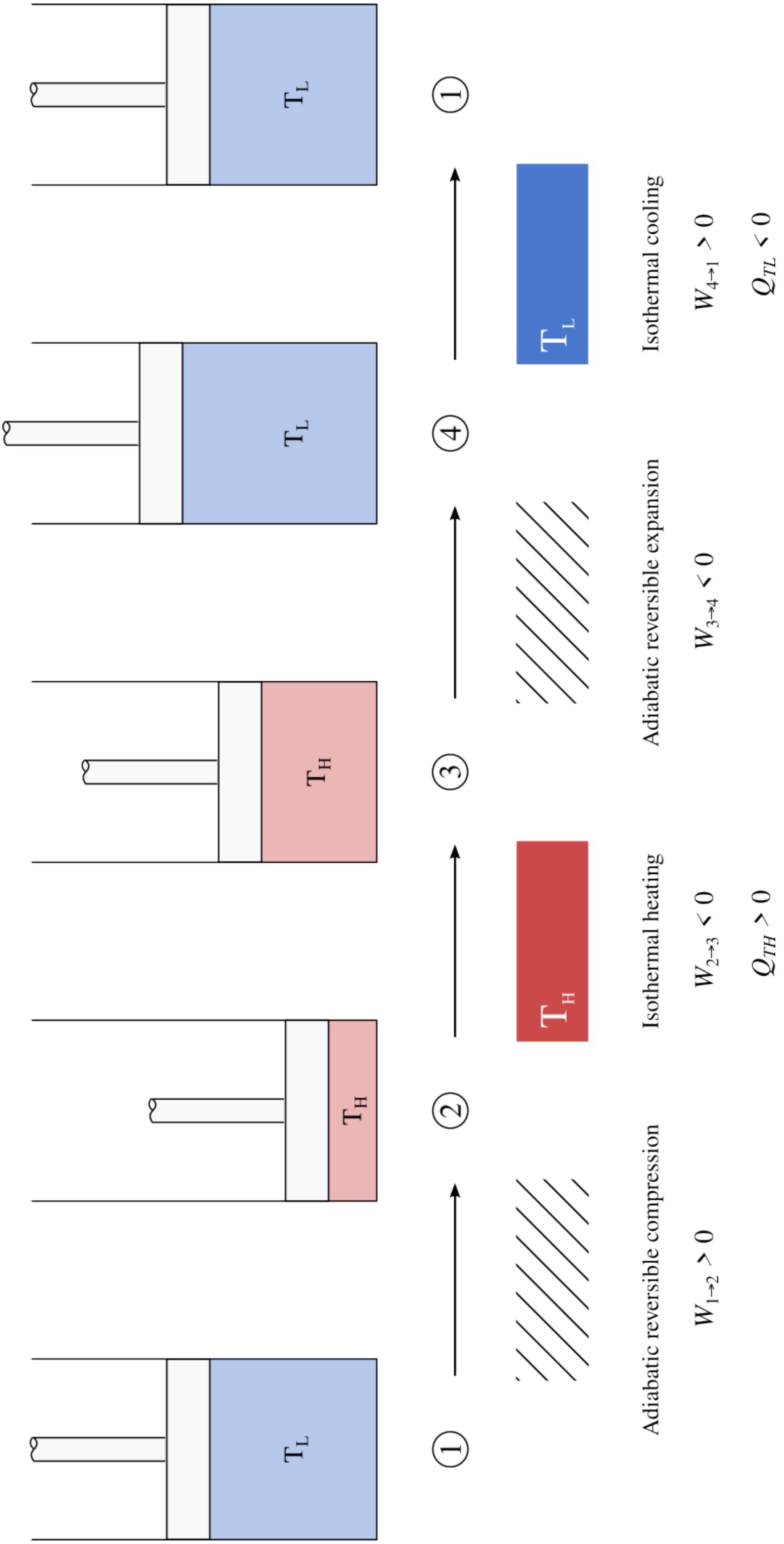


Figure 7.8: The four stages of the Carnot engine, executed with a fixed mass quantity by separating them in time. The cycle is such that when the stages are performed in reverse order (1 → 4 → 3 → 2 → 1), the transfers are exactly opposite.

Diagram CC-0 *Olivier Cleynen*

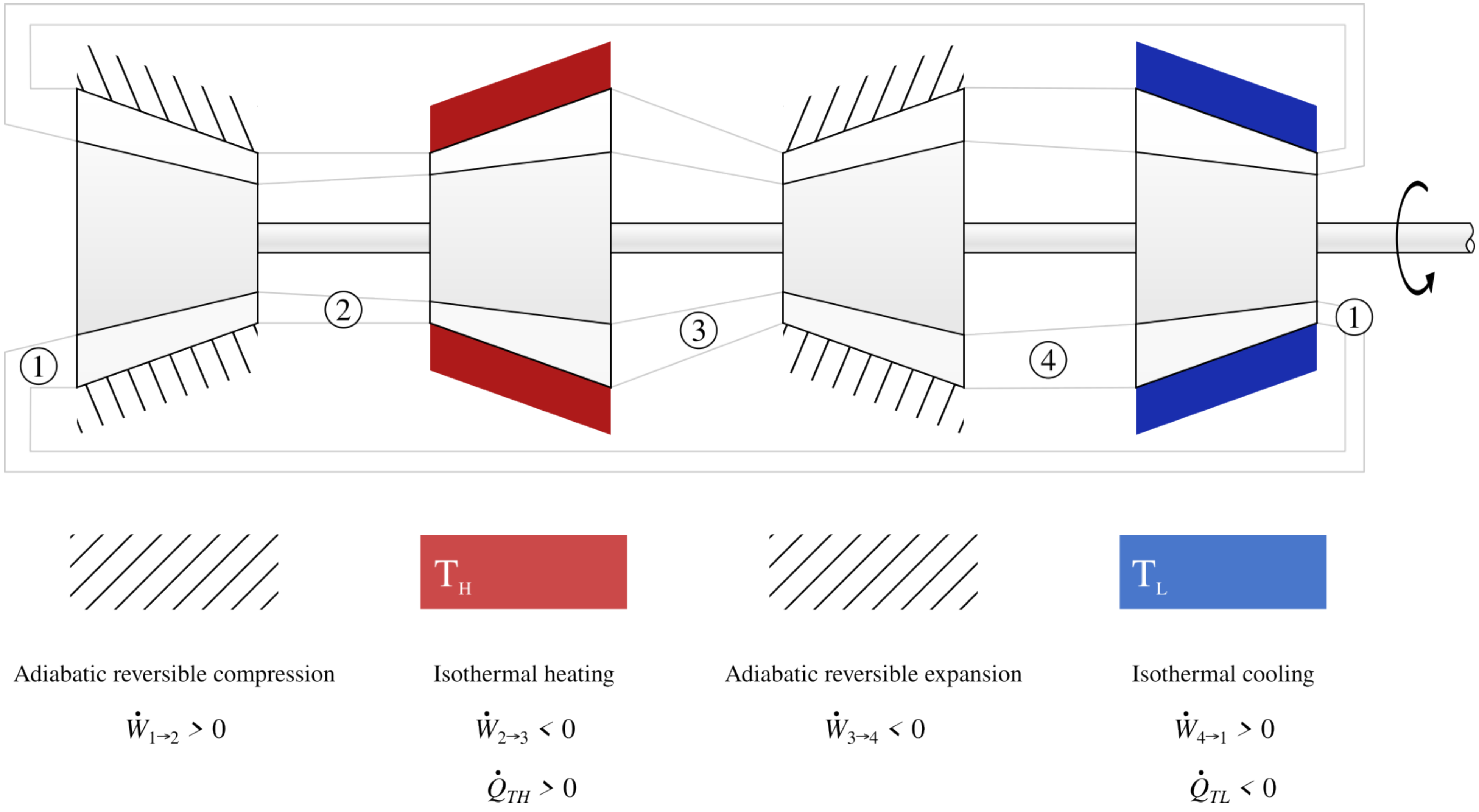


Figure 7.9: The four stages of the Carnot engine, executed with a constant mass flow by separating them in space. Here too, the cycle is such that when the flow direction is reversed (becoming 1 → 4 → 3 → 2 → 1), the transfers are exactly opposite.

Diagram CC-BY-SA *Olivier Cleynen*

The Carnot engine cycle can be plotted on a pressure-volume diagram (for example in figure 7.10 with a perfect gas). It can be observed that the compression phases occur at lower pressure and volume than the expansion phases: the cycle is work-producing. Since all processes are reversible, the area enclosed in the path 1-2-3-4-1 represents the net work quantity W_{net} produced.

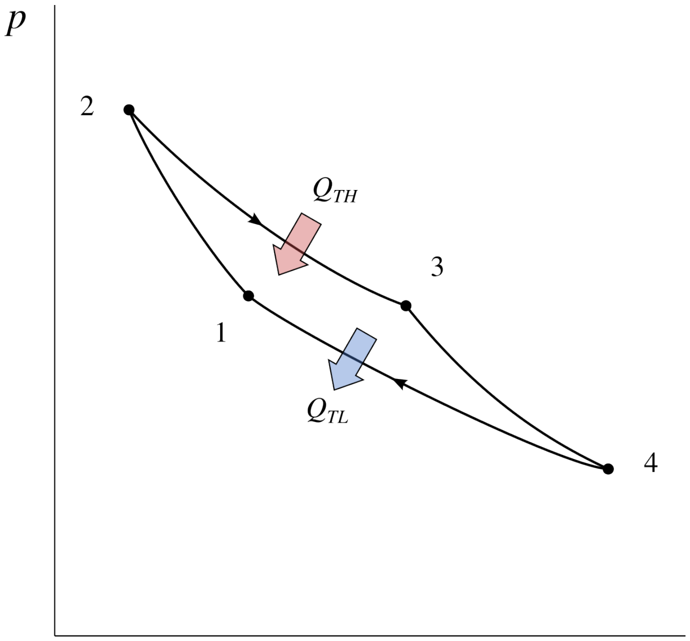


Figure 7.10: Pressure-volume diagram of the Carnot engine performed with a perfect gas. The processes 2 → 3 and 4 → 1 are done at T_H and T_L, respectively.

Diagram CC-0 *Olivier Cleynen*

> « To fully utilize the motive power that can be made available, the expansion would have to be carried out until the temperature of the steam was reduced to that of the condenser; but practical considerations, derived from the manner in which the motive power of heat is employed in the arts, prevent this limit from being reached. »
>
> Émile Clapeyron, 1834 [5]

The perceptive student will have observed that in order for all these phases to be reversible, the piston movement must be infinitely slow, and thus the fluid completes the cycle in an infinite amount of time. The Carnot engine therefore reaches maximum efficiency with infinitely low power.

Example 7.1

Perform a Carnot cycle between temperatures of 600 °C (1112 °F) and 100 °C (212 °F). Use 100 g (0.22 lb) of air trapped in a cylinder at a pressure of 1 bar. What amount of work must be invested? What is the work that will be recovered? What is the efficiency?

We start by raising the temperature to 600 °C with a reversible adiabatic process (1 → 2). The energetic cost for this will be $w_{1\to2} = c_v \, \Delta T = 718 \times (600 - 100) = +359\,\text{kJ}\,\text{kg}^{-1}$ (4/32) and $q_{1\to2} = 0$. Equation 4/37 provides the final pressure: $p_2 = p_1 \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = 1 \times \left(\frac{600+273.15}{100+273.15}\right)^{\frac{1.4}{1.4-1}} = 19.6\,\text{bar} = 284\,\text{psi}$.

☞ This phase's sole purpose is to raise the temperature so that we can later absorb heat reversibly, which would be impossible if the gas temperature were below 600 °C.

We can now absorb heat at constant temperature (2 → 3). We choose to expand the gas to $p_3 = 4\,\mathrm{bar}$. Thus the heat transfer is $q_{2\to3} = -R\,T_2 \ln\left(\frac{p_3}{p_2}\right) = -287 \times (600 + 273.15) \times \ln\left(\frac{4}{19.6}\right) = +398.2\,\mathrm{kJ\,kg^{-1}}$ (4/27, received by the gas) and the work is $w_{2\to3} = -q_{2\to3} = -398.2\,\mathrm{kJ\,kg^{-1}}$ (4/28, supplied by the gas).

☞ A promise has been kept: we had studied isothermal processes in sections §4.4.4 and §5.4.4 precisely in order to be able to use them here…

☞ The choice of the pressure p_3 or its corresponding volume v_3 is entirely arbitrary and does not affect any of the results obtained here.

We proceed with the expansion of the gas, aiming to recover as much work as possible and to reduce its temperature (3 → 4). The process is adiabatic, therefore $w_{3\to4} = c_v\,\Delta T = -w_{1\to2} = -359\,\mathrm{J\,kg^{-1}}$ and $q_{3\to4} = 0$. Using equation 4/37 we obtain the final pressure: $p_4 = p_3\left(\frac{T_4}{T_3}\right)^{\frac{\gamma}{\gamma-1}} = 0.2\,\mathrm{bar} = 2.9\,\mathrm{psi}$.

☞ The work recovered from 3 to 4 is exactly opposite to the one invested from 1 to 2. In a Carnot cycle, it is during the heat transfers that the work leads to the production of a net work.

Finally, to bring the fluid back to its initial state (§6.2.1), we need to cool the gas (4 → 1). This cooling is done at constant temperature, so $q_{4\to1} = -R\,T_4 \ln\left(\frac{p_1}{p_4}\right) = -170.2\,\mathrm{kJ\,kg^{-1}}$ (4/27, supplied by the gas) and the work is $w_{4\to1} = -q_{4\to1} = +170.2\,\mathrm{kJ\,kg^{-1}}$ (4/28, received by the gas).

☞ It is a complex process for such an inglorious step: the rejection of unusable heat! Not only do we need to proceed infinitely slowly with significant volume fluctuations, but we also need to *invest* considerable work to return to 1.

What is the cycle's balance? The compression phases required $w_\text{compression} = w_{1\to2} + w_{4\to1} = +529.2\,\mathrm{kJ\,kg^{-1}}$. In the expansion, we recovered $w_\text{expansion} = w_{2\to3} + w_{3\to4} = -757.2\,\mathrm{kJ\,kg^{-1}}$. The net work is $w_\text{net} = w_\text{compression} + w_\text{expansion} = -228\,\mathrm{kJ\,kg^{-1}}$; $W_\text{net} = m\,w_\text{net} = -22.8\,\mathrm{kJ}$.

☞ We included the gas mass as late as possible. If we had studied a cycle carried out continuously as depicted in figure 7.9, we would only need to replace m with $\dot{m}$ to obtain the sought powers in watts.

☞ Compared to the recovered quantity, a substantial amount of work must be invested, an undesirable characteristic that will be quantified in chapter 10 (*air-based power cycles*) under the name of *work ratio* (10/1).

The efficiency (6/4), finally, is $\eta_\text{engine} = \left|\frac{w_\text{net}}{q_\text{in}}\right| = -\frac{w_\text{net}}{q_{2\to3}} = -\frac{-228}{398.2} = 57.3\,\%$.

☞ Although the cycle is already impracticable in reality, this meager efficiency is the greatest that can physically be attained between temperatures of 600 °C and 100 °C.

☞ We could also have carried out these calculations using a liquid/vapor instead of air: this would not have altered the final results.

7.3.5 Four stages or four strokes?

Piston engines are often classified according to their mode of operation. *Two-stroke engines* perform one expansion per crankshaft revolution (every two piston movements); whereas *four-stroke engines* (figure 6.14) perform one expansion every two revolutions. The distinction lies in the mode of exhaust gas removal and its replacement with fresh air (see §6.4 p. 165).

The transposition of the Carnot engine to reality, where eventually the fluid will need to be drained or transferred to a separate cylinder for cooling, can be done with either two or four strokes at the engineer's discretion. Thus, the Carnot cycle, although it is indeed made up of four *stages*, cannot be specifically associated with either of these two modes of operation.

7.3.6 Carnot refrigerator

By reversing the operating direction of the engine described above, we create a refrigerator, air conditioner, or heat pump of the same efficiency. The fluid then passes through the same states, but by following the reverse path (1-4-3-2-1) as shown in figure 7.11. The heat $Q_{TL} > 0$ is *captured* from the cold source, the work $W_{\text{net}} > 0$ is *received* by the machine, and the heat $Q_{TH} < 0$ is rejected by the machine towards the high-temperature source.

This cycle allows obtaining the maximum efficiency (the "least bad" efficiency, since it is not infinite) of an air conditioning system or a heat pump operating between two given temperatures T_H and T_L.

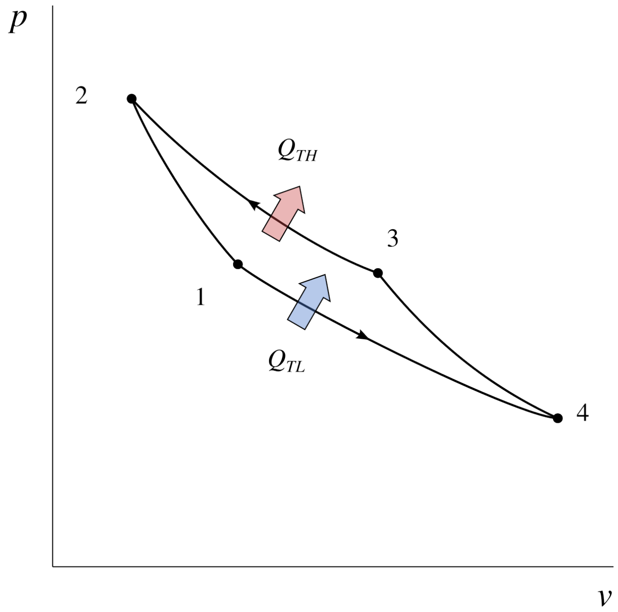


Figure 7.11: Pressure-volume diagram for a reversed Carnot cycle, namely, in refrigeration mode (refrigerator or heat pump), with a perfect gas.

Diagram CC-0 *Olivier Cleynen*

7.4 Thermodynamic Temperature Scale

7.4.1 In a nutshell, for the impatient student

Kelvin defines a temperature scale, called *absolute temperature*. Within a Carnot engine, the ratio of the maximum temperature T_H and minimum temperature T_L is defined to be equal to the ratio of the heat transfer rates, that is:

$$\left|\frac{\dot{Q}_{TH}}{\dot{Q}_{TL}}\right| \equiv \frac{T_H}{T_L} \tag{7/4}$$

by definition, in a Carnot engine,
where $\dot{Q}_{TH}$ is the heat transfer rate absorbed or rejected at high temperature ($\dot{Q}_{TL}$, at low temperature),
and where the temperatures are absolute (measured in K).

This equation 7/4 is a definition. Therefore, we can determine the temperature of a body without needing to use a specific fluid. Kelvin calibrates his scale such that $0\,^\circ\mathrm{C} = 273.15\,\mathrm{K}$.

The rest of this section §7.4 details the path that led to this definition. It is intended for curious readers, and can be safely skimmed by busy students or engineers.

7.4.2 What is a scale in physics?

In order to quantify a property in physics (for example, quantify “mass” or “color”), three things must have been defined:

A zero point which defines what corresponds to zero property (zero mass, zero pressure, etc.);

A standard which serves as a reference gauge (for example, an object of one pound mass, one meter length);

A scale which allows to *define* the property between the zero point and the standard (for example, what is “twice as much” or “half as much” mass, light, etc.).

7.4.3 The limits of the Celsius and Fahrenheit thermometers

At the beginning of the 19th century, the two temperature scales that we use today in everyday life, those of the Swedish Anders Celsius and the German Daniel Gabriel Fahrenheit, were already in use. How are these scales defined from a physical point of view?

- The zero point is rather easy to define (it is the point where bodies are completely frozen, unable to supply heat) but neither Fahrenheit nor Celsius could accurately locate it with certainty;
- The standards of Celsius and Fahrenheit differ significantly. Celsius chose the freezing point of pure water, Fahrenheit of saltwater, at atmospheric pressure, and each assigned it the relative “zero” graduation.
- However, the scales of Celsius and Fahrenheit are strictly identical. In fact, to *measure* temperatures around their standards, both scientists measured

the contraction and expansion of a liquid in a tube. Between his relative zero point and the boiling point of water at atmospheric pressure, Celsius drew 100 graduations; Fahrenheit, 212 graduations.

As always, the history of thermodynamics is full of trivia: Celsius initially used a reversed scale, going from 100 at freezing to 0 at boiling! As for Fahrenheit, he likely chose 212 graduations in order to easily realign with his *first* graduation, calibrated on the freezing point of pure water (32) and the temperature of the human body (96), standards that were quite difficult to reproduce. We must not let ourselves be distracted: in a physical sense, these are only graduations and not a scale, which was already determined by using a liquid thermometer.

The main problem with these two scales is that the temperature is well defined only in the range of existence of liquid thermometers. Whatever fluid is used (mercury, alcohol, water), it always ends up freezing or boiling at some point; and the graduations then no longer provide useful information. For example, Celsius could not *define* or even describe what allows recognizing a temperature of 1200 °C.

In addition to this, neither scale is intuitive in the negative range. If one were to admit, for example, that 40 °C could be "twice as much temperature" as 20 °C, then what temperature would be twice as much as −10 °C? This amounts to asking the question: can we write $\frac{40\,^{\circ}\mathrm{C}}{20\,^{\circ}\mathrm{C}}$ and is it equal to $\frac{80\,^{\circ}\mathrm{C}}{40\,^{\circ}\mathrm{C}}$? As an inquisitive Scott would soon explain, the modern answer to this question is no.

7.4.4 William Thomson's thermometer

Scottish physicist and engineer William Thomson understood these limitations very well. He proposed a temperature scale that does not depend on the behavior of a fluid in a tube.

Thomson took a keen interest in the Carnot cycle and reasoned as follows: the only characteristic that gives maximum efficiency to the Carnot engine is the fact that it is reversible. In other words, all machines based on this cycle and operating between two given temperatures will have the same efficiency — regardless of their fuel, displacement, configuration, or power. One could therefore use the efficiency of a Carnot engine as a measure of temperature.

> « … the absolute values of two temperatures are to one another in the proportion of the heat taken to the heat rejected in a perfect thermo-dynamic engine working with a source and refrigerator at the higher and lower of the temperatures respectively. »
>
> William Thomson, 1854 [15]

Thomson's proposition is as follows: consider a body at a temperature T_1 (for example, a thousand units, as shown in figure 7.12). A Carnot engine is attached to it, which will supply work and reject heat at a lower temperature T_2. This temperature T_2 is half of T_1 if the engine rejects half of the heat it receives; it is one-fourth when it rejects one fourth, and so on. In mathematical terms, Thomson proposed:[1]

$$\left|\frac{\dot{Q}_{TH}}{\dot{Q}_{TL}}\right| \equiv \frac{T_H}{T_L} \tag{7/4}$$

---

[1] In fact, the records are again not that simple. Thomson initially proposed (in 1848 [9]) a scale in which $\frac{\dot{Q}_{TH}}{\dot{Q}_{TL}}$ is proportional to the *difference* of temperatures; making it a logarithmic scale from our current perspective. He revised this with the help of James Prescott Joule to arrive at proposition 7/4 six years later [14].

in a Carnot engine (actually, for any machine performing a reversible transformation),
where $\dot{Q}_{TH}$ is the heat transfer rate absorbed or rejected at high temperature ($\dot{Q}_{TL}$, at low temperature),
and where the temperatures are *absolute* (measured in K).

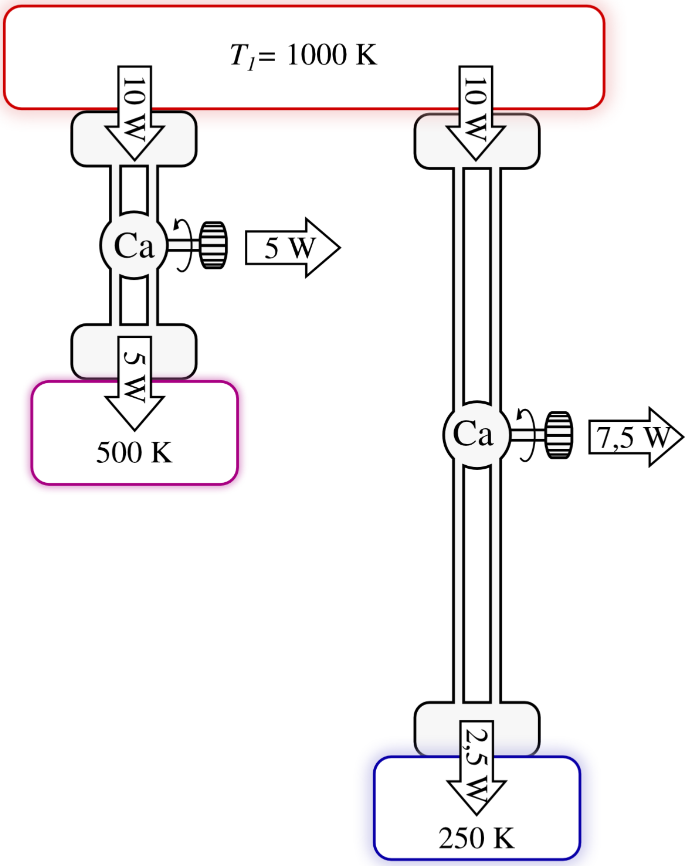


Figure 7.12: Experiment illustrating the absolute temperature scale proposed by William Thomson. A Carnot engine operating between 1000 K and 500 K rejects $\frac{1000}{500}$ = 50 % of the heat it receives. If the low temperature is four times lower, this rejection is four times lower ($\frac{1000}{250}$) than the heat received.

Diagram CC-0 *Olivier Cleynen*

By manipulating equations 7/4 and 7/1, we can reformulate Kelvin's definition as follows:

> Let there be a Carnot engine operating between two thermal
> reservoirs separated by one degree of temperature,
> and to which a quantity of heat Q_{TH} of one unit is supplied;
>
> The temperature of the hot source
> is defined as the inverse of the work produced.

The temperatures in this scale, called the *absolute temperature* scale or *thermodynamic temperature* scale, are always positive and vary from zero to infinity.

7.4.5 Absolute zero and synchronization of scales

Thomson therefore had a *scale* — a method of defining a temperature as “twice as high”.

The zero of this scale corresponds well to the zero temperature point, since with the experiment in figure 7.12 one then has an “abyss” of zero temperature that allows gases to be infinitely expanded down to zero temperature (thus converting all the internal energy of a fluid into work).

> « The particular convention is, that the difference of temperatures between the freezing- and boiling-points of water under standard atmospheric pressure shall be called 100 degrees. »
>
> William Thomson, 1854 [15]

There remained the choice of a standard. Thomson returned to Celsius’s thermometer and took the same reference point (the freezing point of pure water at atmospheric pressure). Observing that the contraction and expansion of fluids remain proportional to the change in their absolute temperature, he assigned a value to this reference point that allows maintaining the same thermometric scale as Celsius. For this, the temperatures 100 °C and 0 °C, which are known to allow a maximum efficiency of 26.8 %, must correspond to temperatures in K spaced by 100 units. The calculation is simple – the student is encouraged to reproduce it – and Thomson obtained the relation:

$$0\,^\circ\mathrm{C} = 273.15\,\mathrm{K} \qquad (7/5)$$

William Thomson, already embarked on a stunning scientific career, was thirty years old when he published his temperature scale in 1854. In 1892, he was ennobled as *First Baron Kelvin* (he even ought to write *The Right Honourable First Lord Kelvin of Largs, of the Order of Merit, the Royal Victorian Order, and of Her Majesty’s Most Honourable Privy Council!*); it is under this name that he is known today. The *Kelvin* unit (K) was officially assigned to absolute temperature in 1948.

Kelvin’s work thus definitively separated the concept of temperature from real or imaginary fluids as previously done with the boiling point of water or the volume of perfect gases (§1.4.1 & §4.1.1); from here on it would be linked to a specific and quantitative physical experience.

Example 7.2

In order to illustrate the nature of the Kelvin scale, we carry out the following conceptual experiment. We have a Carnot engine and a standard: a solid titanium block that we know melts at 1668 °C. We operate the machine between the standard temperature and an object whose temperature we want to measure. The machine absorbs 200 W and produces 45 W as work. What is the temperature of the object?

The engine absorbs $\dot{Q}_{\text{in}} = 200\,\mathrm{W}$, produces $\dot{W}_{\text{net}} = -45\,\mathrm{W}$; thus, it rejects $\dot{Q}_{\text{out}} = -\dot{Q}_{\text{in}} - \dot{W}_{\text{net}} = -155\,\mathrm{W}$.
By definition, the absolute temperatures are such that they correspond to heat transfers of a Carnot engine: $\frac{T_H}{T_L} = \left|\frac{\dot{Q}_{\text{in}}}{\dot{Q}_{\text{out}}}\right|$ (7/4). Therefore, $T_B = -\frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} T_H = -\frac{-155}{200}(1668 + 273.15) = 1504.4\,\mathrm{K} = 1231.2\,^\circ\mathrm{C}$.

☞ Here we see that Kelvin used the Carnot engine as a thermometer. In order to construct the scale, he carried out this experiment between the two standards of 100 °C and 0 °C, and ensured they were separated by one hundred intervals in absolute units.

7.5 Maximum Efficiency of Machines

The final definition of temperature in 7/4 allows us to return to thermal machines and provide a simple answer to the questions raised by Carnot.

7.5.1 Efficiency of the Carnot engine

In the previous chapter (§6.3.2), we saw that the efficiency of an engine was the ratio between the work produced (useful transfer, $\dot{W}_{\text{net}}$) and the heat received (energy expenditure, $\dot{Q}_{\text{in}}$). We had transformed this expression into another that was perhaps less demonstrative:

$$\eta_{\text{engine}} = 1 - \left| \frac{\dot{Q}_{TL}}{\dot{Q}_{TH}} \right| \tag{6/5}$$

for any thermal engine.

« Thus, we are led to establish the following general proposition: *The motive power of heat is independent of the agents employed to produce it; its quantity is determined solely by the temperatures of the bodies between which, as the final result, the transport of caloric takes place.* »

Sadi Carnot, 1824 [4]

« …our aim must always be *to raise the temperatures and pressures to the highest working limit.* »

Rudolf Diesel, 1893 [23, 24]

In the case of a Carnot engine, with relation 7/4, this expression makes sense by becoming:

$$\eta_{\text{Carnot engine}} = 1 - \frac{T_L}{T_H} \tag{7/6}$$

for a reversible thermal engine,
and where the temperatures are absolute (K).

This expression 7/6 is so remarkable that we must pause for a moment.

The answer to the question Carnot was asking, "what is the maximum amount of work, in theory, that can be obtained from the combustion of a given amount of coal?" is here – and it is astonishing: **it depends only on the high and low temperatures of the engine**!

Two important remarks are necessary here.

- First, this efficiency is not 100 %. Yet, we are discussing machines here without friction or leaks, and with infinitely slow movements. By contrast, if we can ignore all practicalities, nothing is there to prevent an electric motor or an alternator from reaching an efficiency of 100 %.

  Therefore, even before addressing the inevitable technological difficulties associated with designing a real-world machine, the engine designer is limited by the fundamental nature of heat in what he or she can obtain from their machine. In the following chapters, we will address the irreversibilities observed in real engines, which will further reduce the efficiency calculated above.

- Secondly, this equation is a strong argument for increasing the combustion temperature in engines.

  In practice, the low temperature T_L is limited by the ambient air temperature. The only remaining parameter to increase the efficiency of an ideal engine is the temperature T_H. This relationship explains the surprising efforts made by engine designers to use high temperatures (and correspondingly, high pressures), even though real engines are far from reversible.

In summary, we can answer Carnot's question as follows: heat loss from the engine (wasted energy) is fundamentally inevitable. The losses are minimized, but not avoided, when the temperature at which the coal is burned is high, and the ambient temperature is low.

Example 7.3

The maximum achievable temperature in an engine is 600 °C (1112 °F), and the temperature at which exhaust gases are discharged is 100 °C (212 °F). What is the maximum efficiency achievable by the engine?

In order to achieve the most efficient conversion, the engine must be reversible. Thus, with equation 7/6: $\eta_{\text{engine}} = \eta_{\text{Carnot engine}} = 1 - \frac{T_L}{T_H} = 1 - \frac{100+273.15}{600+273.15} = 57.3\,\%$.

☞ Make sure to use absolute temperatures here – at this point, using degrees Celsius or Fahrenheit in the calculation would be unforgivable.

☞ The technological specifics of the engine (displacement, injection method, etc.) may possibly bring it close to 57.3 %, but will never bring it beyond that value.

☞ We have indeed found the result obtained previously in example 7.1 p. 190, with a much simpler calculation.

7.5.2 Efficiency of the Carnot refrigerator

We saw in §6.3.3 that the efficiency of a refrigerator is the comparison between the heat extracted from the cold source (useful transfer, $\dot{Q}_{\text{in}}$) and the work input (energy expenditure, $\dot{W}_{\text{net}}$). We had expressed this efficiency with the obscure expression:

$$\eta_{\text{refrigerator}} = \frac{1}{\left|\frac{\dot{Q}_{TH}}{\dot{Q}_{TL}}\right| - 1} \tag{6/7}$$

for all refrigerators.

When it comes to a Carnot refrigerator, this efficiency is a function of temperature only (7/4) and thus we have:

$$\eta_{\text{Carnot refrigerator}} = \frac{1}{\frac{T_H}{T_L} - 1} \tag{7/7}$$

for a reversible refrigerator,
where temperatures are absolute (K).

The same remarks as above apply here: firstly, the efficiency of a refrigerator or an air conditioner never reaches infinity (an infinite COP refrigerator would operate without any work input). Secondly, this efficiency reduces when the refrigeration temperature T_L is reduced. In other words, when cooling an object with an ideal refrigerator, selecting a lower temperature is more expensive, not merely because more heat needs to be extracted from the object, but also because the efficiency of the extraction decreases.

Example 7.4

A refrigerator must bring the cold chamber to 5 °F (−15 °C) in a room at 77 °F (25 °C). What is the maximum achievable efficiency?

The maximum efficiency would be achieved with a reversible refrigerator, allowing us to obtain, with equation 7/7, $\eta_{\text{refrigerator}} = \eta_{\text{Carnot refrigerator}} = \frac{1}{\frac{T_H}{T_L}-1} = \frac{1}{\frac{25+273.15}{-15+273.15}-1} = 6.45$.

☞ By carrying out the same calculation between temperatures of 365 °F and 437 °F (185 °C and 225 °C), we obtain an efficiency of 11.45: this depends not only on the temperature difference but also on their absolute values.

7.5.3 Efficiency of the Carnot heat pump

We saw in §6.3.4 that the efficiency (or Coefficient of Performance, COP) of a heat pump is defined as the ratio of the heat supplied at high temperature to the work input (6/8). We had then transformed this definition with the expression:

$$\eta_{\text{heat pump}} = \frac{1}{1 - \left|\frac{\dot{Q}_{TL}}{\dot{Q}_{TH}}\right|} \tag{6/9}$$

for all heat pumps.

Using relation 7/4, when the machine is reversible, we can express this efficiency solely based on the high and low temperatures:

$$\eta_{\text{Carnot heat pump}} = \frac{1}{1 - \frac{T_L}{T_H}} \tag{7/8}$$

for a reversible heat pump,
where temperatures are absolute (K).

As with a refrigerator, the COP of a heat pump cannot be infinite: it is bounded by the extreme temperatures reached in the cycle. The higher the temperature at which heat Q_{out} is delivered, and the lower the efficiency that can be achieved.

Example 7.5

A heat pump is used to heat water to 120 °C (248 °F) in an environment at −5 °C (23 °F). What is the maximum achievable efficiency?

The maximum efficiency would be achieved with a reversible heat pump, allowing us to obtain, with equation 7/8, $\eta_{\text{heat pump}} = \eta_{\text{Carnot heat pump}} = \frac{1}{1-\frac{T_L}{T_H}} = \frac{1}{1-\frac{-5+273.15}{120+273.15}} = 3.15$.

A Bit of History: Rudolf Diesel's Dream

*

This is the story of an engine born in the margin of thermodynamics lecture notes. "*Kann man Dampfmaschinen konstruieren, welche den vollkommenen Kreisprozess ausführen, ohne zu sehr kompliziert zu sein?*": can we build steam engines that can perform the ideal cycle without being too very complex? The student Rudolf Diesel asked himself this question in the margin of his notes in 1878 in Munich, realizing that the engine cycle followed by the steam engines of his time inevitably condemned them to mediocre efficiencies.

In this way, the concept of a *rational heat engine* would mature over the years, an engine whose characteristics were finally published in 1893 [23, 24]. Rudolf Diesel is unequivocal: "an examination of their operating theory will show that gas and air engines operate on a defective principle, and no improvement will produce better results as long as this principle is retained". He was no kinder to the designers of steam engines.

The main features of the proposed engine strictly stemmed from physical precepts: it was about getting as close as possible to the Carnot cycle, by "*producing the highest temperature of the cycle (the combustion temperature) not through and during combustion, but before and independently of it, entirely through the compression of ordinary air*". This was followed by combustion at constant temperature, controlled by progressive fuel injection. Only the exhaust and intake (done at constant pressure with a four-stroke cycle) deviated from the Carnot cycle.

The characteristics announced on paper give food for thought: the maximum compression pressure must be at least $p_2 = p_1 \left(\frac{T_\text{combustion}}{T_\text{initial}}\right)^{\frac{\gamma}{\gamma-1}}$ (4/37), which lead Rudolf Diesel to 300 bar (4350 psi) – twenty times more than existing engines! The concept of direct injection, which is made necessary by the high temperatures reached during compression to avoid premature combustion, is convincing. However, many details were lacking on how to handle coal dust – the fuel chosen by Rudolf Diesel for its abundance and low cost – so as to allow its direct injection into the cylinders in practice.

Figure 7.13: Rudolf Diesel in 1883.

Photo by unknown author (public domain)

Despite everything, Rudolf Diesel, after a promising start in the design of refrigeration systems, managed to convince the *Maschinenfabrik Augsburg-Nürnberg* company, known today as MAN, to finance his research. They would be challenging: the transition from theory to practice took four years. Many ambitions were scaled back: the maximum pressure decreased to 90 and then 40 bar (580 psi), the coal dust was abandoned in favor of a crude oil for easier handling. Since the structural limits of the engine constrained the cycle, the isothermal combustion was replaced by isobaric at maximum pressure. The second prototype, a single-cylinder engine nearly three meters tall (figure 7.14), was the first to operate autonomously: eight minutes in February 1894. The performance of the third prototype (figure 7.15) was independently measured in 1897: 17 hp at 154 rpm, and an efficiency of 26.2 %. This efficiency was twice that of its contemporaries with internal combustion, and four times that of the best steam engines!

Man quickly started sales of the *rational engine*, renamed as *Diesel engine*, which gradually met success in Europe. Its operational regularity, reliability, and especially its low fuel consumption justified its significant purchase cost: due to the materials and precision manufacturing it required, its price per watt of power was about three times higher than its competitors. The patents filed by Rudolf Diesel brought him a significant income.

The numerous documents left behind make Diesel a compelling figure: cultivated, diligent, and intelligent (he excelled in all his studies), he had a very keen perception of the economic and social upheavals caused by the rapid mechanization of industry and transportation at the end of the 19th century [33, 34, 41]. After a harsh and miserable childhood, expelled from France and then England, he nurtured a strong social ideal that lead him to write *Solidarismus* ("*the rational and economic salvation of humanity*", 1903 [25]). For him, the decentralization of mechanical power production, for small businesses or collectives, for example, would constitute a decisive social advancement.

Despite the remarkable success achieved in fifteen years, Rudolf Diesel struggled to find fulfillment. He was constantly the target of legal disputes, since his critics and competitors argued – not entirely without merit – that the engines he commercialized were ultimately very different from the machine described in his patent. The nationalist tensions leading up to the outbreak of World War I shook him. A poor financial manager, he made multiple unreasonable expenses and ruinous investments, and, to top it all, he was plagued by severe migraines and medical problems. In 1913, the man seemed tormented by his own ethical and philosophical questions. His engines exclusively produced power in factories and power plants: did they ultimately contribute to the emancipation or the servitude of the working classes? He ended his life in September.

The tragic disappearance of its creator would not suffice to slow down the progression of the Diesel engine. The technological obstacles to its adoption in transportation, in particular the delicate fuel injection system, were overcome one by one.

Figure 7.14: The second prototype developed at MAN by Rudolf Diesel, and the first to operate independently, in February 1894. It has only one cylinder with a diameter of 22 cm, and the direct fuel injection is done by a compressed air circuit. The engine is now exhibited at the headquarters of the MAN company.

Photo CC-BY-SA *MAN SE*

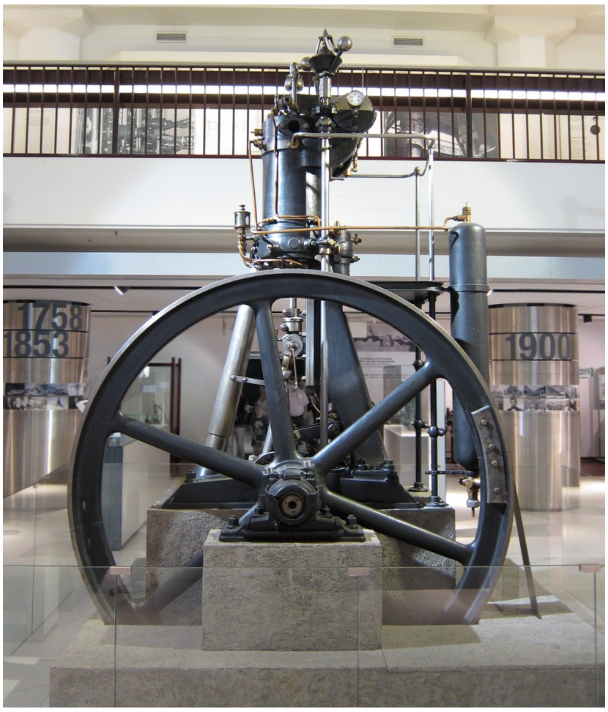


Figure 7.15: Diesel's third prototype, and his first operational engine. The cylinder diameter was 25 cm and the stroke reached 40 cm. It would be tested at the technical university of Munich where it achieved 26.2 % efficiency in 1897. It is exhibited at the *Deutsches Museum.*

Photo CC-BY-SA *Olivier Cleynen*

Figure 7.16: A nine-cylinder *Sulzer RTA76* Diesel engine producing 25 MW of power at 95 rpm. This unit is installed here in a factory but the model is commonly used to propel merchant ships.

Photo CC-BY-SA *by de:Wikipedia User:Sleipnir*

Today, it is used wherever constraints of economy and durability take precedence over lightness and responsiveness. In merchant ships, Diesel engines several stories high operate on highly turbocharged two-stroke cycles. In these engines exceeding 2000 tons and 18 000 hp (figure 7.16), the cylinders move slowly on strokes of over two meters, allowing for nearly isothermal combustion at 80 rpm and an efficiency exceeding 50 %.

At the other end of the spectrum, Diesels powering the smallest utility vehicles benefit from numerous systems to increase their responsiveness and extend their power and torque ranges. In these tiny machines of 80 hp rotating beyond 2000 rpm, electronic control systems perform up to four fuel injections at 2000 bar directly into the cylinder for each combustion, optimizing combustion based on the demanded power [46].

Ultimately, there is probably not a product in today's industry that we manufacture whose materials or components have not been extracted, assembled, and transported without the contribution of power from a Diesel engine. A great achievement for a curious student!

Problems

The properties of water are tabulated in Steam Tables 1, 2, and 3 (see Appendix A1 p. 307)

Air is considered an ideal gas.

$$c_{v\,(\text{air})} = 718\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1} \qquad R_{\text{air}} = 287\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$$
$$c_{p\,(\text{air})} = 1005\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1} \qquad \gamma_{\text{air}} = 1.4$$

We assume that for a reversible adiabatic process (without heat transfer and infinitely slow), the properties of air are linked according to the following three relationships:

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{v_2}{v_1}\right)^{\gamma-1} \tag{4/36}$$

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \tag{4/37}$$

$$\left(\frac{p_1}{p_2}\right) = \left(\frac{v_2}{v_1}\right)^{\gamma} \tag{4/38}$$

We also assume that during a reversible isothermal process (at constant temperature and infinitely slow) of an ideal gas, the work done in an open or closed system is:

$$w_{1\to2} = R\,T_{\text{cst.}} \ln\left(\frac{p_2}{p_1}\right) = R\,T_{\text{cst.}} \ln\left(\frac{v_1}{v_2}\right) \tag{4/29}$$

Finally, we assume that the efficiencies of thermal machines based on a Carnot cycle are expressed as a function of the absolute temperatures as follows:

$$\eta_{\text{Carnot engine}} = 1 - \frac{T_L}{T_H} \tag{7/6}$$

$$\eta_{\text{Carnot refrigerator}} = \frac{1}{\frac{T_H}{T_L} - 1} \tag{7/7}$$

$$\eta_{\text{Carnot heat pump}} = \frac{1}{1 - \frac{T_L}{T_H}} \tag{7/8}$$

7.1 Maximum Efficiency of an Engine

What is the maximum efficiency that a steam power plant can reach when operating in the atmosphere at room temperature (15 °C or 59 °F), with a maximum temperature of 800 °C (1472 °F)?

7.2 Maximum Efficiency of a Refrigerator

What is the theoretical maximum efficiency that a household freezer could reach when operating between temperatures of −6 °C and 20 °C (21.2 °F and 68 °F)?

For what reason(s) is the cop reached by conventional freezers (around 3) lower than this value?

7.3 Maximum Efficiency of a Heat Pump

A person wants to install a heat pump to heat their home with a power of 10 kW.

1. Explain briefly why the performance of a heat pump is expressed as:

$$\eta_{\text{heat pump}} = \left|\frac{\dot{Q}_{\text{out}}}{\dot{W}_{\text{net}}}\right| \tag{7/9}$$

2. Estimate the theoretical minimum consumption of the pump on a very cold evening ($T_{\text{ext.}} = -12\,^\circ\text{C} = 10.4\,^\circ\text{F}$; $T_{\text{int.}} = 20\,^\circ\text{C} = 68\,^\circ\text{F}$).
3. What will be the minimum consumption of the heat pump when the internal and external temperatures are 17 °C and 16 °C respectively?
4. What will be the theoretical minimum consumption of the heat pump in the case where the internal and external temperatures are identical? What happens in theory if the external temperature is higher than inside?

7.4 Carnot Cycle

Since this cycle plays a central role in thermodynamics, it is useful to be able to describe it precisely:

1. Describe briefly the four phases of a Carnot engine cycle, describing the direction of heat transfers.
2. Why are heat transfers isothermal?
3. Is it preferable to use a perfect gas or a liquid-vapor mixture to perform this cycle?
4. What practical problems does the Carnot cycle pose?

7.5 Carnot Steam Engine

An attempt is made to set up a steam power plant based on the Carnot cycle to generate electricity (figure 7.17). The boiler operates at a maximum temperature of 527 °F (275 °C) and admits water in the state of saturated liquid. When the water leaves the boiler and enters the turbine, it is in the state of saturated vapor.

1. Draw the processes undergone by the water during one cycle on a pressure-volume diagram, qualitatively (that is, without showing numerical values).
2. At what pressure would the water need to be cooled to achieve an efficiency of 40 %?
3. What would be the power supplied by the plant if its mass flow rate was $9\,\mathrm{kg\,s^{-1}}$ (19.8 lb/s)?
4. What would the cycle and the machine look like if the steam were to be further heated at a constant temperature of 527 °F at the outlet of the boiler? How would the efficiency of the engine vary then?

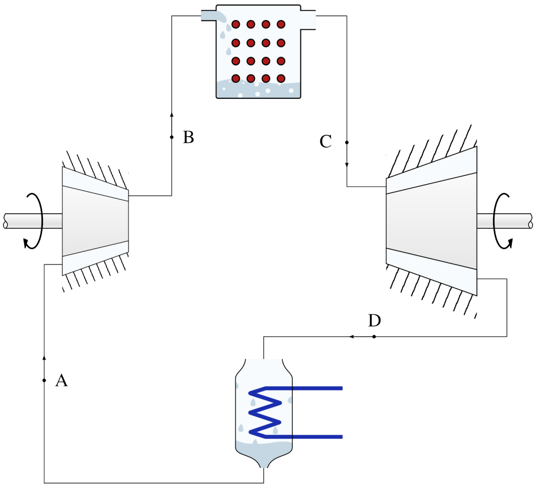


Figure 7.17: Schematic representation of a steam power plant operating with the Carnot cycle.

Diagram CC-BY-SA *Olivier Cleynen*

7.6 Reversibility of Machines

Briefly show that it is impossible to design a heat pump with efficiency higher than that achieved by a reversible machine, for example in the same way we did with an engine in fig. 7.7 p. 185.

7.7 Ideal Turbine Engine

A group of engineers in an engineering company is working on an air engine concept, operating in a steady-flow state using turbines and compressors.

The engineers are using the Carnot cycle as a starting point. They plan to be able to supply heat at a temperature of 600 °C and reject heat at a temperature of 20 °C. The pressure is 1 bar at the inlet of the adiabatic compressor and 30 bar at the inlet of the adiabatic turbine. These characteristics give the engine a specific net power of $70\,\mathrm{kJ\,kg^{-1}}$.

1. Represent schematically the general arrangement of this hypothetical engine, showing the path followed by the air, and all heat and work transfers.
2. Starting from the definition of the efficiency of an engine, show that the efficiency of a reversible engine can be quantified by the equation

$$\eta_{\text{Carnot engine}} = 1 - \frac{T_L}{T_H} \tag{7/6}$$

3. What power will need to be supplied to the engine in the form of heat?
4. What will be the power rejected in the form of heat?

Of course, the Carnot cycle is impractical in an industrial application and the group of engineers immediately adopts a modification. In order to be able to supply heat by internal combustion, it is necessary to later vent the air from the engine. Thus, in the modified engine, the expansion in the adiabatic turbine is interrupted when the pressure reaches 1 bar, and the “used” air is then rejected into the atmosphere. The rest of the engine is not affected.

5. Draw the new engine cycle on a pressure-volume diagram, qualitatively, comparing it to that of the Carnot cycle.
6. What is the temperature of the air when it is rejected from the engine?
7. What is the reduction in power of the adiabatic turbine compared to the ideal engine?
8. What amount of power is saved by removing the compressor that was rejecting heat?
9. What is now the efficiency of the engine?

7.8 Irreversibilities in a Refrigerator

We propose to study the operation of a refrigerator starting from a theoretical cycle allowing for maximum efficiency.

The refrigerator operates strictly on a Carnot cycle, in a steady-flow state, with a liquid-vapor mixture.

1. Represent the refrigeration cycle on a pressure-volume diagram, indicating the direction of heat and work transfers.

Of course, in practice, adiabatic compression and expansion cannot be carried out reversibly.

2. Represent the irreversible cycle on the pressure-volume diagram.
3. How will each of the heat and work transfers vary compared to the theoretical case?
4. Briefly show that these changes lead to a decrease in the efficiency (the COP) of the refrigerator.

7.9 Refrigeration in Stages

A chemical plant uses a refrigeration system to control the temperature of hazardous products. We aim to study the least inefficient refrigeration system to equip it, here based on the Carnot cycle with a perfect gas.

The minimum refrigeration temperature is $-50\,^{\circ}\mathrm{C}$ and the heat is rejected at $40\,^{\circ}\mathrm{C}$.

While studying the characteristics of the Carnot cycle, a beginner engineer observes that the efficiency of the refrigerator increases if the heat rejection temperature is lowered (eq. 7/7 p. 198).

S/he proposes to configure the refrigerator in such a way that it rejects heat at only $10\,^{\circ}\mathrm{C}$. This heat at $10\,^{\circ}\mathrm{C}$ would then be captured by a heat pump which would in turn bring it to $40\,^{\circ}\mathrm{C}$.

1. Show that if the refrigerator operates on a reversible cycle, the proposed modification can only increase (or at best, keep the same) the total consumption of the refrigeration system.
2. Draw the cycle of the refrigerator and of the heat pump as proposed by the engineer on the same pressure-volume diagram, qualitatively.
3. Draw, on a pressure-volume diagram, the cycles that would be followed in the two machines if their expansion phases were adiabatic (without heat transfer), but non-reversible.

7.10 Gasoline Engine Based on a Carnot Cycle

We aim to quantify the minimum gasoline consumption that could result from a piston-cylinder automotive engine generating $100\,\mathrm{kW}$ of power (about $130\,\mathrm{hp}$), given some practical constraints imposed by the limited available volume and weight limits:

- The compression ratio (namely, the ratio $\frac{v_{\text{max.}}}{v_{\text{min.}}}$) is 12 during the adiabatic phases (to limit mechanical constraints);
- The maximum temperature is $1300\,\mathrm{K}$ (imposed by material resistance);
- The engine has four cylinders, each performing 400 cycles per minute.

The engine is fueled by gasoline with a specific heat of combustion of $40\,\mathrm{MJ\,kg^{-1}}$.

If we consider the best engine that can be designed:

1. At what temperature would the heat be rejected?
2. What would be the efficiency of the engine?
3. What would be the amount of heat to be supplied for each combustion, and the corresponding fuel mass?
4. What would be the hourly gasoline consumption?

Answers

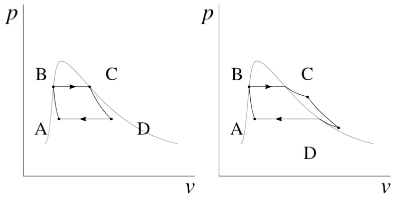
7.1 $\eta_{\text{max.}} = 73.1\,\%$ (eq. 7/6, see example 7.3 p. 198).

7.2 1) $\text{COP}_{\text{max.}} = 10.3$ (eq. 7/7, see example 7.4 p. 199);
2) Non-reversible compressions and expansions (thus $w_{4\to3} + w_{2\to1} > 0$), especially if a valve is used (§6.2.3); Non-isothermal heat transfers.

7.3 1) see §6.3.4;
2) $\dot{W}_{\text{net}} = \frac{\dot{Q}_{\text{out}}}{\eta_{\text{max.}}} = +1.09\,\text{kW}$; 3) $\dot{W}_{\text{net}} = +34.5\,\text{W}$ (!), $\dot{W}_{\text{net}} = 0\,\text{W}$; and when $T_{\text{ext.}} > T_{\text{int.}}$, $\dot{W}_{\text{net}}$ becomes negative: the heat pump operates as an engine…

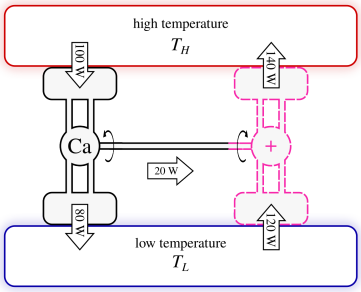
7.4 1) see §7.3.4, especially figures 7.8 and 7.9;
2) see §7.3.3;
3) It doesn't matter at all!
4) Its volume and its power ratio are very large, and its power is infinitely small…

7.5



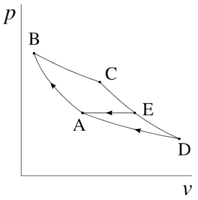
We see that when performed under the saturation curve, the Carnot cycle is already easier to perform in practice, since heat transfers occur at constant pressure (and thus do not require a moving part);
2) $T_B = (1 - \eta)T_H = 55.74\,^\circ\text{C}$; thus by interpolation between 55 and $60\,^\circ\text{C}$ we get $p_{\text{sat. }55.74\,^\circ\text{C}} = 0.1285\,\text{bar}$. The condenser is thus depressurized;
3) $q_{\text{in}} = h_{LV\,275\,^\circ\text{C}} = +1574.3\,\text{kJ}\,\text{kg}^{-1}$: we obtain $\dot{W}_{\text{net}} = -\dot{m}\eta q_{\text{in}} = -566.7\,\text{kW}$;
4) The power ratio and the complexity of the machine will increase, but the efficiency will remain unchanged!

7.6



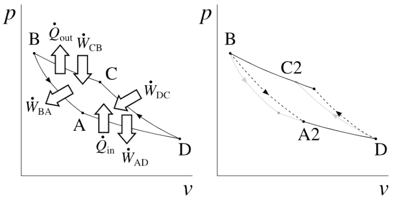
A heat pump like this could be powered by a Carnot engine; together, they would form a machine capable of carrying heat from T_L to T_H without the need for external work (with the arbitrary values shown here, $\dot{Q}_{\text{out together}} = -40\,\text{W}$).

7.7



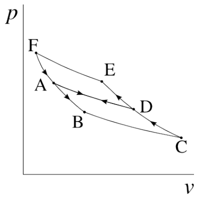
1) This is the arrangement shown in fig. 7.9 p. 189;
2) By starting from the definition 6/4: $\eta_{\text{engine}} \equiv \left|\frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}}\right| = -\frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = -\frac{-\dot{Q}_{\text{in}}-\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} = 1 + \frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} = 1 - \left|\frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}}\right| = 1 - \left|\frac{\dot{Q}_{TH}}{\dot{Q}_{TL}}\right|$; With definition eq. 7/4 p. 194 we arrive at the requested equation 7/6;
3) $q_{\text{in}} = -\frac{w_{\text{net}}}{\eta_{\text{engine}}} = +105.4\,\text{kJ}\,\text{kg}^{-1}$;
4) $q_{\text{out}} = -w_{\text{net}} - q_{\text{in}} = -35.4\,\text{kJ}\,\text{kg}^{-1}$;
6) The isothermal compressor is removed, the adiabatic turbine is truncated. $T_E = T_C \left(\frac{p_E}{p_C}\right)^{\frac{\gamma-1}{\gamma}} = 330.4\,\text{K} = 57.3\,^\circ\text{C}$ (eq. 4/37);
7) $w_{\text{lost}} = w_{\text{isentropic turbine 1}} - w_{\text{isentropic turbine 2}} = -37.54\,\text{kJ}\,\text{kg}^{-1}$;
8) $w_{\text{saved}} = w_{\text{isentropic compressor}} = +35.4\,\text{kJ}\,\text{kg}^{-1}$;
9) $\eta_{\text{engine 2}} = 64.4\,\%$, which is a -2 point change, very honorable given the considerable simplification of the machine!

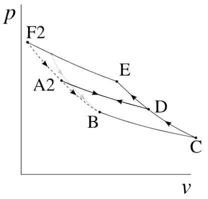
7.8



3) $w_{1\to4}$ decreases, $w_{4\to3}$ and $w_{3\to2}$ increase, $w_{2\to1}$ decreases; $q_{3\to2}$ increases and $q_{1\to4}$ decreases;
4) Since w_{net} increases and $q_{\text{in}} = q_{1\to4}$ decreases, the $\text{COP} = \frac{q_{\text{in}}}{w_{\text{net}}}$ necessarily decreases.

7.9





Let's bet that the student did better than the beginner engineer in the problem: with two reversible systems in series pumping a quantity q_{in} of heat at temperature $T_1 = -50\,^{\circ}\text{C}$, with exchange temperature $T_2 = 10\,^{\circ}\text{C}$ and final high temperature $T_3 = 40\,^{\circ}\text{C}$, the necessary work is $w_{\text{total}} = w_{\text{net1}} + w_{\text{net2}} = \eta_1 q_{\text{in}} + \eta_2 (q_{\text{in}} + w_{\text{net1}}) = \left(\frac{T_2}{T_1} - 1\right) q_{\text{in}} + \left(\frac{T_3}{T_2} - 1\right)(q_{\text{in}} + w_{\text{net1}}) = \left(\frac{T_2}{T_1} - 1\right) q_{\text{in}} + \left(\frac{T_3}{T_2} - 1\right)\left(q_{\text{in}} + \left(\frac{T_2}{T_1} - 1\right) q_{\text{in}}\right) = \left(\frac{T_2}{T_1} - 1\right) q_{\text{in}} + \left(\frac{T_3}{T_2} - 1\right)\left(\frac{T_2}{T_1} q_{\text{in}}\right)$; thus $\eta_{\text{together}} \equiv \frac{q_{\text{in}}}{w_{\text{total}}} = \frac{1}{\frac{T_2}{T_1} - 1 + \frac{T_3 T_2}{T_2 T_1} - \frac{T_2}{T_1}} = \frac{1}{\frac{T_3}{T_1} - 1}$, which is the efficiency of a single reversible machine operating between T_1 and T_3. Stacking two machines in series therefore brings no theoretical advantage.

7.10

1) At the outlet, $T_1 = 208\,^{\circ}\text{C}$ (eq. 4/37);
2) $\eta_{\text{engine}} = 63\,\%$;
3) $\dot{Q}_{\text{in}} = 158.8\,\text{kW}$ so $Q_{\text{combustion one cylinder}} = 5.955\,\text{kJ}$ at each combustion. We obtain, for a cylinder, $m_{\text{fuel combustion}} = \frac{Q_{\text{combustion one cylinder}}}{q_{\text{fuel}}} = 0.149\,\text{g}$;
4) $\dot{m}_{\text{fuel}} = 14.3\,\text{kg/h} = 31.5\,\text{lb/h}$.

CHAPTER 8
Entropy

or

The Terrifying Prophecy of Professor Clausius

Chapter 8 – Entropy

Executive summary for chapter 8

Entropy is a property of bodies. We quantify its changes in order to measure the irreversibility of energy transfers (always undesirable for the engineer). The total entropy always increases during irreversible transfers: of work (with abrupt motion), of heat (with temperature gradient).

Introduction

We present here the most powerful and difficult concept of thermodynamics. This chapter 8 (*entropy*) aims to approach it in the most pragmatic way possible, starting from two questions:

- What does entropy represent?
- Why do physicists and engineers quantify its variations?

8.1 The Concept of Entropy

8.1.1 What is entropy used for?

At the very beginning of chapter 1 (*fundamental concepts*), we had seen that we could conceptualize *energy* as "a quantity that never changes when things evolve". Thus, we quantify energy to determine the limits of what is possible: for example, we know that with 500 J, a stationary body of mass 10 kg cannot reach a speed greater than $10\,\mathrm{m\,s^{-1}}$ (eq. 1/5 p. 15).

However, our intuition and daily experience teach us that many transformations can only occur in one direction (figure 8.1). For example, there is as much energy in a glass of water on the edge of a table as in the same glass shattered with the water spilled on the floor. Yet we know, or more precisely, we have a deep conviction, that it is *possible* for the glass to fall and break, but *impossible* for the shards and water on the floor to spontaneously gather together into a full glass on the table.
Thus, the quantification of energy is not entirely sufficient to determine *what is possible*. We would also like to be able to predict absolutely and quantitatively the direction in which energy can or cannot be transformed.

Entropy was conceived to address this question. By the end of this chapter, we will have a tool to *calculate* the direction of a process, that is, to mathematically predict which of two situations separated in time must have occurred before the other.

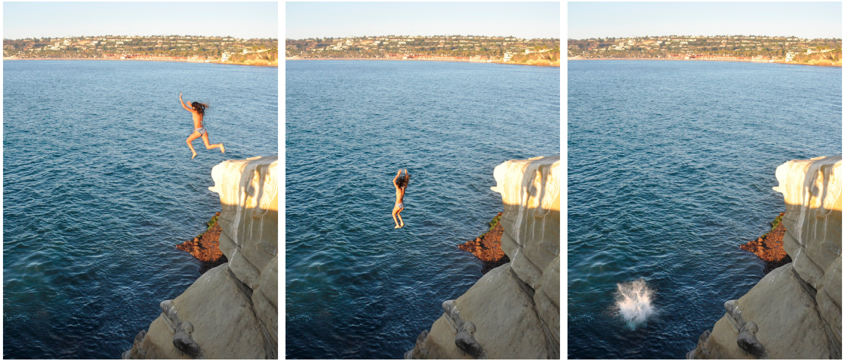


Figure 8.1: We have the intuition and an inner conviction that these three photos were taken in a very particular order. A calculation of the *entropy* in these three situations, in which the *energy* is the same, allows us to determine this order by associating a calculable quantity with our intuition.

images derived from Photos CC-BY-SA *by Jarosław W. Tuszyński*

8.1.2 How can we determine the direction of a process?

In the vocabulary of thermodynamics, the concept of a “one-way change” is of course called *irreversibility*. We had already discussed irreversibility in section §2.4.3, where we had determined that it had two main causes:

- The conversion of work into heat, through friction and turbulence;
- The transfer of a quantity of heat between two bodies at different temperatures.

Irreversible processes in fluids invariably lead to states where the temperature, pressure, or volume are greater than they would have been with a reversible process.

> « For this purpose let us conceive the matter, after the changes of condition which has to be examined in this matter, reduced to its original condition by any reversible operation. We shall thereby obtain a small cyclical process, to which the equation (II) will be just as applicable as to the whole. Consequently, if we know the quantities of heat which the matter has received during the process, and the temperatures which correspond thereto, the negative integral $-\int \frac{\mathrm{d}Q}{T}$ will give the uncompensated transformation involved therein. »
>
> Rudolf Clausius, 1856 [16, 18, 20]

In order to quantify the irreversibility of a process, we will quantify *the amount of heat that one would need to remove from the body to bring it back to its initial state reversibly.* By subtracting to this amount the heat that was actually transferred, we obtain the heat that was somehow needlessly created during the process. Moreover, the lower the temperature at which this heat is created, and the less of it can be transformed into work (§7.5.1). We will thus “penalize” the heat cost by dividing it by the temperature.

In this way, we will obtain a quantity in joules per kelvin – the entropy created during the process – which will be zero during reversible processes and will always be positive during irreversible processes. It is this creation that will be the unmistakable sign that the process is possible only in one direction.

8.2 Definition

8.2.1 Entropy is a property

Let us begin by acknowledging the fact that entropy is a *physical property*, meaning something that characterizes the state of a system. Put another way: if we consider a portion of the universe at a given moment (a system), we find that this system has a mass, a volume, a temperature: these quantities describe its current state (which is why they are called *state quantities*). Entropy is one of these quantities.

By contrast, we could say that heat, work, or electric current are not properties: they are not quantities that describe an object, but rather a transfer between two objects (*path quantities*).

Therefore, we will always think of entropy as the entropy “of something” (perhaps as we would say the color, the temperature “of something”). For example, we will say “this body has entropy” or “the entropy of this body is increasing/decreasing”, and not “we are taking/giving entropy to this body”. Rigorously, we say that entropy is an additive state quantity (see the appendices A3 and A4).

8.2.2 Definition

Entropy, noted S, is a physical property.

- When a system undergoes a reversible process, its entropy varies such that:

$$\mathrm{d}S \equiv \left(\frac{\delta Q}{T}\right)_{\text{rev.}} \tag{8/1}$$

where the subscript *rev.* indicates the calculation is done along a reversible path;
$\mathrm{d}S$ is the infinitesimal change in entropy ($\mathrm{J\,K^{-1}}$);
δQ is the infinitesimal amount of (reversibly) supplied heat (J);
and T is the temperature at which the heat transfer occurs (K).

When it transitions from a state A to a state B reversibly, the entropy of a system therefore varies by an amount ΔS:

$$\Delta S = \int_{\mathrm{A}}^{\mathrm{B}} \left(\frac{\delta Q}{T}\right)_{\text{rev.}} \tag{8/2}$$

where the subscript *rev.* indicates the calculation is done along a reversible path.

- When a system undergoes an irreversible process between A and B (as is the case for the majority of real processes), then *a reversible path between these two states must be found* and the integration 8/2 must be performed along it to calculate ΔS.

There is always a reversible way (in fact, there is even an infinity of such ways) to make a system undergo a process between two arbitrary states. For this, the work transfers must be carried out infinitely slowly and the heat transfers must be carried out with infinitesimal temperature differences.

We must be careful here: if one integrates the quantity $\frac{\delta Q}{T}$ along a process where the temperature or pressure are not homogeneous (for example during a rapid expansion, or in a body that has an internal temperature gradient, see §2.4.3), then a result will be obtained which is lower than the actual entropy change ΔS.

The SI unit of entropy S is the $\mathrm{J\,K^{-1}}$ (joule per kelvin); and correspondingly, the *specific entropy* s is defined as:

$$s \equiv \frac{S}{m} \tag{8/3}$$

where m is the considered mass (kg),
and s is its specific entropy ($\mathrm{J\,K^{-1}\,kg^{-1}}$).

In practice, the term “entropy” is often used even if it refers to specific entropy; the symbol and context determine which variable is being referred to.

Example 8.1

We extract 2000 J as heat, reversibly, from a mass of air while maintaining its temperature constant at 30 °C. How much does its entropy change?

Since the process is reversible, we immediately apply equation 8/2:

$\Delta S = S_B - S_A = \int_{\mathrm{A}}^{\mathrm{B}} \left(\frac{\delta Q}{T}\right)_{\text{rev.}} = \left[\frac{1}{T_{\text{cst}}} \int_{\mathrm{A}}^{\mathrm{B}} \delta Q\right]_{\text{rev.}} = \left[\frac{1}{T_{\text{cst}}} Q_{\mathrm{A}\to\mathrm{B}}\right]_{\text{rev.}} = \frac{1}{30+273.15}(-2000) = -6.6\,\mathrm{J\,K^{-1}}$.

☞ We have already explored reversible processes at constant temperature (isothermal processes) in sections §4.4.4 p. 98 and §5.4.4 p. 137. Here, the gas loses 2 kJ of heat and receives 2 kJ of work.

☞ We do not know the value of entropy, but we know that it decreases by 7 joules per kelvin.

☞ The change in entropy should not be confused with the thermal capacity, $c \equiv \frac{\delta q}{\mathrm{d}T} = \frac{1}{m}\frac{\delta Q}{\mathrm{d}T}$ (1/16), where we divide the heat by the *change* in temperature. In this process, the thermal capacity is infinite since $\mathrm{d}T = 0\,\mathrm{K}$.

Once the cooling is done, the air is allowed to expand suddenly: the expansion is irreversible. During this process, only 1000 J is supplied as heat and only 1000 J is recovered as work. At the end of the expansion, the gas is in the same state (same temperature, pressure, and internal energy) as at the very beginning of the experiment. What is the entropy change?

This time, the process is not reversible. We should not consider the heat that is actually transferred, but the heat *that would have been transferred* in a reversible process leading to the same final state.

Fortunately, we know that the gas returns from B to its initial state A; and from A to B the process was reversible. By performing the exact reverse process, we would reverse all heat and work transfers. Thus, along this imaginary process from B to A: $\Delta S = S_{\mathrm{A}} - S_{\mathrm{B}} = \int_{\mathrm{B}}^{\mathrm{A}} \left(\frac{\delta Q}{T}\right)_{\text{rev.}} = -\int_{\mathrm{A}}^{\mathrm{B}} \left(\frac{\delta Q}{T}\right)_{\text{rev.}} = -(S_{\mathrm{B}} - S_{\mathrm{A}}) = +6.6\,\mathrm{J\,K^{-1}}$.

☞ The ΔS corresponds to the actual entropy change; but it is calculated along an imaginary path.

☞ Here we see that the heat actually transferred is not important. It is the heat “that would have needed to be transferred” that interests us.

☞ Here, to simplify the exercise, the gas returns exactly to its initial state A. If it ended up in a different state, we could still calculate the entropy change, as we will learn to do in section §8.3.3.

☞ In this irreversible process from B to A, it is the difference between $\int_{\mathrm{B}}^{\mathrm{A}} \left(\frac{\delta Q}{T}\right)_{\text{rev.}} = +6.6\,\mathrm{J\,K^{-1}}$ and $\int_{\mathrm{B}}^{\mathrm{A}} \left(\frac{\delta Q}{T}\right)_{\text{actual}} = \frac{+1000}{30+273.15} = +3.3\,\mathrm{J\,K^{-1}}$ that will allow us to measure the irreversibility, in other words, to show that with a transfer of 1 kJ we can go from B to A but not from A to B.

8.2.3 Remarks

Let's add three remarks before moving on.

1. Equation 8/2 does not allow for the calculation of the entropy of a system, but only *its change* during the process. In fact, we do not know how to calculate the entropy of an arbitrary body! We will see that this is not important for the engineer.
2. Just like energy, entropy is invisible, odorless, intangible, and inaudible. There is no instrument capable of measuring it. We can only calculate its changes.
3. Entropy changes can only be calculated along reversible processes, which is a very important limitation (no real process of interest to engineers is reversible). However, there are always multiple reversible ways, all equivalent, to reproduce the final state of an irreversible process.

8.3 Changes in Entropy

8.3.1 Analogy with volume

We have seen in chapter 2 (*closed systems*) that when the process is reversible, the work done by a fluid as its volume changes is expressed by equation 2/14:

$$W_{\mathrm{A}\to\mathrm{B}} = -\int_{\mathrm{A}}^{\mathrm{B}} p \, \mathrm{d}V \tag{8/4}$$

for a closed system when the volume changes are infinitely slow.

We could thus propose to *define* volume as being "what varies with pressure when work is done, when the process is reversible", which would amount to the following definition:

$$\mathrm{d}V = -\left(\frac{\delta W}{p}\right)_{\mathrm{rev.}} \tag{8/5}$$

where the subscript *rev.* indicates the calculation is done along a reversible path.

or even the following, more approachable expression, which can be visualized on a pressure-volume diagram (figure 8.2):

$$\Delta V = -\int_{\mathrm{A}}^{\mathrm{B}} \left(\frac{\delta W}{p}\right)_{\mathrm{rev.}} \tag{8/6}$$

where the subscript *rev.* indicates the calculation is done along a reversible path.

We can see that entropy is defined in a similar way, that is, as the variable S that, during a reversible heat transfer, allows us to relate heat to temperature with the relation 8/2:

$$\Delta S = \int_{\mathrm{A}}^{\mathrm{B}} \left(\frac{\delta Q}{T}\right)_{\mathrm{rev.}}$$

where the subscript *rev.* indicates the calculation is done along a reversible path.

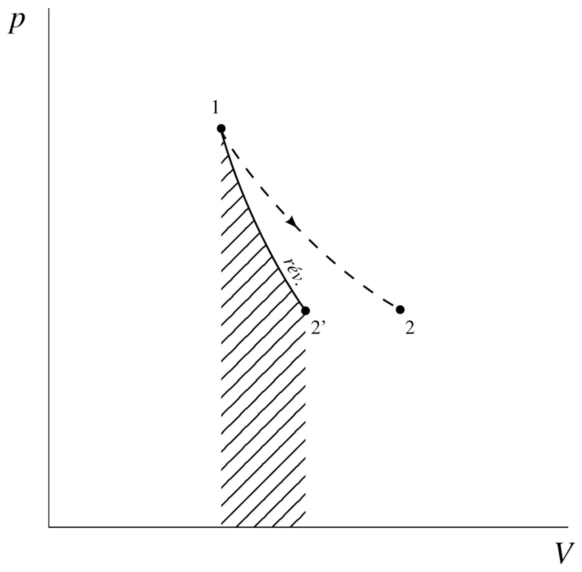


Figure 8.2: Volume changes during adiabatic expansions. The increase in volume is calculable by integrating $\delta W/p$ along a reversible path ($1 \to 2'$), but not along an irreversible path ($1 \to 2$). *Diagram* CC-0 *Olivier Cleynen*

Then we have

$$Q_{\text{A}\to\text{B}} = \int_{\text{A}}^{\text{B}} (T \, \text{d}S)_{\text{rev.}} \tag{8/7}$$

$$q_{\text{A}\to\text{B}} = \int_{\text{A}}^{\text{B}} (T \, \text{d}s)_{\text{rev.}} \tag{8/8}$$

for any process,
where the subscript *rev.* indicates the calculation is done along a reversible path.

In this way, we are able to represent the processes on a *temperature-entropy diagram*. As shown in figure 8.3, the area under the curve of a process will represent the heat transferred in cases where the process is reversible; but in cases where the process is irreversible, it does not.

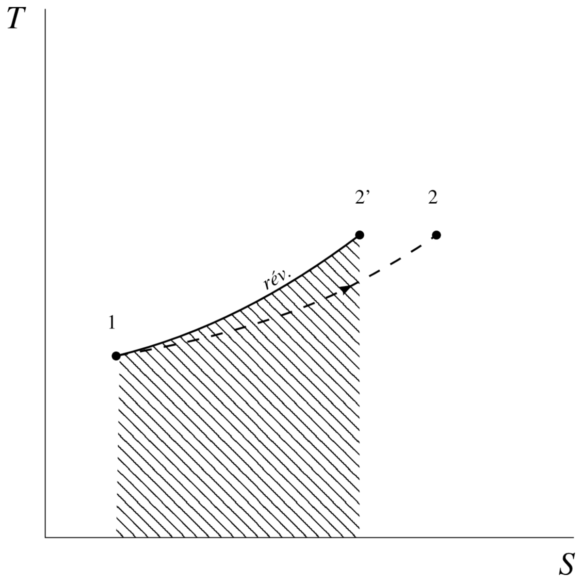


Figure 8.3: Temperature-entropy diagram. During a reversible process, the area under the curve of a T-s diagram represents the transmitted heat $Q_{1\to2'}$; but not when it is irreversible. *Diagram* CC-0 *Olivier Cleynen*

8.3.2 Temperature-entropy diagrams

After six chapters of loyal and dedicated service, the time has come to honorably discharge the pressure-volume diagram, because it is time to make use of our new tool: the temperature-entropy diagram. Even though it is a bit more abstract, the T-s diagram is very useful for describing what happens inside machines because it is easy to plot and allows us to visualize directly the *irreversibility*, which is always undesirable for the engineer.

When a fluid receives or supplies work in a reversible adiabatic manner, then $\Delta s = \int \frac{\delta q}{T} = 0$, since the process is reversible and $\delta q = 0$. A reversible adiabatic process thus occurs at constant entropy – it is iso-entropic, and we call that *isentropic*. We will represent it as a vertical path on temperature-entropy diagrams (figure 8.4).

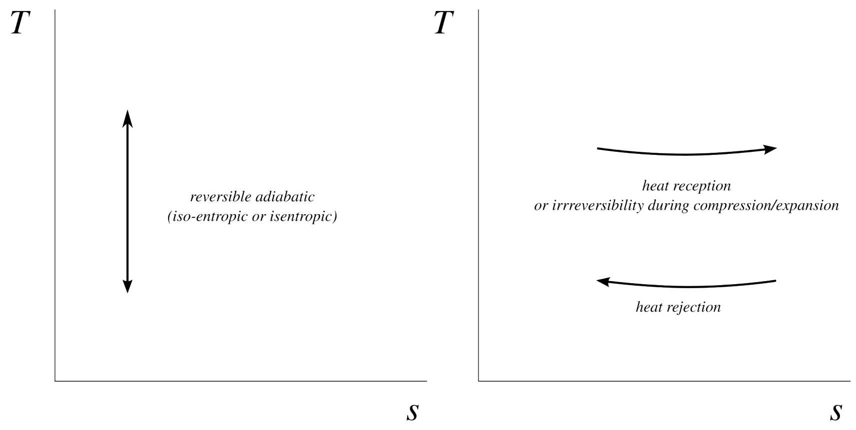


Figure 8.4: elementary processes on temperature-entropy diagrams.

Diagrams CC-0 *Olivier Cleynen*

Transfer of heat, on the other hand, causes a change in the system's entropy (positive when heat is received and negative when it is rejected). On T-s diagrams, we move from left to right while receiving heat or when there is irreversibility in a compression or expansion.

When the system loses heat, its entropy decreases and we move from right to left on the T-s diagrams (figure 8.4).

Note also that when a fluid completes a cycle, the temperature at which entropy decreases can be lower than the temperature at which it increases (just like volume with pressure). The net heat transfer is then negative: the fluid has *absorbed* heat which has been converted into work. If it were to follow the reverse path, the fluid cycle would be a source of heat: this is the operating principle of the refrigerator (§6.2.3).

When the processes are reversible, this net heat is represented by the area enclosed by the path taken by the fluid on a temperature-entropy diagram (figure 8.5).

Finally, we are pleased to note that the Carnot cycle, consisting of two isothermal phases (T = constant) separated by two isentropic phases (s = constant), benefits greatly from being represented on a temperature-entropy diagram, as shown in figure 8.6.

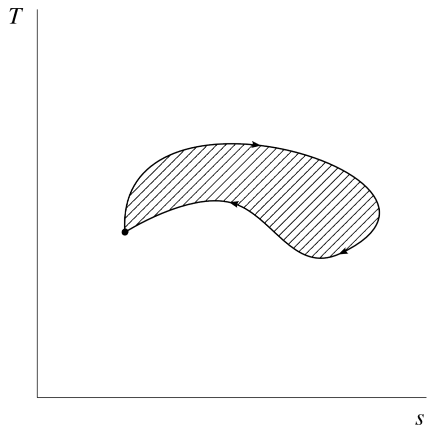


Figure 8.5: Thermodynamic cycle during which heat has been absorbed, and therefore transformed into work. When the path is reversed, heat is rejected (and work absorbed) by the fluid.

Diagram CC-0 *Olivier Cleynen*

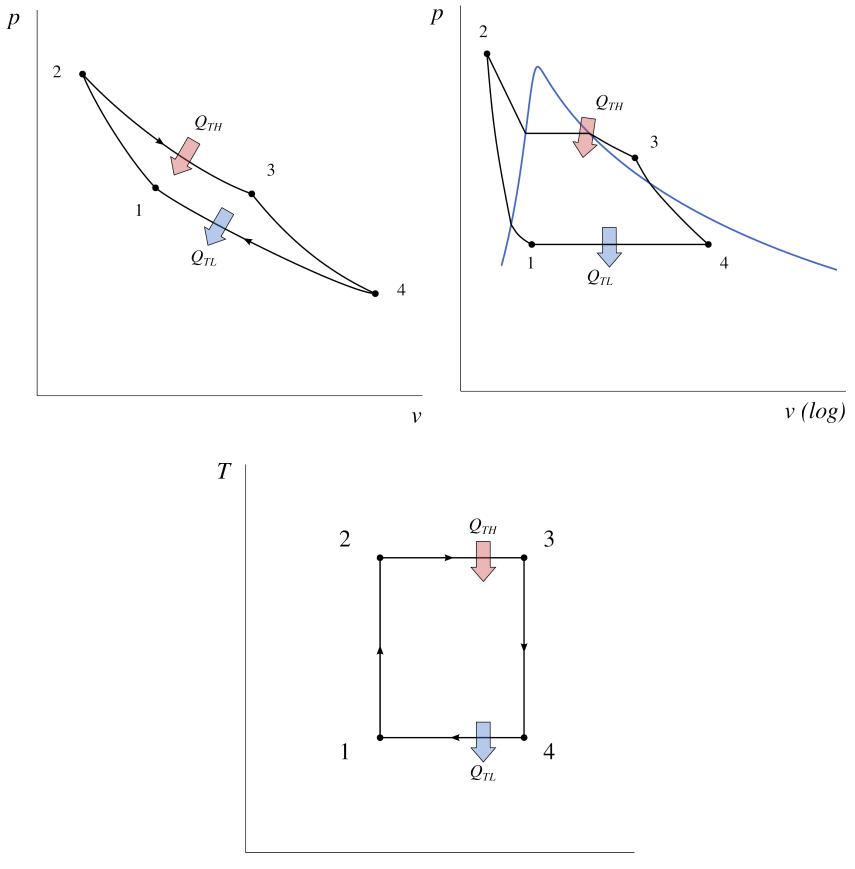


Figure 8.6: Carnot engine cycle, on a p-v diagram for an ideal gas (left), on a p-v diagram for a liquid-vapor system (right), and on a T-s diagram (bottom). Regardless of the fluid used, the temperature-entropy diagram remains the same.

Diagrams 1, 2 and 3 CC-0 *Olivier Cleynen*

8.3.3 Entropy changes of an ideal gas

From now on, we wish to quantify the entropy change in fluids for any arbitrary process. For an ideal gas, the quantification of this change is surprisingly simple.

For any process undergone by a fixed quantity of fluid, we have (2/2):

$$q_{1\to 2} + w_{1\to 2} = \Delta u$$

If we imagine a reversible path between 1 and 2, we can quantify $q_{1\to 2} = -\int_1^2 T\,\mathrm{d}s$ (8/8) and $w_{1\to 2} = -\int_1^2 p\,\mathrm{d}v$ (2/15) along it, and we can write:

$$\int_1^2 T\,\mathrm{d}s - \int_1^2 p\,\mathrm{d}v = \Delta u$$

$$T\,\mathrm{d}s - p\,\mathrm{d}v = \mathrm{d}u$$

$$\mathrm{d}s = \frac{\mathrm{d}u}{T} + \frac{p}{T}\,\mathrm{d}v \tag{8/9}$$

along any reversible process.[1]

Now, if we use an ideal gas, we have $u = c_v T$ (4/11) and $\frac{p}{T} = \frac{R}{v}$ (4/1), thus:

$$\mathrm{d}s = \frac{c_v\,\mathrm{d}T}{T} + \frac{R}{v}\,\mathrm{d}v$$

$$= c_v\frac{\mathrm{d}T}{T} + R\frac{\mathrm{d}v}{v}$$

$$\Delta s = s_2 - s_1 = c_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1} \tag{8/10}$$

$$\Delta s = s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} \tag{8/11}$$

for an ideal gas,
for any process from 1 to 2, reversible or not.

This equation is interesting because it indicates that the entropy change Δs during a process from 1 to 2 depends only on the initial and final states. Even though we started this demonstration along a reversible process, we obtain an expression 8/10 in which the path used does not appear.

It is therefore possible to easily calculate the change entropy of an ideal gas if its other properties are known. Unlike the internal energy u which depends only on temperature, changes in entropy (Δs) also depend on the gas pressure.

« In the deduced expression, the difference $S - S_0$ is again perfectly determined when the initial and final conditions are given, and it is only when forming the integral $\int \frac{\mathrm{d}Q}{T}$ that the manner in which the passage from one to the other took place must be taken into consideration. »

Rudolf Clausius, 1865 [17, 18, 19]

In the case where pressure or specific volume is kept constant, these equations 8/10 and 8/11 become respectively:

$$\Delta s_{v_{\text{const.}}} = c_v \ln\frac{T_2}{T_1} \tag{8/12}$$

$$\Delta s_{p_{\text{const.}}} = c_p \ln\frac{T_2}{T_1} \tag{8/13}$$

for an ideal gas,
for any process at constant volume or respectively at constant pressure.

These two equations 8/12 and 8/13 allow us to plot isochoric (at constant volume) and isobaric (at constant pressure) curves for an ideal gas on a T-s diagram, as shown in figure 8.7.

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[1] This equation 8/9 is even true for any process, but this generalization is simpler to address after equations 8/10 and 8/11.

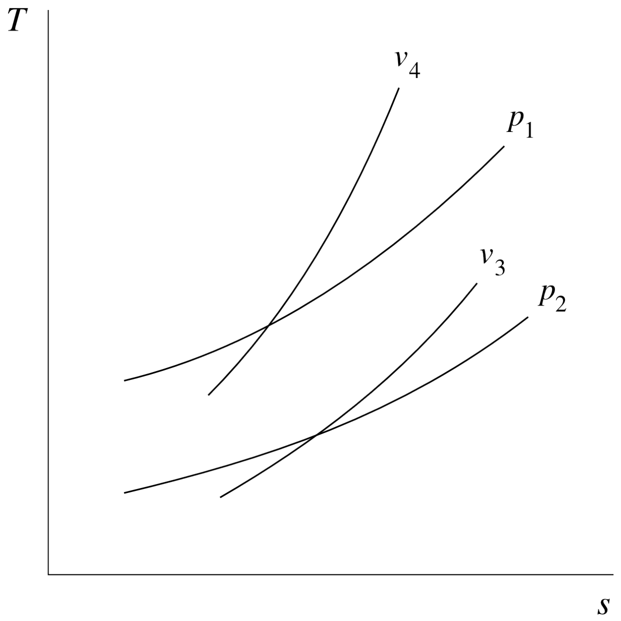


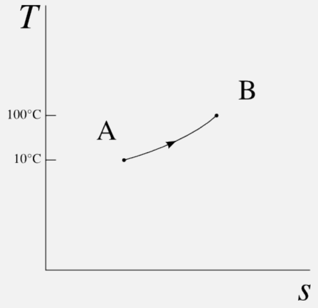
Figure 8.7: Isobaric and isochoric curves on a T-s diagram, for an ideal gas. Here $p_1 > p_2$ and $v_3 > v_4$.

Diagram CC-0 *Olivier Cleynen*

Example 8.2

What is the change in specific entropy of a mass of 2 kg (4.4 lb) of air, when it is heated at constant pressure of 2 bar, from 50 °F to 212 °F (10 °C to 100 °C)?

The process can be drawn qualitatively (that is, without showing numerical values) on a temperature-entropy diagram as follows:



In order to calculate ΔS, we start from equation 8/11, $\Delta s = c_p \ln \frac{T_B}{T_A} - R \ln \frac{p_B}{p_A}$ and here $p_B = p_A$. We thus have $\Delta s = c_p \ln \frac{T_B}{T_A} = 1005 \ln \frac{100+273{,}15}{10+273{,}15} = +277.4\,\mathrm{J\,K^{-1}\,kg^{-1}}$.
The change in entropy is $\Delta S = m\,\Delta s = 2 \times 277.4 = +554.8\,\mathrm{J\,K^{-1}}$.

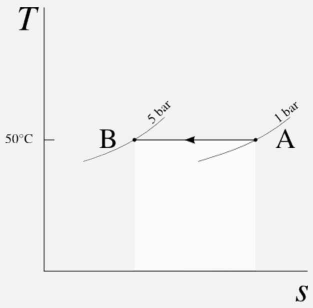
☞ It does not matter whether the process is reversible or not: we just need to know the initial and final states.

☞ We correctly state that "the entropy of the air increases" and not that "entropy is given to it" (§8.2.1).

Example 8.3

How much does the entropy of a mass of air of 0.5 kg change when it is slowly cooled at constant temperature from 1 bar and 50 °C to 5 bar? How much heat needs to be removed for this?

The process can be drawn qualitatively on a temperature-entropy diagram as follows:



In order to quantify ΔS we start from equation 8/11, $\Delta s = c_p \ln \frac{T_B}{T_A} - R \ln \frac{p_B}{p_A}$ and here $T_B = T_A$. We thus have $\Delta s = -R \ln \frac{p_B}{p_A} = 287 \ln \frac{5}{1} = -461.9\,\mathrm{J\,K^{-1}\,kg^{-1}}$.

The change in entropy is $\Delta S = m\,\Delta s = 0.5 \times -461.9 = -231\,\mathrm{J\,K^{-1}}$.

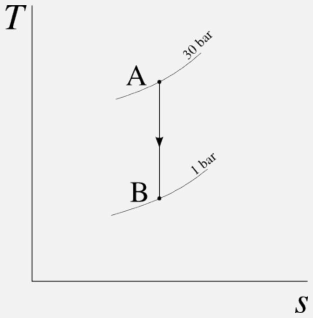
Since the process is reversible, the heat removed is obtained using equation 8/7 : $Q_{\mathrm{A}\to\mathrm{B}} = \int_{\mathrm{A}}^{\mathrm{B}} T\,\mathrm{d}S = T_{\text{cst.}} \int_{\mathrm{A}}^{\mathrm{B}} \mathrm{d}S = T_{\text{cst.}} \Delta S = (50 + 273,15) \times -231 = -74.6\,\mathrm{kJ}$.

☞ We already knew how to quantify $Q_{\mathrm{A}\to\mathrm{B}}$ without using entropy, using equations 4/27 and 4/28.

Example 8.4

How much does the temperature of air change when adiabatically and reversibly expanded from 30 bar and 600 K down to 1 bar?

The process can be drawn qualitatively on a temperature-entropy diagram as follows:



In order to quantify ΔS we again start from equation 8/11 : $\Delta s = c_p \ln \frac{T_\text{B}}{T_\text{A}} - R \ln \frac{p_\text{B}}{p_\text{A}}$ and here $\Delta s = 0$:

$$0 = c_p \ln \frac{T_\text{B}}{T_\text{A}} - R \ln \frac{p_\text{B}}{p_\text{A}}$$

$$\ln \frac{T_\text{B}}{T_\text{A}} = \frac{R}{c_p} \ln \frac{p_\text{B}}{p_\text{A}}$$

$$\left(\frac{T_\text{B}}{T_\text{A}}\right) = \left(\frac{p_\text{B}}{p_\text{A}}\right)^{\frac{R}{c_p}} = \left(\frac{p_\text{B}}{p_\text{A}}\right)^{\frac{\gamma-1}{\gamma}}$$

Here we recognize the dreaded equation 4/37 which we have wielded in the past: $T_\text{B} = 600 \times \frac{1}{30}^{\frac{0.4}{1.4}} = 227\,\text{K}$, approximately $-46\,^\circ\text{C}$ or $-51\,^\circ\text{F}$.

☞ Using the reasoning "adiabatic reversible = isentropic" did not actually bring us anything we did not already know here, as the ideal gas model is already extremely simple and powerful. This will not be the case with liquids/vapors.

8.3.4 Entropy changes of a liquid/vapor mixture

For a liquid/vapor mixture, the changes of s cannot be predicted with a calculation because there is no simple mathematical model to describe temperature as a function of other properties. The saturation curve and the path of process at constant pressure are represented in figure 8.8; this figure closely resembles the temperature-volume diagram we plotted in figure 5.7.

In order to quantify changes in entropy, we we will proceed exactly as we did with internal energy u in chapter 5 (*liquids and vapors*): by tabulating values for s. Since there is no way to measure s directly, all the tabulated values of entropy are relative to a reference point for which s is arbitrarily set to $0\,\text{J}\,\text{K}^{-1}\,\text{kg}^{-1}$; in our case, it is the triple point of water. This does not matter for our calculations, since we are only interested in the changes of entropy.

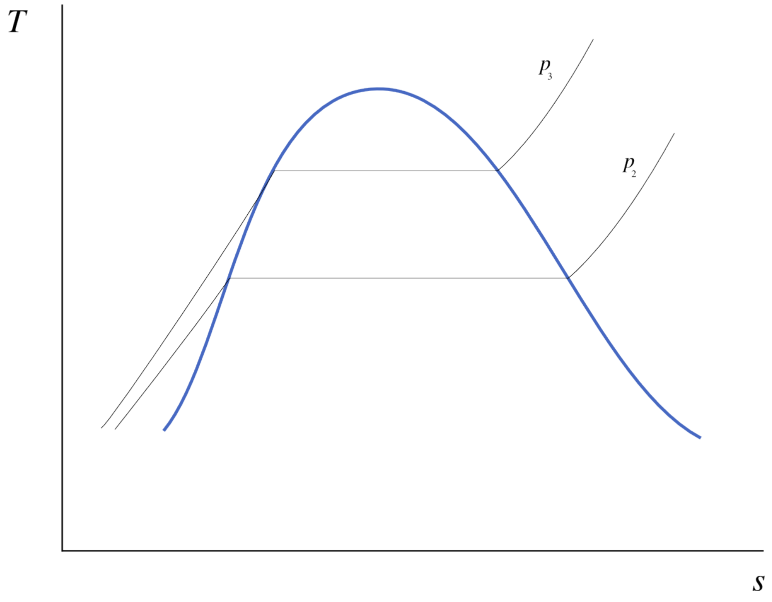


Figure 8.8: Temperature-entropy diagram of a liquid/vapor mixture. This figure closely resembles fig. 5.7 p. 123.

Diagram CC-0 *Olivier Cleynen*

When water is either in a saturated liquid or dry steam state, values of entropy can simply be read in the last column in Steam Table 1 (see Appendix A1 pp. 308-311, and section §5.3 p. 125), an extract of which is repeated in table 8.1.

°C	$\frac{\text{m}^3}{\text{kg}}$	$\frac{\text{kJ}}{\text{kg}}$	$\frac{\text{kJ}}{\text{kg}}$	$\frac{\text{kJ}}{\text{K kg}}$
T	v	u	h	s
	$p = 1.6\,\text{MPa}$ $(T_{\text{sat.}} = 201.37\,°\text{C})$			
10	0.001	42	43.6	0.1509
20	0.001001	83.8	85.4	0.2962
50	0.001011	209.1	210.7	0.7031
100	0.001043	418.6	420.3	1.306
200	0.001156	850.4	852.3	2.3305
300	0.15866	2 781.5	3 035.4	6.8863
500	0.22029	3 120.1	3 472.6	7.5409
600	0.24999	3 293.9	3 693.9	7.81
700	0.2794	3 473.5	3 920.5	8.0557
800	0.30865	3 659.5	4 153.3	8.2834
900	0.3378	3 852.1	4 392.6	8.4965
1 000	0.36687	4 051.2	4 638.2	8.6974
1 100	0.39589	4 256.6	4 890	8.8878
1 200	0.42487	4 467.9	5 147.7	9.0689
1 500	0.51169	5 133.7	5 952.4	9.5656
2 000	0.65615	6 326.8	7 376.6	10.272

Table 8.1: An extract from Steam Table 1 (see pp. 308-311). Values for entropy can be read in the last column, and its values are interpolated like the other properties.

Within the saturation curve, in other words, when a mix of phases is present, we interpolate values for entropy between those of s_L (saturated liquid) and s_V (saturated vapor) using the concept of *dryness fraction*, exactly as with equation 5/5:

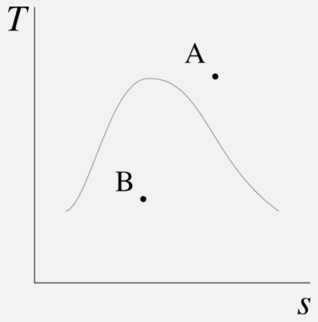
$$s_x = s_L + x\, s_{LV} \tag{8/14}$$

Example 8.5

How much does the entropy of water change when it goes from a state at 240 °C and 6 bar, to 130 °C with an internal energy of $1000\,\text{kJ}\,\text{kg}^{-1}$?

A quick look at the steam tables allows us to draw the process qualitatively on a temperature-entropy diagram as shown below.
We read s_A by interpolation in Steam Table 1 at 0.6 MPa between 200 °C and 300 °C: $s_A = 6.9683 + \frac{40}{100} \times (7.374 - 6.9683) = 7.1306\,\text{kJ}\,\text{K}^{-1}\,\text{kg}^{-1}$.
Upon arrival, the water is in a liquid-vapor mixture (because $u_B < u_{V\,130\,°\text{C}}$), so we read from Steam Table 2 (5/5): $x_B = \frac{u_B - u_L}{u_{LV}} = \frac{1000-546.1}{1993.5} =$

0.228; thus with equation 8/14 we can calculate the entropy: $s_B = s_L + x_B\, s_{LV} = 1.6346 + 0.228 \times 5.3918 = 2.8623\,\mathrm{kJ\,K^{-1}\,kg^{-1}}$.



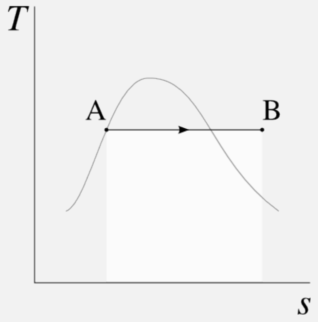
We see that the entropy has decreased: $\Delta s = s_B - s_A = -4.268\,\mathrm{kJ\,K^{-1}\,kg^{-1}}$.

☞ We do not know what process took place. The less reversible it was, and the more heat had to be removed from the steam in order to bring it from A to B.

Example 8.6

We slowly heat 2 kg (4.4 lb) of saturated liquid water at 300 °C, maintaining its temperature constant, until the volume reaches $2\,\mathrm{m^3}$ (440 imp gal). How much heat needs to be added for this?

A quick look at the steam tables allows us to draw the process qualitatively on a temperature-entropy diagram as follows:



Since the process is reversible, we can calculate $Q_{A\to B}$ by integrating the term $T\,\mathrm{d}s$ between A and B.

We read s_A in Steam Table 2: $s_A = s_{L300\,^\circ\mathrm{C}} = 3.2552\,\mathrm{kJ\,kg^{-1}}$.

At the end, the specific volume is $v_B = \frac{V_B}{m} = \frac{2}{2} = 1\,\mathrm{m^3\,kg^{-1}}$; in order to obtain s_B we must interpolate between two blocks of Steam Table 1 (between 0.2 MPa and 0.4 MPa at 300 °C). Let $y \equiv \frac{v_B - v_{300\,^\circ\mathrm{C}\ \&\ 0.2\,\mathrm{MPa}}}{v_{300\,^\circ\mathrm{C}\ \&\ 0.4\,\mathrm{MPa}} - v_{300\,^\circ\mathrm{C}\ \&\ 0.2\,\mathrm{MPa}}} = \frac{1 - 1.3162}{0.654\,89 - 1.3162} = 0.4781$ and correspondingly, $s_B = s_{300\,^\circ\mathrm{C}\ \&\ 0.2\,\mathrm{MPa}} + y(s_{300\,^\circ\mathrm{C}\ \&\ 0.4\,\mathrm{MPa}} - s_{300\,^\circ\mathrm{C}\ \&\ 0.2\,\mathrm{MPa}}) = 7.8941 + 0.4781(7.5677 - 7.8941) = 7.738\,\mathrm{kJ\,K^{-1}\,kg^{-1}}$.

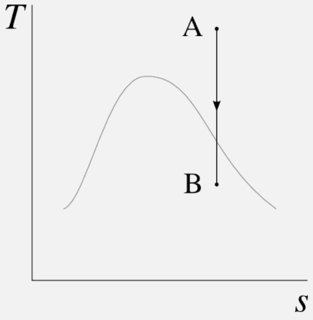
We can finally calculate $q_{A\to B}$ with equation 8/8: $Q_{A\to B} = \int_1^2 T\,dS = m\,T\,\Delta s = 2 \times (300 + 273.15) \times (7.738 - 3.2552) = +5.139\,\text{kJ}$.

☞ This somewhat laborious calculation may not be spectacular, but it is important to realize that without the use of entropy, we had *no way* to quantify $Q_{A\to B}$ without conducting an experiment. We had gotten stuck on this issue in example 5.12 page 139.

Example 8.7

In a turbine, steam undergoes a reversible adiabatic (isentropic) expansion. The steam enters at 40 bar and 500 °C; it is expanded to 0.5 bar. What is the specific power output?

A final look at the steam tables allows us to draw the process qualitatively on a temperature-entropy diagram as follows:



Here $q_{A\to B} = 0$ because the turbine is adiabatic, and we are looking for $w_{A\to B} = \Delta h$. Therefore, we need to find a way to quantify h_B.

In A, we read from Steam Table 1, at 4 MPa: $h_A = 3446\,\text{kJ}\,\text{kg}^{-1}$ and $s_A = 7.0922\,\text{kJ}\,\text{K}^{-1}\,\text{kg}^{-1}$.

At B, we know that $s_B = s_A$ because the process is isentropic; this information will allow us to calculate h_B. Using Steam Table 3 at 0.05 MPa, with equation 8/14, we obtain $x_B = \frac{s_B - s_L}{s_{LV}} = \frac{7.0922-1.0912}{6.5018} = 0.923$. Using the dryness fraction, we can simply calculate the enthalpy (equation 5/4): $h_B = h_L + x_B\,h_{LV} = 340.5 + 0.923 \times 2304.7 = 2467.7\,\text{kJ}\,\text{kg}^{-1}$.

The specific power of the turbine is therefore $w_{A\to B} = \Delta h = -978.3\,\text{kJ}\,\text{kg}^{-1}$.

☞ The calculation we have just carried out is extremely useful for engineers. The idea that entropy remains constant during a reversible adiabatic process allows us – at last! – to predict the state of a liquid-vapor mixture at the outlet of a compressor or turbine. Until now, this could be done with an ideal gas (and the cumbersome relationships like $\frac{T_1}{T_2} = ...$), but not for a liquid/vapor mixture.

8.4 Predicting the Direction of Processes

Here we come to the central concept that has opened the doors of physics to thermodynamics. Based on the quantifications of entropy changes, we are able to describe the direction of processes, that is, to prove for example that a state B comes *after* a state A.

8.4.1 Irreversibilities during heat transfers

> « If two transformations which, without necessitating any other permanent change, can mutually replace one another, be called equivalent, then [...] the passage of the quantity of heat Q from the temperature t_1 to the temperature t_2, has the equivalence-value $Q\left(\frac{1}{T_2}-\frac{1}{T_1}\right)$ [...] »
>
> Rudolf Clausius, 1854 [13, 18, 22]

In order to reward ourselves for already making it halfway through the chapter, we make ourselves a cup of tea. Because no one can resist a little thermodynamics, we press our mug closely against a bottle of cold water. O wonder, o joy! We have before our eyes a source of entropy. Let's investigate.

Our mug A is at temperature T_A, higher than T_B, the temperature of the water bottle (figure 8.9). The two bodies are brought into contact, and an infinitesimal amount of heat δq passes from A to B.

If we consider only a small amount of time, the temperature of body A is uniform, and its heat loss occurs reversibly. Thus, the entropy change of A is:

$$\mathrm{d}s_A = -\frac{\delta q}{T_A}$$

The temperature of body B is also uniform: the process there is also internally reversible, and the change of its entropy is:

$$\mathrm{d}s_B = +\frac{\delta q}{T_B}$$

However, the temperature of the entire system [A+B] is not uniform at all: the process *there* is not internally reversible. Even if the system receives no heat from the external surroundings, it does not have "a" temperature, and we cannot apply integral 8/2, $\int_1^2 (T\,\mathrm{d}s)_{\text{rev.}}$ to calculate its entropy change. The entropy change of system [A+B] is the sum of those of its components, namely:

$$\mathrm{d}s_{[A\&B]} = \mathrm{d}s_A + \mathrm{d}s_B = \frac{\delta q}{T_B} - \frac{\delta q}{T_A} \tag{8/15}$$

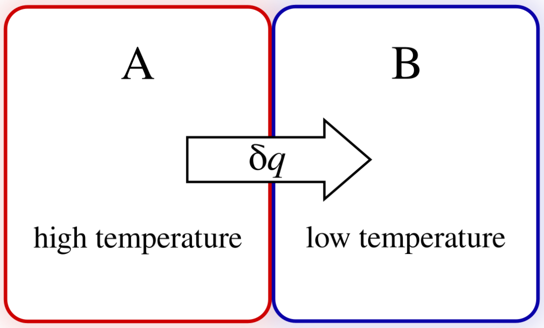


Figure 8.9: Creation of entropy by heat transfer. The process is internally reversible for each of the two bodies A and B, but irreversible for the system [A+B].

Diagram CC-0 *Olivier Cleynen*

Since $T_\mathrm{A} > T_\mathrm{B}$, this change is *positive and non-zero*; entropy *has been created* during the irreversible heat transfer. The irreversibility occurs neither in cup A nor in bottle B, but at the thin material boundary separating them. The process can be represented rather convincingly on a T-s diagram (figure 8.10).

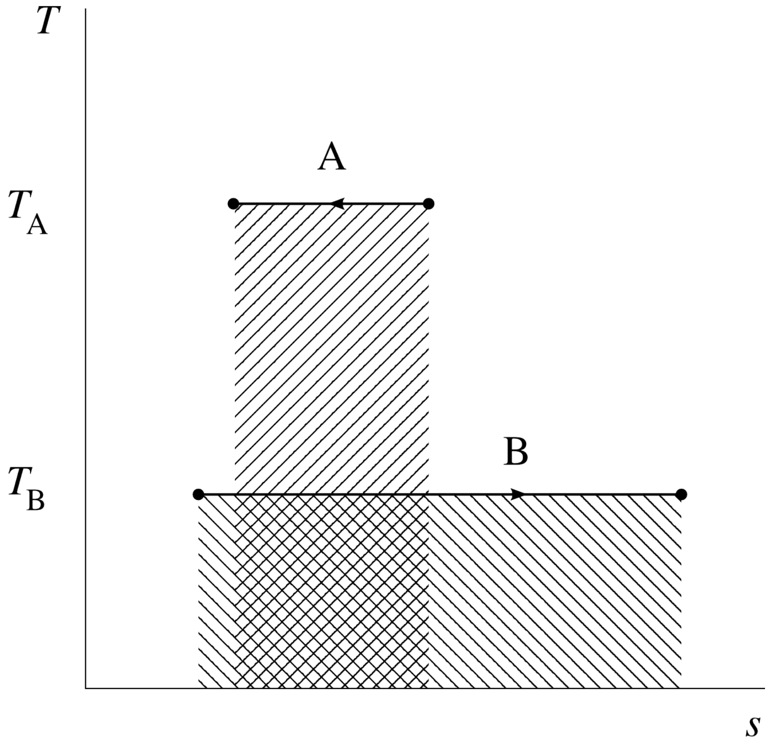


Figure 8.10: Entropy changes for bodies A and B. The two shaded areas are equal (representing the heat quantity δq), but the sum of the two entropies increases.

Diagram CC-0 *Olivier Cleynen*

This small investigation shows us that every temperature gradient leads to irreversibility, resulting in an increase in total entropy. Any heat transfer between two objects of different temperatures can be viewed as a missed opportunity to do work – likely a source of anxiety for both students and engineers. By placing a Carnot engine between bodies A and B, no irreversibility would occur, and $\mathrm{d}s_{[\mathrm{A\&B}]}$ would be zero. By placing a thermal engine with low efficiency, $\mathrm{d}s_{[\mathrm{A\&B}]}$ would be small; the case above where heat transfer occurs without a machine is the limiting case where no work is produced.

> « *Wherever there exists a difference in temperature, there can be a production of motive power.* Conversely wherever this power can be consumed, it is possible to generate a difference in temperature, it is possible to cause a disruption of equilibrium in the caloric. »
>
> Sadi Carnot, 1824 [4]

8.4.2 Irreversibilities during adiabatic compressions and expansions

Another type of process leads to irreversibilities, and thus to an increase in total entropy: it is the transfer of work in fluids.

In practice, any expansion or compression occurs in the presence of internal irreversibilities. Since the duration of the process is finite (unlike Carnot's idealized processes), there will necessarily be pressure imbalances within the fluid. These imbalances lead to internal turbulence, which causes the conversion of mechanical energy into internal energy through friction and heat.

Thus, a real adiabatic compression causes the fluid to reach a higher temperature than a reversible adiabatic compression (figure 8.11): part of the supplied work is completely converted into heat due to internal friction. Accordingly, during a real adiabatic expansion, the temperature decreases less than during a reversible adiabatic expansion. Each time, entropy is increased even though no heat transfer δq has occurred.

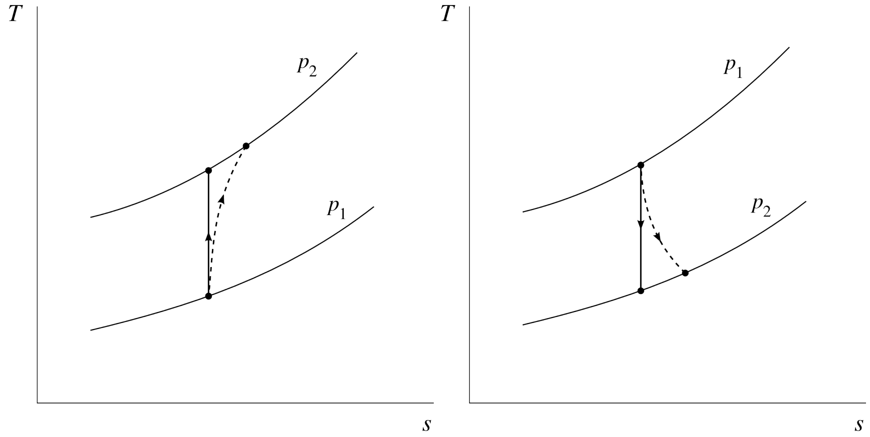


Figure 8.11: Theoretical (isentropic, solid lines) and real (dotted lines) adiabatic expansions and compressions.
It is important to note that the increase in entropy is not related to a heat transfer "δq". The path on the T-s diagram is not continuous, and the area underneath does not represent a heat flow across the system boundaries.

Diagrams CC-0 *Olivier Cleynen*

8.4.3 The second law and entropy

« *Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time.* »

Rudolf Clausius, 1854 [13, 18, 22]

We have stated in chapter 7 (*the second law*) that heat spontaneously moves only towards a lower temperature – a postulate we call the *second law*. We can now formulate this statement with a mathematical expression.

During a heat transfer from a body at temperature T_A to another at temperature T_B, the overall entropy change $\Delta s = \frac{-q}{T_A} + \frac{q}{T_B}$ is necessarily zero or positive because T_A is necessarily equal to or greater than T_B.

During a work transfer any irreversibility results in a higher final temperature than it could have been (see §2.4.3). Achieving the same final state with a reversible path thus requires a heat input, in other words, a positive term $\int \left(\frac{\delta Q}{T}\right)_{\text{rev.}}$. An irreversibility therefore leads to an increase in total entropy.

Thus, we can translate the second law as follows:

When an energetically-isolated system undergoes a process,
its entropy s changes in such a way that

$$\Delta s \geqslant 0 \tag{8/16}$$

We can always decrease the entropy of a system to bring it back to its initial value (by returning the system itself to its initial state, whatever the method used), but this will necessarily be at the expense of an increase *at least as large* in the entropy of another system.

We could also say, in the same way that we described energy as “a quantity that does not change during transformations” (§1.1.1), that entropy is conceptualized as “a quantity that always increases during transformations.” This is the indicator we were looking for in §8.1.1 to determine the direction of processes.

8.4.4 Predicting the direction of processes

In order to demonstrate that a system can only go from state A to state B, in other words, that the process is irreversible, we need to proceed as follows:

1. We need to find a reversible path A → B, namely, a process to go from A to B while keeping pressure and temperature internally homogeneous even if they vary;
2. Along this reversible path, we calculate Δs (that is, we carry out the integral $\int \frac{\delta q}{T}$ for this path).
3. We compare the integral $\int \frac{\delta q}{T}$ along the reversible path with the integral for the *real* path.

There are three possibilities:

- If the two integrals are equal, then the real process is *reversible*: it can take place in both directions.
- If $\int \left(\frac{\delta q}{T}\right)_{\text{real path}} < \int \left(\frac{\delta q}{T}\right)_{\text{rev.}}$, then the real process is *irreversible*. It can only take place from A to B.
- If $\int \left(\frac{\delta q}{T}\right)_{\text{real path}} > \int \left(\frac{\delta q}{T}\right)_{\text{rev.}}$, then the described “real” process is *impossible*. It can only take place in the reverse direction (B → A).

Thus, we can mathematically determine the direction of time, at least for some simple cases – a subtlety that one would not expect from engineers concerned about their fuel consumption!

Example 8.8

A mass of air undergoes a process without any heat exchange. There are two states:

- State X at 5 bar and 100 °C (72.5 psi and 212 °F);
- State Y at 1 bar and 5 °C (14.5 psi and 41 °F).

In which direction (X → Y or Y → X) can the process take place?

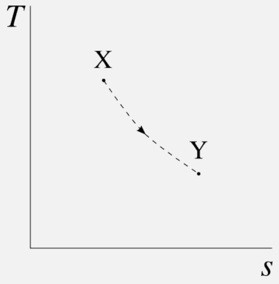
Let’s suggest the direction X → Y and check if it is physically possible.

The entropy change is equal to the integral $\int_{\text{X}}^{\text{Y}} \left(\frac{\delta q}{T}\right)_{\text{rev.}}$. Using equation 8/11, $\Delta s = c_p \ln \frac{T_{\text{Y}}}{T_{\text{X}}} - R \ln \frac{p_{\text{Y}}}{p_{\text{X}}} = 1005 \ln \frac{5+273{,}15}{100+273{,}15} - 286 \ln \frac{1}{5} = +166.6\,\text{J}\,\text{K}^{-1}\,\text{kg}^{-1}$.

On the other hand, the integral $\int_{\mathrm{X}}^{\mathrm{Y}} \left(\frac{\delta q}{T}\right)_{\text{real path}}$ is equal to zero because in reality there was no heat transfer ($\delta q = 0$).

Therefore, we have $\Delta s > \int_{\mathrm{X}}^{\mathrm{Y}} \left(\frac{\delta q}{T}\right)_{\text{real path}}$ and the process is irreversible. If we wanted to reverse the process, from Y to X, we would have to remove heat.

The process can be drawn qualitatively on a temperature-entropy diagram as follows:



Example 8.9

Water undergoes a process during which $1\,\mathrm{MJ\,kg^{-1}}$ of heat is added to it at while its temperature is fixed at $130\,^{\circ}\mathrm{C}$ ($266\,^{\circ}\mathrm{F}$). There are two states, one at the beginning and the other at the end:

- State X as liquid at $130\,^{\circ}\mathrm{C}$ and saturated;
- State Y as vapor at $170\,^{\circ}\mathrm{C}$ ($338\,^{\circ}\mathrm{F}$) and saturated.

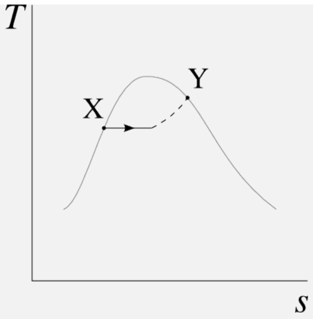
In which direction (X → Y or Y → X) can the process take place?

Let's consider the direction X → Y and verify if it is physically possible.

We read the values of entropy in Steam Table 2: $s_{\mathrm{X}} = s_{L130\,^{\circ}\mathrm{C}} = 1.6346\,\mathrm{kJ\,K^{-1}\,kg^{-1}}$ and $s_{Y} = s_{V170\,^{\circ}\mathrm{C}} = 6.665\,\mathrm{kJ\,K^{-1}\,kg^{-1}}$.
Thus, $\Delta s = \int_{\mathrm{X}}^{\mathrm{Y}} \left(\frac{\delta q}{T}\right)_{\text{rev.}} = +5.03\,\mathrm{kJ\,K^{-1}\,kg^{-1}}$.

Separately, we can calculate the integral $\int_{\mathrm{X}}^{\mathrm{Y}} \left(\frac{\delta q}{T}\right)_{\text{real path}}$ because we know that heat was supplied when the temperature was fixed at $130\,^{\circ}\mathrm{C}$. Thus $\int_{\mathrm{X}}^{\mathrm{Y}} \left(\frac{\delta q}{T}\right)_{\text{real path}} = \frac{1}{T} \int_{\mathrm{X}}^{\mathrm{Y}} (\delta q)_{\text{real path}} = \frac{q_{\mathrm{X}\to\mathrm{Y}}}{T} = \frac{1\times10^{6}}{170+273{,}15} = +2.26\,\mathrm{kJ\,K^{-1}\,kg^{-1}}$.

The process can be drawn qualitatively on a temperature-entropy diagram as follows:



Here, we have $\Delta s > \int_X^Y \left(\frac{\delta q}{T}\right)_{\text{real path}}$ and the process is irreversible. If we wanted to go back from Y to X, we would need to cool the water by removing a quantity *necessarily* greater than $1\,\mathrm{MJ\,kg^{-1}}$.

8.5 Entropy, Time, and the Universe

8.5.1 Entropy for the engineer

We have seen that entropy, just like energy, is a concept which was designed in order to back with calculations an intuition that we have about the world: quantifying their changes enables us to determine the transformations that are *possible*. It is therefore fundamentally a concept for physicists. For engineers, entropy is:

- "what does not change when compressing and expanding fluids ideally". Thus, quantifying Δs allows us to quantify the properties that a fluid should have at the outlet of a compressor or a turbine;
- "what does not change when transferring heat within a system ideally". Thus, quantifying Δs allows us to calculate the irreversibility that occurs during heat transfers.

Whenever we produce an increase in overall entropy, we have to proceed ultimately to an unwanted heat rejection. Thus, these quantifications of Δs allow us to measure the quality of expansions, compressions, cooling, and heating that we carry out with fluids in our machines.

8.5.2 Context: the direction of time

The examples we have studied in this chapter to determine the direction of processes are very academic, however the approach remains valid for any process: a stone thrown into a pond, a food plate breaking when it falls, etc. If we go back to the three photos in figure 8.1, we could determine their order by finding the initial and final states of the water around the diver, and comparing Δs with the integral $\int \left(\frac{\delta q}{T}\right)_{\text{real path}}$ carried out during the entry into the water.

This desire to find the absolute order in which states succeed one another, in other words, the direction of time, led the German physicist Rudolf Clausius to propose the concept of *entropy* in 1865 in a masterly publication — *Über verschiedene für die Anwendung bequeme Formen der Hauptgleichungen der mechanischen Wärmetheorie* [17, 18, 19]. Concluding a decade of work around the quantity $\frac{Q}{T}$, he formalized a concept that his French colleague Frédéric Reech and Scottish counterpart William Rankine had only touched upon [36], and synthesized all the contemporary knowledge of his discipline.

> « I have intentionally formed the word *entropy* so as to be as similar as possible to the word *energy*; for the two magnitudes to be denoted by these words are so nearly allied in their physical meanings, that a certain similarity in designation appears to be desireable. »
>
> Rudolf Clausius, 1865 [17, 18, 19]

Clausius created the word *entropy* based on the ancient Greek *tropè* τροπή (revolution, change), which, coupled with his authoritarian tone, did nothing to win the enthusiasm of his contemporaries. But the concept is so powerful, and equation 8/16 so simple, that they were universally accepted.

After a century of efforts, the physics of heat had caught up with engine technology. We were finally able to fully and quantitatively describe the behavior of bodies without having to delve into that of their constituents, such as molecules, atoms, or subatomic particles: entropy was the last missing piece of what we now call *macroscopic thermodynamics*.

8.5.3 Entropy at the microscopic scale

After Clausius, the development of thermodynamics is no longer of great interest to engineers, but physicists may still be longing for more. Indeed, although we had *described* the irreversibility phenomenon, we had not yet *explained* its origin within bodies made up of particles whose movements (an incessant buzzing of collisions, based on attractive and repulsive forces), *themselves*, are perfectly reversible.

It would take only ten years for the answer to be formalized: in 1875, Austrian physicist Ludwig Boltzmann proposed a *microscopic definition* of entropy:

$$S \equiv k \ln \lambda \tag{8/17}$$

where λ is the number of possible configurations of the system which correspond to its state,
and k is a constant.

> « We measure "disorder" by the number of ways that the insides can be arranged, so that from the outside it looks the same. *The logarithm of that number of ways is the entropy.*
> … So with the above technical definition of disorder we can understand the proposition. First, the entropy measures the disorder. Second, the universe always goes from "order" to "disorder," so entropy always increases. Order is not order in the sense that we like the arrangement, but in the sense that the number of different ways we can hook it up, and still have it look the same from the outside, is relatively restricted. »
>
> Richard Feynman, 1963 [30, 35]

Thus, for Boltzmann, entropy is a measure of the probability that the system is in the state in which it is observed. The more probable the configuration (homogeneity of pressure and temperature), the greater the entropy.

At the macroscopic scale, we had described the second law as an impossibility (§7.1.1): for example, an object at one temperature cannot spontaneously have one of its ends cool down while the other heats up. According to Boltzmann, such an event is not strictly impossible, but only very improbable. The state where the fastest molecules are all gathered at one end, and the slowest at the other, is much less probable (lower entropy) than a state where they are distributed homogeneously (higher entropy).

This approach not only has the merit of reconnecting our discipline with atomic theory – and hence we will talk about *microscopic thermodynamics* and *statistical thermodynamics* – but it also opened the door to information theory. Indeed, the resolution and precision with which we evaluate the state of a system affect the number of possible configurations that can be attributed to it. Here, the concept of *information* became linked to other physical properties: an impressive result for a discipline that was only intended to explore what "hot" meant!

8.5.4 Entropy and the universe

We leave entropy on an open question. To the extent that we think of the universe as a finite set, in other words, as an isolated system containing a fixed amount of energy, can we apply equation 8/16: $\Delta s_{\text{universe}} > 0$ as time passes? Is the universe moving towards a final homogeneous minimum temperature? Clausius was unequivocal: he immediately concluded his 1865 article with the affirmation:

> If for the entire universe we conceive the same magnitude to be determined [...] which for a single body I have called *entropy*, and if at the same time we introduce the other and simpler conception of *energy*, we may express in the following manner the fundamental laws of the universe which correspond to the two fundamental theorems of the mechanical theory of heat.
>
> 1. *The energy of the universe is constant.*
> 2. *The entropy of the universe tends to a maximum.*
>
> Rudolf Clausius, 1865 [17, 18, 19]

> « Within a finite period of time past, the earth must have been, and within a finite period of time to come the earth must again be, unfit for the habitation of man as at present constituted, unless operations have been, or are to be performed, which are impossible under the laws to which the known operations going on at present in the material world are subject. »
>
> William Thomson, 1852 [12]

Is the theory of refrigerators and engines capable of predicting the end of the world? To explore this question in a fun way, students may read *The Last Question* by Isaac Asimov [29, 32] or *Entropy and all that* by Philippe Depondt [43]. For a more formal answer, one must refer to a good physics textbook.

A Bit of History: Rumford, an Adventurer Who "Weighs" Heat

*

By Philippe Depondt
Pierre and Marie Curie University, Paris

He was actually named Benjamin Thompson (1753-1814) and was an American from Woburn, Massachusetts [40]. He initially had many diverse occupations: clerk in a store, teacher, medical student… However, when the American Revolution broke out, New England where he lived was at the heart of the conflict; Thompson chose the Loyalist side and became a secret agent for the British. At the time of the Declaration of Independence of the United States of America in 1776, cautiously, he left for England.

He then undertook research on projectiles and joined the *Royal Society* in 1779. He nevertheless returned to America in 1782 still to fight on the British side, but peace was declared the following year. He was ennobled as Baron Rumford by King George III in reward for his services. He then became an advisor to the Elector of Bavaria, and later the Minister of War. During his 14 years spent in Bavaria, he reformed the army, established assistance for the needy, and created the famous *Englischer Garten* in Munich. He then returned to England, and finally settled in France in 1805 where he had a –disastrous– marriage with Marie Lavoisier, the widow of the chemist who was guillotined during the French Revolution.

During his adventurous life, he managed to make at least two important discoveries. Within the framework of the caloric theory, he attempted to weigh heat by measuring the weight of bodies at different temperatures. From these unsuccessful attempts, he concluded that heat has no weight. Later, during his tenure as Minister of War, he oversaw the drilling of cannons and observed in 1798 that the drilling process heated the barrels to the point of boiling water. He realized that the amount of heat released was not limited by the quantity of matter containing it. From this, he deduced that the heat was not stored in the material but rather *produced* by the work of the horses driving the drill—a foreshadowing of Joule's first law.

The history of science also includes some rather romantic figures!

Figure 8.12: Sir Benjamin Thompson of Rumford, militia fighter, secret agent, architect, Minister of War, bold experimenter, cosmopolitan seducer, and of course, thermodynamicist.

Engraving by J. P. P. Rauschmayr, 1797 (public domain)

Problems

The properties of water are tabulated in Steam Tables 1, 2, and 3 (see Appendix A1 p. 307)

Air is considered an ideal gas.

$$c_{v\,(\text{air})} = 718\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1} \qquad R_{\text{air}} = 287\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$$

$$c_{p\,(\text{air})} = 1005\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1} \qquad \gamma_{\text{air}} = 1.4$$

We assume that for a reversible adiabatic process (without heat transfer and infinitely slow), the properties of air are linked according to the following three relationships:

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{v_2}{v_1}\right)^{\gamma-1} \tag{4/36}$$

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \tag{4/37}$$

$$\left(\frac{p_1}{p_2}\right) = \left(\frac{v_2}{v_1}\right)^{\gamma} \tag{4/38}$$

We also assume that the change in entropy of an ideal gas, for any evolution, is quantified by the following relations:

$$\Delta s = s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \tag{8/10}$$

$$\Delta s = s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \tag{8/11}$$

Finally, we assume that the efficiencies of thermal machines based on a Carnot cycle are expressed as a function of the absolute temperatures as follows:

$$\eta_{\text{Carnot engine}} = 1 - \frac{T_L}{T_H} \tag{7/6}$$

$$\eta_{\text{Carnot refrigerator}} = \frac{1}{\frac{T_H}{T_L} - 1} \tag{7/7}$$

$$\eta_{\text{Carnot heat pump}} = \frac{1}{1 - \frac{T_L}{T_H}} \tag{7/8}$$

8.1 Knowledge Recap Questions

In order to tackle problems on such a substantial topic, it is necessary to first master the basics!

1. How is the change in the entropy of a body calculated during any real process?
2. Can the entropy of a body ever be reduced?
3. What is the difference between specific entropy and thermal capacity (both have the same units!)?
4. What would figure 8.10 p. 227 look like if the heat transfer was continued beyond an infinitesimal amount of heat δQ until the mug A and the water bottle B were at the same temperature?

8.2 Elementary Processes Undergone by an Ideal Gas

Among the processes of an ideal gas described in figure 8.13, identify the process at constant temperature, constant pressure, isentropic, and constant volume (*this exercise is parallel to exercise 4.10 p. 109*).

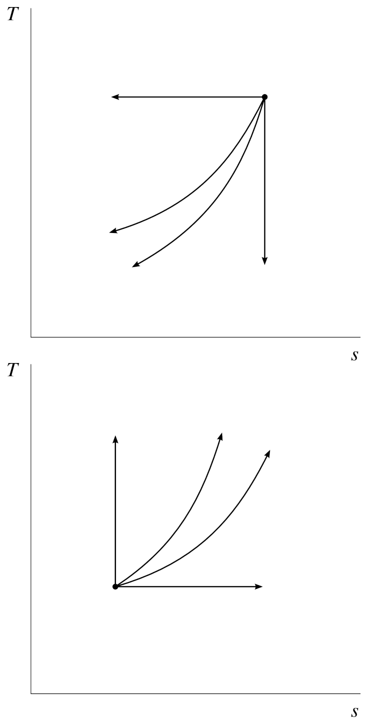


Figure 8.13: Reversible elementary processes of an ideal gas, represented on a temperature-entropy diagram.

Diagram CC-0 *Olivier Cleynen*

8.3 Expansion of a Liquid/Vapor

We have 10 kg (22 lb) of water at 45 bar and 600 °C (653 psi and 1112 °F).

1. What is the maximum amount of work that can be extracted from this mass of water without supplying heat, if it can expand to 4 bar (58 psi)?
2. If the expansion were continued to a lower pressure, at what temperature would the water condense?
3. Draw the process qualitatively (that is, without showing numerical values) on a temperature-entropy diagram, showing the saturation curve.

8.4 Heating at Constant Temperature

A quantity of heat of 3000 kJ kg^{-1} is slowly supplied to a mass of water saturated at 200 °C. The temperature is kept constant throughout the process.

What is the amount of work delivered by the water during the process? Draw the process qualitatively on a temperature-entropy diagram, showing the saturation curve.

8.5 Temperature-Entropy Diagrams

Draw qualitatively, on a temperature-entropy diagram (including the saturation curve when relevant), the processes that we studied in previous chapters:

1. Simple processes: problems 4.6 and 4.7 on page 109, 5.3 and 5.4 on page 145;
2. Thermodynamic cycles: problems 7.5 and 7.7 on page 204.

8.6 Carnot Cycle

Draw the cycle undergone by the fluid inside a heat pump operating according to the Carnot cycle on a temperature-entropy diagram, qualitatively, showing also the two heat transfers.

How would the cycle be modified if the compression and expansion remained adiabatic but were not reversible? How would the two heat transfers be affected?

8.7 Steam Turbine

In the engine room of a large ship (figure 8.14), a steam flow rate of 250 t/h (~550 000 lb/h) enters the turbine at 55 bar and 660 °C (798 psi and 1220 °F).

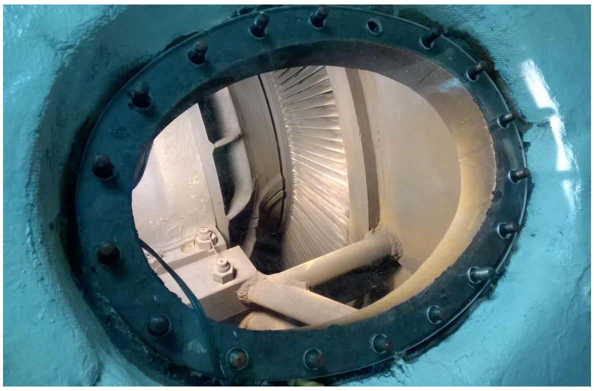


Figure 8.14: Inspection window of one of the low-pressure turbines (power approximately 25 MW) of the aircraft carrier *USS Hornet* launched in 1943.

Photo CC-BY-SA *by Tony Kent*

In the turbine, steam expands following an approximately reversible adiabatic process. When the pressure reaches 1 bar, steam is extracted at a low flow rate (1 kg s^{-1}) to heat another part of the power plant. The remaining steam in the turbine is expanded to a pressure of 0.18 bar (2.6 psi).

What is the power delivered by the turbine?

8.8 Direction of Processes (1)

A mass of air undergoes a process without any heat transfer. There are two states:

- State X: at 1 bar and 300 °C;
- State Y: at 5 bar and 500 °C.

In which direction (X → Y or Y → X) can the process take place?

Draw the process on a pressure-volume diagram and on a temperature-entropy diagram, qualitatively.

8.9 Direction of Processes (2)

Water undergoes a process during which 2 MJ kg^{-1} of heat is extracted from it, while its temperature is fixed at 250 °C. There are two states, one at the beginning and the other at the end:

- State X: in the saturated vapor state at 200 °C;
- State Y: in the saturated liquid state at 240 °C.

Which of the two states must have occurred before the other?

8.10 Expansion of Compressed Air

The air in a thermally insulated cylinder is expanded from 6.8 bar and 430 °C to 1 bar.

At the end of the expansion, the temperature is measured at 150 °C.

Is the expansion reversible? Draw the process qualitatively on a temperature-entropy diagram.

8.11 Air Pump

Air enters a small centrifugal pump with a flow rate of 4 kg/min (figure 8.15). The pump is not isentropic, but its heat losses can be neglected.

At the inlet, the air is at 1 bar and 15 °C.
At the outlet, the pressure is 2 bar and the temperature is measured at 97 °C.

1. What is the power required to operate the compressor?
2. What would be the power if the compression were isentropic?

Figure 8.15: Public air compressor in Stockholm for cyclists. A heat exchanger integrated under the bodywork fortunately ensures the temperatures calculated in this problem are never attained.

Photo CC-BY-SA *Jakob Voß (cropped)*

3. What would be the heat and work transfers required to return the air to its initial conditions (minimizing heat transfers)?

8.12 Theoretical Power Plant

During the design of an power plant, a group of enthusiastic engineers is studying the possibility of having water follow a Carnot cycle. The heat released by coal combustion is transferred to a steam boiler. The steam is expanded in a turbine, which powers an electric generator.

From A to B Water is compressed in an isentropic pump.
At A, the liquid-vapor mixture is at a pressure of 0.04 bar (0.58 psi).
At B, the water is in the saturated liquid state, at a pressure of 40 bar (580 psi).

From B to C Water is heated at constant pressure (40 bar) in the boiler. At C, the water is in the saturated vapor state.

From C to D Water is expanded in an isentropic turbine. At D, the water is at the initial pressure, that is, 0.04 bar.

From D to A Water is cooled in a condenser at constant pressure (0.04 bar).

1. Sketch the elements of the circuit followed by the steam, and draw the process qualitatively on a temperature-entropy diagram, showing the saturation curve.
2. What is the dryness fraction of the water when condensation is interrupted (at A)? What is the specific enthalpy at that point?
3. What is the dryness fraction at the turbine outlet (at D) and the specific enthalpy at this point?
4. What is the power delivered by the turbine?
5. What is the power of the boiler?
6. What is the power of the pump?
7. What is the efficiency of the power plant?

8.13 Irreversible Heat Transfers

A steam engine operates on a Carnot cycle, with a steady flow rate of $2\,\mathrm{kg\,s^{-1}}$, between the saturation points of water. The engine is designed to exploit a heat source at a moderate temperature ($300\,^\circ\mathrm{C}$), from the combustion of industrial waste, and it rejects heat into a river at a low temperature ($5\,^\circ\mathrm{C}$).

The boiler has thick walls to reduce the impact of manufacturing imperfections and to withstand the high pressure of the water. This thickness imposes a significant temperature gradient across the walls, adding up to $10\,^\circ\mathrm{C}$. The same applies to the condenser (also $10\,^\circ\mathrm{C}$ across the walls).

1. What is the rate of increase of entropy of the {heat source + water} system?
2. What is the rate of increase of entropy of the {heat sink + water} system?
3. What is the power loss associated with this increase in entropy?
4. Which physical properties of the boiler wall material are most desirable to minimize this issue?

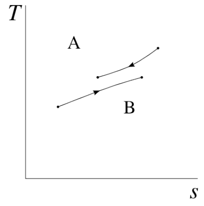
8.14 Irreversible Compressions and Expansions

The team of engineers in charge of the engine from the previous problem (Carnot cycle operating between $290\,^\circ\mathrm{C}$ and $15\,^\circ\mathrm{C}$, problem 8.13) discovers that the compression and expansion phases are not reversible.

The compressor does bring the water to a high temperature, but its energy consumption is $10\,\%$ higher than expected. The turbine does bring the water to a low temperature, but it supplies $10\,\%$ less work than expected.

1. What is the rate of increase of entropy of the steam in each of these two components?
2. By how much do the heat rejections increase?
3. What is the efficiency loss of the system compared to a reversible system?

Answers



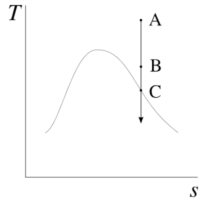
8.1 1) See §8.2.2 page 213;

2) Yes, a simple heat extraction is enough: see example 8.1 page 213;

3) Specific thermal capacity: heat per unit mass δq needed to generate a temperature change $\mathrm{d}T$ (equation 1/16 page 21: $c \equiv \frac{\delta q}{\mathrm{d}T}$). Specific entropy: specific heat divided by the temperature at which it is supplied during a reversible process (equation 8/2 page 213);

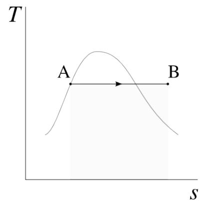
4) Both temperatures change until they equalize; $\Delta s_\text{A} + \Delta s_\text{B} > 0$.

8.2 *Clockwise starting from the vertical, on both graphs:* isentropic, isochoric, isobaric, isothermal.

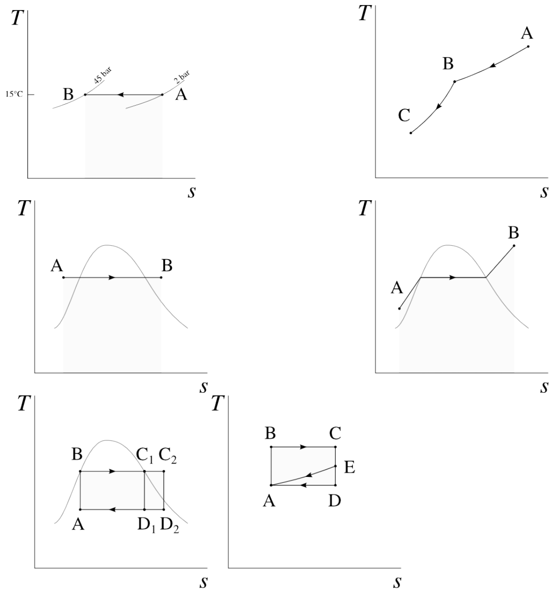


8.3 1) $u_1 = 3276.4\,\mathrm{kJ\,kg^{-1}}$ and $u_2 = 2703.3\,\mathrm{kJ\,kg^{-1}}$: $W_\text{max.} = -5.731\,\mathrm{MJ}$;

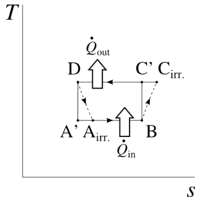
2) $T_3 = 103.51\,^\circ\mathrm{C} = 218.32\,^\circ\mathrm{F}$.



8.4 $s_2 = 8.671\,\mathrm{kJ\,K^{-1}\,kg^{-1}}$; thus $u_2 = 2660.89\,\mathrm{kJ\,kg^{-1}}$; finally $w_{1\to2} = -1.19\,\mathrm{MJ\,kg^{-1}}$.

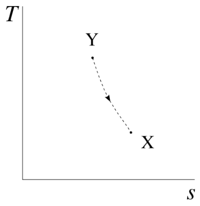


8.5



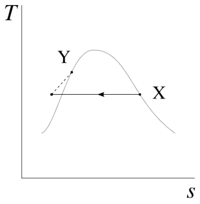
8.6 2) In this case $W_{\text{B}\to\text{C}_\text{irr.}} > W_{\text{B}\to\text{C}'}$ and, as negative values, $W_{\text{D}\to\text{A}_\text{irr.}} > W_{\text{D}\to\text{A}'}$. Thus the rejected heat $Q_{\text{C}\to\text{D}}$ increases (which may initially seem like an interesting result) and the heat intake $Q_{\text{A}\to\text{B}}$ decreases (and we see that the increase in $Q_{\text{C}\to\text{D}}$ is actually only due to the inefficiencies of the compressor and the turbine and only serves to reduce the efficiency).

8.7 The process is as described in example 8.7 page 225: $h_1 = 3803.5\,\mathrm{kJ\,kg^{-1}}$ (dry steam); $h_2 = 2677.7\,\mathrm{kJ\,kg^{-1}}$ (dry steam); $h_3 = 2413.6\,\mathrm{kJ\,kg^{-1}}$ (mixture with 91.9% dryness fraction); thus $\dot{W}_\text{turbine} = -96.26\,\mathrm{MW}$.



8.8 With equation 8/11 we find $s_\text{Y} - s_\text{X} = -161.08\,\mathrm{J\,K^{-1}\,kg^{-1}} < \int_\text{X}^\text{Y} \left(\frac{\delta q}{T}\right)_\text{real path} = 0\,\mathrm{kJ\,K^{-1}\,kg^{-1}}$ (since the process is adiabatic). Therefore, the direction is Y → X.

8.9



Assuming X → Y, then $\Delta s = -3.728\,\mathrm{kJ\,K^{-1}\,kg^{-1}}$ but $\int_{\mathrm{X}}^{\mathrm{Y}} \left(\frac{\delta q}{T}\right)_{\text{real path}} = -3.823\,\mathrm{kJ\,K^{-1}\,kg^{-1}}$, so we are reassured: the direction is indeed X → Y.

8.10

With equation 8/11 we get $\Delta s = +39.77\,\mathrm{J\,K^{-1}\,kg^{-1}}$ but –*aha*!– $\int_1^2 \left(\frac{\delta q}{T}\right)_{\text{real path}} = 0\,\mathrm{kJ\,K^{-1}\,kg^{-1}}$, thus the process is irreversible. We could also have used the very classic equation 4/37 p. 104 to find that $T_{2\text{isentropic}} < 150\,^{\circ}\mathrm{C}$.

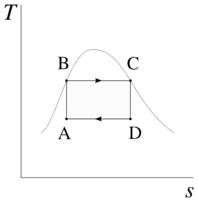
8.11

1) $\dot{W}_{\text{pump}} = \dot{m} c_p \Delta T = +5.493\,\mathrm{kW}$ (equations 3/15 & 4/13);

2) Using equation 4/37 $T_{2\text{is.}} = 351.3\,\mathrm{K}$, that is, $78.1\,^{\circ}\mathrm{C}$ or $172.7\,^{\circ}\mathrm{F}$, thus $\dot{W}_{\text{ideal}} = +4.231\,\mathrm{kW}$;

3) One possibility: isentropic expansion to obtain $\dot{W}_{2\to1} = -4.231\,\mathrm{kW}$, then a necessary cooling without work of $\dot{Q}_{2\to1} = -1.262\,\mathrm{kW}$. All reversible processes with a net sum of transfers taking these values (for example during a cooled expansion) will allow to return to 1.

8.12



The diagram is presented in figure 7.17 p. 204;

2) $x_{\mathrm{A}} = \frac{s_{\mathrm{B}} - s_L}{s_{LV}} = 0.2949$; thus $h_{\mathrm{A}} = h_L + x_{\mathrm{A}} h_{LV} = 838.7\,\mathrm{kJ\,kg^{-1}}$;

3) Same process: $x_{\mathrm{D}} = 0.7014$ thus $h_{\mathrm{D}} = 1827.5\,\mathrm{kJ\,kg^{-1}}$;

4) $w_{\text{turbine}} = h_{\mathrm{D}} - h_{\mathrm{C}} = -973.3\,\mathrm{kJ\,kg^{-1}}$;

5) $q_{\text{boiler}} = h_{\mathrm{C}} - h_{\mathrm{B}} = +1713\,\mathrm{kJ\,kg^{-1}}$;

6) $w_{\text{pump}} = h_{\mathrm{B}} - h_{\mathrm{A}} == +248.8\,\mathrm{kJ\,kg^{-1}}$;

7) $\eta_{\text{plant}} = \left|\frac{w_{\text{net}}}{q_{\text{inn}}}\right| = \frac{-w_{\text{turbine}} - w_{\text{pump}}}{q_{\text{boiler}}} = 42.29\,\%$. Since all phases are reversible and heat transfers are isothermal, we have $\eta_{\text{plant}} = \eta_{\text{Carnot engine}} = 1 - \frac{T_{\text{consenser water}}}{T_{\text{boiler water}}}$ (7/6).

8.13

1) $\dot{S}_{\text{high temp. wall}} = \dot{m}\left(\Delta s_{\text{combustion}} + \Delta s_{\text{water}}\right) = +91.77\,\mathrm{J/(K\,s)} = +91.77\,\mathrm{W\,K^{-1}}$;

2) $\dot{S}_{\text{low temp. wall}} = +188.3\,\mathrm{W\,K^{-1}}$, and we see that a gradient of $10\,^{\circ}\mathrm{C}$ is more penalizing at low temperature than at high temperature;

3) $\dot{W}_{\text{lost}} = \dot{Q}_i nn \left(\eta_{\text{high temp.}} - \eta_{\text{low temp.}}\right) = 77.9\,\mathrm{kW}$

4) To reduce temperature gradients, materials with very high thermal conductivity are needed (this is of course not the only quality required of them…).

8.14

1) $\Delta s_{\text{compressor}} = +67\,\mathrm{J\,K^{-1}\,kg^{-1}}$; $\Delta s_{\text{turbine}} = +382\,\mathrm{J\,K^{-1}\,kg^{-1}}$;

2) $\Delta q_{\text{out}} = -110.2\,\mathrm{kJ\,kg^{-1}}$, or $+14.6\,\%$;

3) $\eta_{\text{real plant}} = 39.82\,\%$, or $-9\,\mathrm{pt}$.

CHAPTER 9
Steam Power Cycles

or

The Understated Charm of Thermal Power Plant Water Circuits

Chapter 9 – Steam Power Cycles

Executive summary for chapter 9

Steam engines are used in stationary applications. Many modifications are made to ideal cycles to reduce cost and increase machine power.

Introduction

Now that we have acquired solid theoretical concepts, we can take a closer look at the thermodynamic cycles used in industry. This chapter 9 (*steam power cycles*) aims to answer two questions:

- Why and how are steam engines used today?
- Why are we moving away from ideal cycles and how do we quantify these compromises?

9.1 Why Use a Steam Engine?

The use of water as a working fluid in a machine undeniably has many disadvantages. In particular, unlike internal combustion engines:

- It is necessary to either recycle the water in the machine (and therefore cool it), or find a continuous source of pure water to operate it;
- There is an inevitable loss of some of the heat supplied to the machine, above the boiler.

Why, then, be interested in the operation of steam engines? The answer is that many sources of heat do not allow heat to be brought directly inside the working fluid. At the end of combustion, coal, wood, household or agricultural waste, for example, leave significant residues that cannot be circulated in a turbine. As for nuclear reactions, they cannot be carried out directly within the air. The exploitation of these sources, which accounts for a significant share of the world's sources of mechanical or electrical energy, therefore requires extracting heat from outside the engine.

Liquids have an excellent volumetric thermal capacity compared to that of air (as the student will find using both chapter 4 and 5, that of liquid water is approximately a thousand times higher): they are compact mediums for extracting heat from an external source. Among them, water is the most abundant and certainly the least difficult to handle. Thus, almost every time the heat input cannot be made within air, water is chosen as the engine's working fluid.

The sources of heat used by steam engines make their use in transportation difficult, so they are most often used in static installations to generate electricity: a configuration that allows economies of scale in energy storage and transport. All these factors justify the development of steam power plants with several gigawatts of electrical power ($1\,\text{GW} = 10^9\,\text{W}$), making them the most powerful thermal engines in the world.

> « Attempts have been made, on various occasions, to make heat act upon atmospheric air in order to give birth to motive power. This gas presents, in comparison to water vapor, some advantages and some disadvantages which we shall now examine.
> 1°. It offers, relative to water vapor, a notable advantage in that, possessing at equal volume a far lesser capacity for heat, it would cool to a greater degree by a similar expansion in volume. [...] Now, we have seen how important it is to induce, by changes in volume, the greatest possible variations in temperature.
> 2°. Water vapor can only be formed through the intermediary of a boiler, whereas atmospheric air could be heated directly by combustion executed within it. One would thus avoid a considerable loss, not only in the quantity of heat, but also in its thermometric degree. »
>
> Sadi Carnot, 1824 [4]

9.2 Engine Evaluation Criteria

Several parameters are taken into account in the evaluation of the performance and value of steam engines.

9.2.1 Thermal efficiency and overall efficiency

The parameter that we have learned to quantify so far is of course the *thermal efficiency* of the engine $\eta_{\text{engine}} \equiv \left|\frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}}\right|$ (6/4) that we always strive to maximize towards its theoretical maximum, $\eta_{\text{Carnot engine}} = 1 - \frac{T_{\text{min.}}}{T_{\text{max.}}}$ (7/6). However, it is important not to forget that the conversion of heat into work is just one of the many operations involved in electricity production:

- Fuel preparation (refinement and heating of oil, coal pulverization, separation of oil sands) itself may require energy, which we can evaluate with an efficiency $\eta_{\text{preparation}}$;
- In the boiler, the energy transfer from the heat source to the water can be incomplete (with some heat potentially being rejected with the exhaust gases), which we can evaluate with an efficiency η_{boiler};
- The transmission of mechanical energy from the turbine to the generator, possibly using a gearbox, incurs friction losses that we evaluate with an efficiency $\eta_{\text{transmission}}$;
- The transformation of shaft power into electrical power also involves losses resulting in the production of heat, which we evaluate with an efficiency $\eta_{\text{generator}}$.

Thus, the *overall efficiency* η_{overall} of electricity production at the output of the power plant, which compares the electrical energy produced to the actual heat spent to generate it (in other words, its energy cost), is the product of all these efficiencies:

$$\eta_{\text{overall}} \equiv \eta_{\text{preparation}}\,\eta_{\text{boiler}}\,\eta_{\text{engine}}\,\eta_{\text{transmission}}\,\eta_{\text{generator}} \qquad (9/1)$$

Engineers are expected to work on increasing the overall efficiency rather than just η_{engine}. It may be acceptable to intentionally reduce thermal efficiency if it allows, for example, an increase in η_{boiler} (with better heat extraction from the flue gases) or $\eta_{\text{generator}}$ (with an increase in turbine speed).

9.2.2 Power and specific consumption

The efficiency of an engine is not the only parameter we consider in the economic evaluation of its use: the costs associated with its maintenance or supervision of its operation, and of course the acquisition costs, are also crucial. These expenses can only be calculated if we delve into technological details that go beyond the scope of this book.

Despite this, we can already crudely assess the size and acquisition cost of an engine by calculating its *specific net power* w_{net}. In order to be compact, it is indeed desirable for an engine to generate a large net power for a given mass flow rate: it may even be desirable to compromise thermal efficiency for this purpose.

In industry, it is more common to measure the inverse parameter, which we call *specific steam consumption*. This consumption indicates the steam flow rate required to supply one watt of useful power. We denote it as ssc:

$$\text{ssc} \equiv \frac{1}{|w_\text{net}|} \tag{9/2}$$

where ssc is the specific consumption ($\text{kg}\,\text{J}^{-1}$),
and w_net is the specific power delivered by the machine ($\text{J}\,\text{kg}^{-1}$).

The unit of specific consumption is $\text{kg}\,\text{J}^{-1}$ (representing $\text{kg}\,\text{s}^{-1}$ of water per W of power), but the common practice in industry is to measure it in kg/(kW h) (kilograms per kilowatt-hour).

> « [...] The appropriate load for the production of the *absolute* maximum of useful effect, in expansive machines, is not the maximum load of which the machine is capable. [...] If one desires that the machine set in motion the greatest load it is capable of, it must be made to work without expansion; but this load is not the one that produces the *absolute* maximum of useful effect. »
>
> François-Marie Guyonneau de Pambour, 1839 [7]

9.2.3 Environmental impact

The production of heat in thermodynamic engines is at the heart of the major ecological challenges of our century. A comprehensive study goes beyond the scope of this book, so we will only note that the environmental impact can be divided into three main categories:

- Pollution by emission of harmful particles from combustion, particularly concerning the combustion of solids (especially coal as well as household and agricultural waste). With a filtering system, these emissions can generally be reduced to a very low level;
- Emission of greenhouse gases, especially CO_2, an inevitable product of the combustion of hydrocarbons which is now known to be a major contributor to global climate change. These emissions concern all heat sources based on combustion;
- Emission of radioactive waste, which concerns nuclear heat sources. These wastes are in small quantities but remain harmful for time periods counted in millennia.

Thus, apart from a few rarely-available heat sources (geothermal or solar concentration), thermodynamic engines are always powered by sources with major disadvantages. However, they are still the only ones that provide us with abundant energy in mechanical and electrical form, and this abundance is tightly coupled with the economic and societal progress our civilization made in the last two centuries. It is up to responsible engineers and citizens to judiciously assess their flaws and qualities.

9.3 Components of Steam Power Plants

Before studying the construction of steam thermodynamic cycles, we briefly review the operation mode of the most common components of power plants.

9.3.1 Calculation of component powers

All steam systems used today operate with a continuous flow rate. Also, in these machines, the changes in the steam's kinetic and potential energy are small compared to heat and work transfers. We will therefore exclusively use the concepts covered in chapter 3 (*open systems*) and we will be able to relate the powers and the thermodynamic state of the steam using the simple equation:

$$q_{1\to 2} + w_{1\to 2} = \Delta h \tag{3/15}$$

for all processes (reversible or not) in a steady-flow open system ($\dot{m}$ = const.), when changes in mechanical energy are neglected.

From our study of chapter 3, we recall that when the process is reversible, the work $w_{A\to B}$ between two points A and B is expressed as an integral:

$$w_{A\to B} = \int_A^B v \, dp \tag{3/22}$$

in an open system,
and when the process is reversible.

Generally, for equipment operating with steady flow, the heat transfers and work transfers are separated in space. This greatly reduces the complexity of the machines.

- The supply or extraction of heat preferably occurs without work transfer, that is, at constant pressure (isobaric processes). Ideally, these transfers occur at constant temperature (isothermal processes).
- The supply or extraction of work, which requires a change in pressure and the movement of mechanical parts within the fluid, preferably occurs without heat transfer (adiabatic processes). Ideally, these transfers occur without an increase in entropy (isentropic processes).

9.3.2 Compressors and pumps

Compressing a fluid without heat transfer in a steady flow requires a work transfer:

$$\dot{W}_{\text{compression}} = \dot{m}\,(h_2 - h_1) \tag{9/3}$$

In general, the more the volume of a fluid varies during compression, and the more complex the geometry and operation of the compressor. Two-phase mixtures are especially challenging because of the starkly differing behavior of the two fluids, and engineers generally prefer to compress either dry steam or subcooled liquid.

Since the specific volume of liquid water is about a thousand times smaller than that of water vapor, a brief rereading of equation 3/22 leads us to prefer the compression of liquids to that of gases. This is why the compression phases in industrial plants are always done in the liquid state, using pumps (figures 9.1 and 9.2). These are more compact and geometrically simple compared to gas compressors.

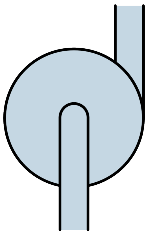


Figure 9.1: Schematic diagram of a water pump

Diagram CC-0 *Olivier Cleynen*

Figure 9.2: A pump from the manufacturer KSB delivering 2500 t/h of water at 350 bar in a steam power plant. Liquid pumps are usually powered by an electric motor, but this model is mechanically driven by the turbine and must therefore operate over a wider speed range. Its maximum power is 38 MW; the power of the driven turbine exceeds 800 MW.

Photo CC-BY-SA *KSB Aktiengesellschaft, Frankenthal*

The specific power required to compress a fluid flow from a pressure p_A to a pressure p_B, in a reversible process, is expressed from the relation 3/22. Since the specific volume v_L of pure saturated liquid water (approximately $v_L = 1 \times 10^{-3}\,\text{m}^3\,\text{kg}^{-1}$) varies very little with its pressure, we can write:

$$w_\text{liquid pump} \approx v_L \int_\text{A}^\text{B} \mathrm{d}p = v_L(p_\text{B} - p_\text{A}) \tag{9/4}$$

in the case of an approximately reversible pump operating with liquid water.

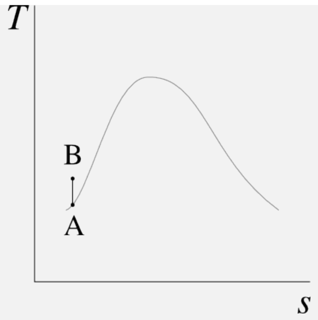
Example 9.1

In a power plant, a pump is fed with a mass flow of $35\,\text{kg}\,\text{s}^{-1}$ (77.2 lb/s) of liquid saturated water at 0.5 bar (7.25 psi). The water is compressed in an approximately isentropic manner until 40 bar (580.2 psi. What is the required power?

The process can be drawn qualitatively on a temperature-entropy diagram as shown below.

In Steam Table 3 at 0.05 MPa we read the inlet enthalpy $h_\text{A} = h_{L0.05\,\text{MPa}} = 340.5\,\text{kJ}\,\text{kg}^{-1}$. The specific volume will remain almost constant in the pump, at the value $v_\text{A} = v_{L0.05\,\text{MPa}} = 0.001\,03\,\text{m}^3\,\text{kg}^{-1}$.

Using equation 9/4 we calculate $\dot{W}_\text{pump} \approx \dot{m}\, v_L(p_\text{B} - p_\text{A}) = 35 \times 0.001\,03(40 \times 10^5 - 0.5 \times 10^5) = +142.4\,\text{kW}$.



☞ Since the compression is assumed to be isentropic, we could also start from the fact that $s_A = s_B$ to obtain h_B by interpolation in Steam Table 1 and thus calculate the power of the pump. A calculation of v_B using this method allows us to see that the specific volume varies imperceptibly (less than 0.1 %) during this process.

☞ Since we have calculated the pump power, we are able to calculate $h_B = \frac{\dot{W}_{pump}}{\dot{m}} + h_A = 482.9\,\mathrm{kJ\,kg^{-1}}$, which is the enthalpy of the water at the boiler inlet, a very useful information for later calculating the boiler power.

9.3.3 Boiler

In steam power plants, heat inputs occur at constant pressure. The water in the thermodynamic cycle is heated by contact with another pipeline: air in the case of combustion plants (waste, coal, gas), or water from a secondary circuit in the case of nuclear power plants (where the secondary circuit is used to avoid passing the high-pressure water from the thermodynamic cycle directly through the reactor core).

The wondrous behavior of fluids when they change phase works to our advantage here: in two-phase mixtures, a process at constant pressure also occurs at constant temperature (§5.2.2), allowing us to approach the conditions prescribed by Carnot without the need for any moving parts.

Because it operates at high pressure (beyond 60 bar in modern power plants) and is the scene of significant heat transfer and temperature gradients, the boiler is an expensive and heavy component (figures 9.3 and 9.4), even though its operating principle is simple.

When the heat in the power plant comes from combustion, the thermal energy of the gases can only be transferred to the water in the circuit when the temperature of the water is lower. Thus, the higher the minimum temperature of the water, and the greater the amount of heat lost above the boiler. The efficiency $\eta_{boiler} = \frac{\dot{Q}_{water}}{\dot{Q}_{heat\ source}}$ of a high-performance gas boiler is typically around 80 %.

Since no work is supplied in the boiler, the power $\dot{Q}_{boiler}$ supplied to the water by the boiler is expressed as:

$$\dot{Q}_{boiler} = \dot{m}\,(h_2 - h_1) \qquad (9/5)$$

The difference in density between the two phases in the boiler makes it difficult to superheat the steam in the presence of liquid (the liquid, being

Figure 9.3: Transport of the boiler for a wood power plant capable of withstanding a pressure of 100 bar.

Photo CC-BY-SA *by Commons User:Sensenschmied*

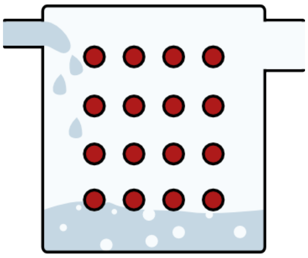


Figure 9.4: Schematic representation of a fire-tube boiler. Water enters in the liquid state on the left and exits at the top right in the form of saturated steam. In *fire-tube boilers*, the gas pipes pass through the heat exchanger filled with water. In more recent *water-tube boilers*, it is the water pipes that pass through the heat exchanger filled with hot gases. The latter technique is not detailed in this book, but its thermodynamic working principle is identical.

Diagram CC-BY-SA *by Olivier Cleynen*

denser and therefore at the bottom of the boiler, is more likely to absorb heat at high temperature). We will therefore always consider that the water is in the form of saturated vapor (index V) at the outlet of the boiler.

9.3.4 Turbine

The turbine (figures 9.5 and 9.6) is the centerpiece of any steam power plant. Several tens of meters long in modern power plants, it is carefully balanced, installed in its casing, and, if given adequate attention (minimization of temperature gradients, advanced lubrication), can deliver mechanical power for several decades without any interruption.

The efficiency of a turbine is measured by comparing its power with that of an ideal turbine (a turbine that would be isentropic). We call this parameter the *isentropic efficiency* η_T:

$$\eta_T \equiv \frac{\dot{W}_{\text{actual turbine}}}{\dot{W}_{\text{isentropic turbine}}} \tag{9/6}$$

where $\dot{W}_{\text{actual turbine}}$ is the actual power supplied by the turbine,
and $\dot{W}_{\text{isentropic turbine}}$ is the power of an isentropic turbine operating with the same mass flow rate and between the same pressures.

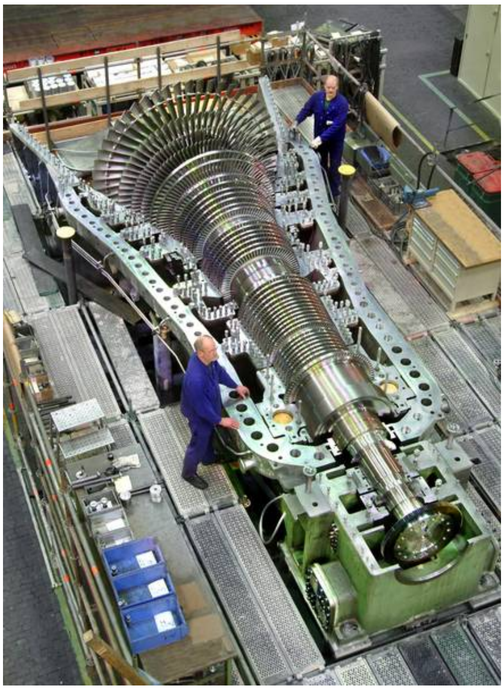


Figure 9.5: Turbine of a medium-sized steam power plant. As the water passes through the turbine, it loses energy in the form of work and its specific volume increases, requiring increasingly larger blades.

Photo CC-BY-SA MAN SE

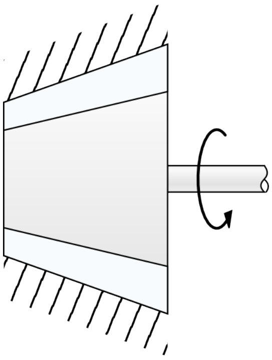


Figure 9.6: Schematic representation of a steam turbine.

Diagram CC-BY-SA *by Olivier Cleynen*

The actual power is expressed in terms of the properties of the fluid at the inlet and outlet of the turbine:

$$\dot{W}_{\text{actual turbine}} = \dot{m}\,(h_{2\text{ actual}} - h_1) = \dot{m}\,\eta_T\,(h_{2'} - h_1) \qquad (9/7)$$

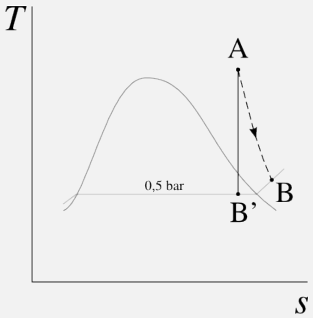
We use equation 9/7 to predict the state of the steam at the outlet of any turbine whose power and isentropic efficiency are known.

An important parameter that must be monitored is the dryness fraction of the water, especially in the final stages. Indeed, as we have already seen in section §5.4.5 p. 140, the isentropic curves always end up crossing the saturation curve: in an isentropic expansion, the steam always ends up condensing. The liquid droplets, much denser than the steam surrounding them, then violently impact the blades, causing erosion. The thermodynamic engineer will therefore ensure to maintain a high dryness fraction, typically not dropping below 95 %.

Example 9.2

A turbine with an isentropic efficiency of 85 % receives 35 kg s^{-1} of water at 40 bar and 600 °C. It expands the water to 0.5 bar. What is the delivered power?

The process can be drawn qualitatively on a temperature-entropy diagram as follows:



We start by imagining that we are equipped –a thermodynamicist's irrepressible dream!– with an isentropic turbine between these same pressures. In Steam Table 1 at 4 MPa, we read $h_\text{A} = 3674.9\ \text{kJ}\,\text{kg}^{-1}$ and $s_\text{A} = 7.3705\ \text{kJ}\,\text{K}^{-1}\,\text{kg}^{-1}$. The steam at the outlet of this hypothetical turbine (at B') has the same entropy: $s_{\text{B}'} = s_\text{A}$, and we observe that $s_{\text{B}'} < s_{V0.05\,\text{MPa}}$: the water is partially condensed. The calculation of the dryness fraction (§5.3.3) allows us to calculate the value of its enthalpy: $h_{\text{B}'} = h_{L0.05\,\text{MPa}} + \frac{s_{\text{B}'} - s_{L0.05\,\text{MPa}}}{s_{LV0.05\,\text{MPa}}} h_{LV0.05\,\text{MPa}} = 2566.3\ \text{kJ}\,\text{kg}^{-1}$.

We can now return to the the painful reality: the actual turbine delivers only 85 % of the power of this hypothetical turbine, so $\dot{W}_\text{real turbine} = \dot{m}\, \eta_\text{T}\, (h_{\text{B}'} - h_\text{A}) = 35 \times 0.85 \times (2566.3 \times 10^3 - 3674.9 \times 10^3) = -32\,980\ \text{kW} = -32.98\ \text{MW}$.

☞ Equation 9/7 allows us to calculate the enthalpy h_B actually obtained at the outlet of the turbine: $h_\text{B} = \frac{\dot{W}_\text{turbine}}{\dot{m}} + h_\text{A} = 2732.6\ \text{kJ}\,\text{kg}^{-1}$, which is very useful for later calculating the power of the condenser. A glance at Steam Table 2 shows us that $h_\text{B} > h_{V0.05\,\text{MPa}}$: the steam is dry throughout its expansion.

☞ The 15 % of missing power in the mechanical shaft of the turbine is transferred as heat (through turbulence) to the water during its expansion in the turbine.

☞ The power of the turbine is two hundred times greater than the power supplied to the pump in example 9.1 on page 247 between these same pressures.

9.3.5 Condenser

The condenser (figures 9.7, 9.8 and 9.9), the least glorious component of the power plant, is responsible for rejecting all the heat that the engineer no longer knows how to use (§7.2). The water is always cooled at constant pressure, which does not require any moving parts.

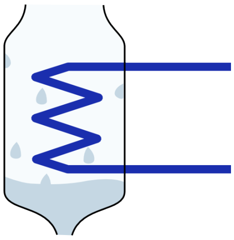


Figure 9.7: Schematic representation of a condenser. Water from the thermodynamic cycle enters at the top, in a state close to saturated vapor. It exits at the bottom in the liquid state. The heat extraction is usually ensured by a secondary water circuit (illustrated in dark blue) which is in contact with the atmosphere.

Diagram CC-BY-SA *Olivier Cleynen*

Figure 9.8: A condenser in which the heat is dissipated directly into the atmosphere, by forced convection using fans.

Photo CC-BY-SA *Cenk Endustri*

Technologically, the condenser is a simple element: the steam pipeline is simply brought into contact with a low-temperature circuit. Usually, this cooling circuit consists of external water from a river or the sea, which is then itself cooled by evaporation in the large towers seen around power plants. Using a secondary cooling circuit has two benefits. Firstly, the pressure in the condenser can be lowered to a level lower than atmospheric pressure, thereby reducing the minimum temperature of the cycle. Secondly, the water from the thermodynamic cycle, purified at considerable effort, is not lost to the atmosphere.

Since the pressure of the steam inside the condenser is often very low (down to 0.1 bar or 0.15 psi) so as to reduce the minimum temperature of the power plant cycle, care must be taken to ensure the tightness of the condenser to prevent external air or water from entering the main circuit.

Figure 9.9: Cooling towers of the Eggborough coal power plant (1967, 1960 MW) in the United Kingdom. In these towers, the heat extracted from the water in the condenser is dissipated into the atmosphere. This cooling is done through a secondary water circuit, which is brought into contact with the atmosphere and partially evaporates.

Photo CC-BY-SA *Steve Fareham*

The power rejected by the steam in the condenser is expressed as:

$$\dot{Q}_{\text{condenser}} = \dot{m}\,(h_2 - h_1) \tag{9/8}$$

9.4 Steam Engine Cycles

9.4.1 The Carnot cycle

Since the Carnot cycle which we studied in §7.3 serves as a reference in the design of engines, we start our study with it. The temperature of a liquid-vapor mixture remains constant when heated at constant pressure, so achieving isothermal heat transfers (an important characteristic of the Carnot cycle) is relatively easy with steam. A steam engine based on a Carnot cycle is schematized in figures 9.10 and 9.11.

The efficiency of the Carnot engine cycle (7/6) is only reached if the turbine and compressor operate isentropically. In practice, as we have seen, the power of the turbine is always lower, and that of the compressor always higher, than if they were isentropic.

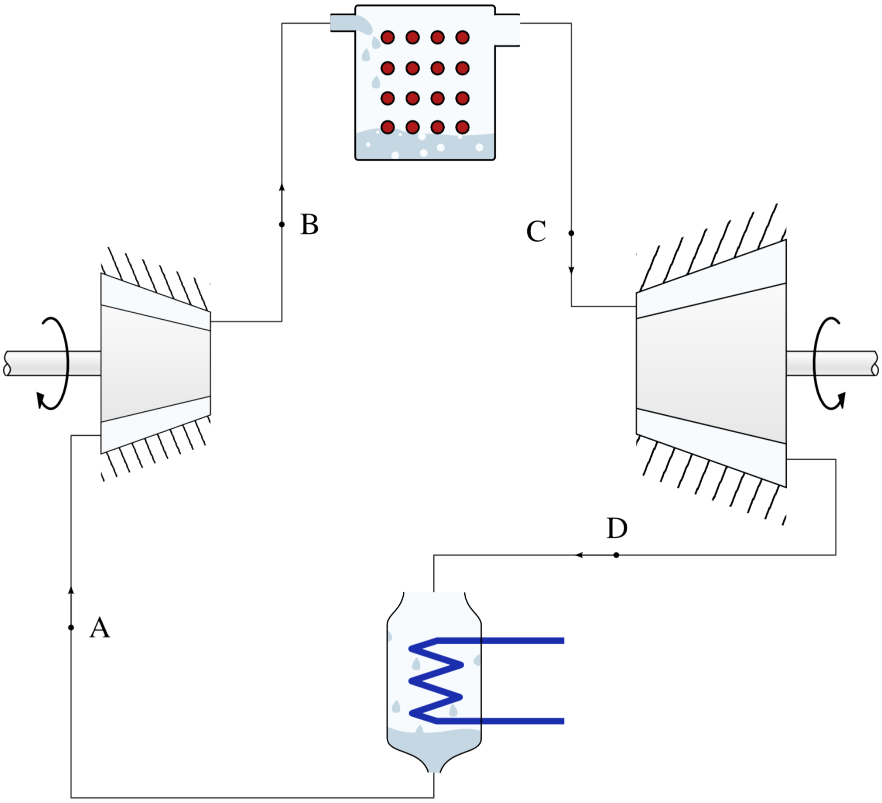


Figure 9.10: Diagram of a steam power plant operating on a Carnot cycle.

Diagram CC-BY-SA *Olivier Cleynen*

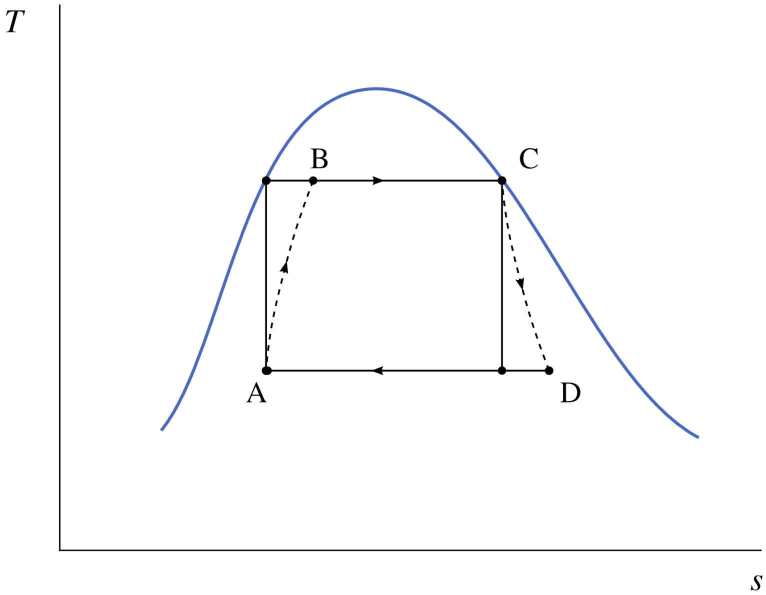


Figure 9.11: Temperature-entropy diagram of a steam power plant operating on a Carnot cycle. The dashed paths represent the real (irreversible) processes of the fluid during compressions and expansions.

Diagram CC-0 *Olivier Cleynen*

9.4.2 The Rankine cycle

In practice, using the Carnot cycle as described above poses several difficulties:

- Compressing a two-phase mixture is challenging (§9.3.2 p. 246);

- In the condenser, it is difficult to stop condensation at a specific point (point A in figures 9.10 and 9.11 above), where the dryness fraction is close but not equal to zero).

In 1859, English engineer William Rankine proposed a modification of the cycle by continuing the condensation until saturation and by compressing the water only in the liquid state. A machine based on this cycle is described in figures 9.12 and 9.13.

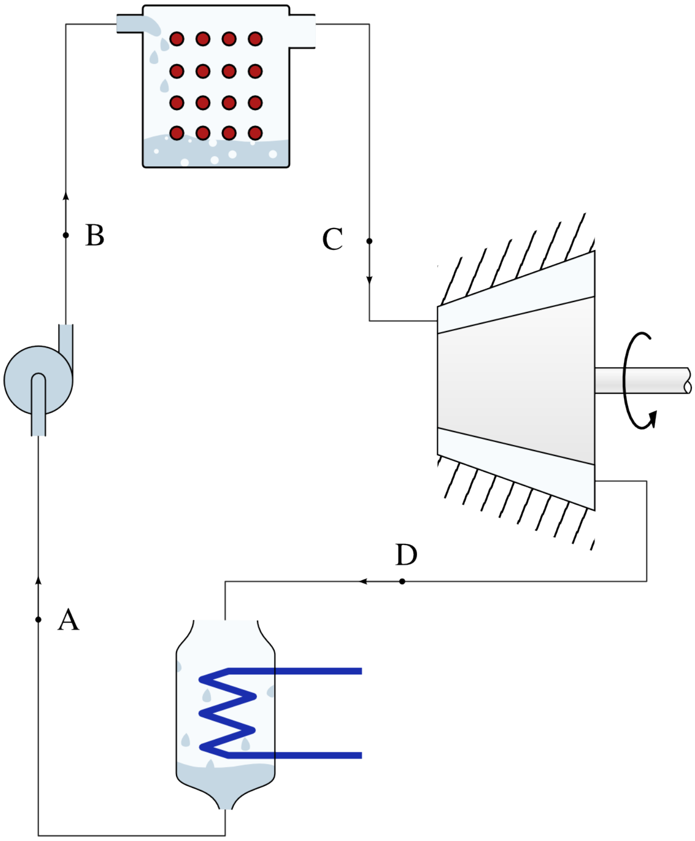


Figure 9.12: Diagram of a steam power plant operating on a Rankine cycle. The water at the outlet of the condenser is in the form of saturated liquid; it enters the boiler at a lower temperature.

Diagram CC-BY-SA *Olivier Cleynen*

The *Rankine cycle* thus uses a liquid water pump instead of a compressor dealing with a liquid-vapor mixture. Technologically, a pump is simpler to design, manufacture, and operate than a compressor. Another advantage is that compressing a liquid is several tens of times more energy-efficient than compressing a mixture (§9.3.2).

This energy saving, however, is not without consequence: at the outlet of the pump (point B), the water is at a much lower temperature than it was at the outlet of the compressor in figure 9.10. It is *the boiler* that will have to return the water to the state of saturated liquid. In other words, a considerable additional expenditure of heat must be supplied to compensate for the decrease in compression power.

It can be noticed that a significant part of the heat supplied by the boiler (that is, $q_{\text{boiler}} = h_C - h_B$) is no longer supplied at the maximum temperature of the cycle. We saw in chapters 7 and 8 that supplying heat at low temperature always results in a lower efficiency.

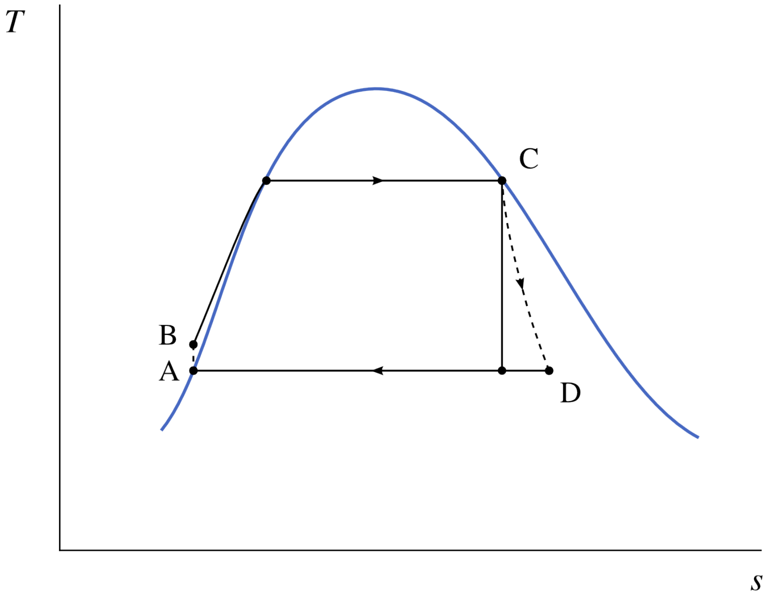


Figure 9.13: Temperature-entropy diagram of a steam power plant operating on a Rankine cycle.

Diagram CC-0 *Olivier Cleynen*

However, in practice, this heat input can make it possible to exploit low-temperature heat sources, such as exhaust gases that were previously discharged above the boiler. Thus, in some cases, the drop in thermodynamic efficiency (η_{engine}) can be compensated by an increase in the boiler efficiency (η_{boiler}), which can extract more energy from the fuel to transfer it to the steam.

Rankine thus deliberately deviated from the Carnot cycle and, in doing so, reduced the thermodynamic efficiency (although this decrease can often be offset by an increase in boiler efficiency). On the other hand, by eliminating the compressor, his modification greatly reduces the size and complexity of the machine.

9.4.3 Superheating

In order to reduce the specific steam consumption (SSC, see §9.2.2 p. 244) of a power plant, it is desirable to increase the power delivered by the turbine for a given steam flow rate. To achieve this, there are several options:

- Increase the enthalpy at the inlet of the turbine (in other words, increase the saturation pressure in the boiler).
  Unfortunately, this requires the boiler to be more resistant and more expensive; moreover, it reduces the amount of specific heat that can be supplied in it, since the enthalpy of vaporization h_{LV} decreases with temperature;
- Reduce the enthalpy at the outlet of the turbine (in other words, decrease the pressure in the condenser).
  This requires a larger turbine, promotes the entry of air bubbles into the steam circuit, and above all, reduces the steam dryness fraction at the turbine outlet;
- Increase the enthalpy (and thus the temperature of the steam) *after* its exit from the boiler.

This allows for fully utilizing the turbine's capacities, whose metallurgical limits (generally around 1000 K) often already exceed those of the boilers.

It is this last option that is very often chosen. This modification is called *superheating*: the steam is superheated at the outlet of the boiler, at constant pressure, through a series of tubes heated to a higher temperature (figures 9.14 and 9.15). Superheating could theoretically be done in the boiler itself; however, since the density of dry steam is relatively low, it is easier to bring it in contact with the hottest gases outside (and below) the boiler.

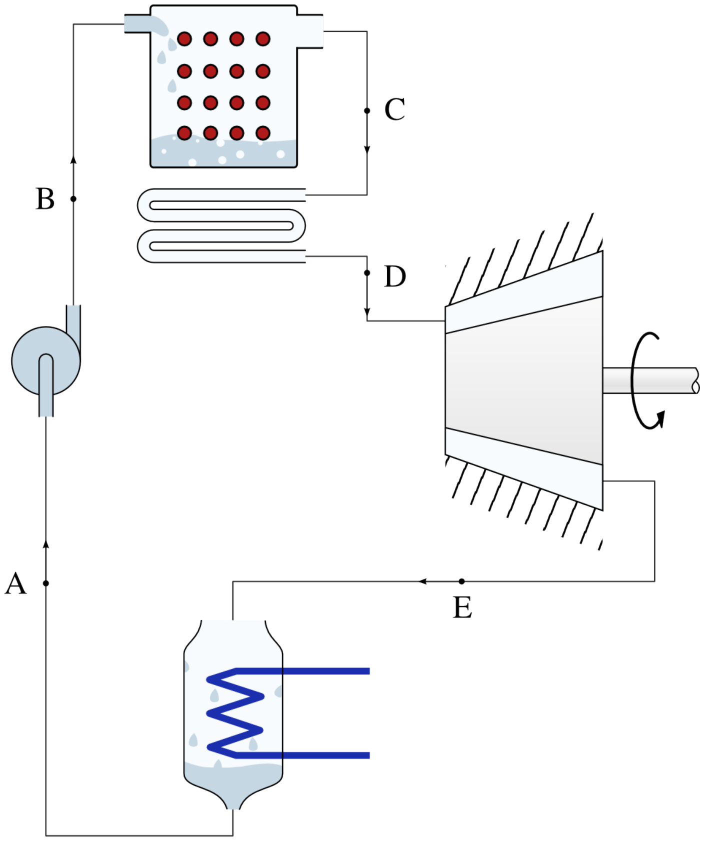


Figure 9.14: Diagram of a steam power plant operating on a superheated Rankine cycle. The water at the outlet of the boiler is heated to a higher temperature (section C → D) before entering the turbine.

Diagram CC-by-sa *Olivier Cleynen*

The main advantage of this modification is that it allows for a reduction in specific consumption that is relatively simple to implement. Additionally, increasing the average temperature at which heat is supplied tends to increase thermodynamic efficiency. Finally, it becomes possible to shift the operating range of the turbine entirely into the realm of dry steam: erosion of the blades by liquid water is thus avoided. All modern steam power plants now use a superheating circuit.

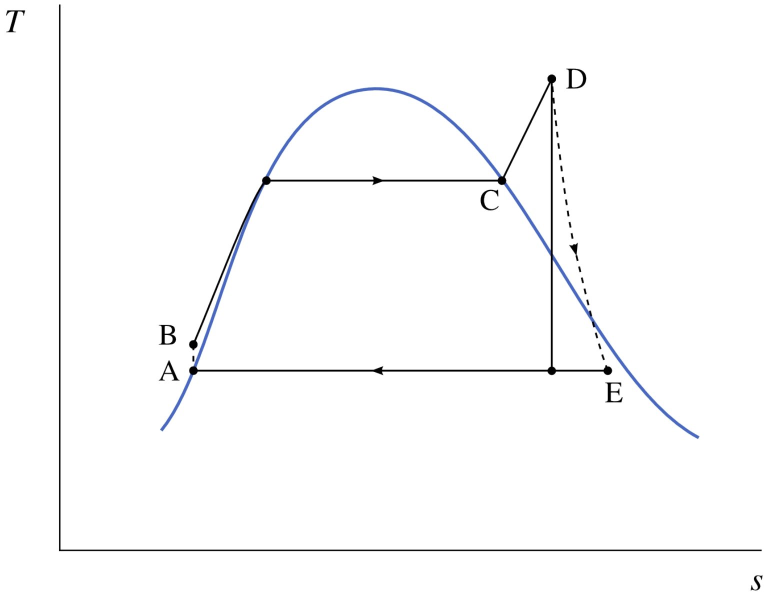


Figure 9.15: Temperature-entropy diagram of a steam power plant operating on a superheated Rankine cycle.

Diagram CC-0 *Olivier Cleynen*

9.4.4 Reheat

In order to again increase the power of the machine without increasing the steam flow rate (and therefore its overall size and the cost of the boiler), it is possible to heat the steam a second time before its exit from the turbine (figures 9.16 and 9.17). This is called *reheat*.

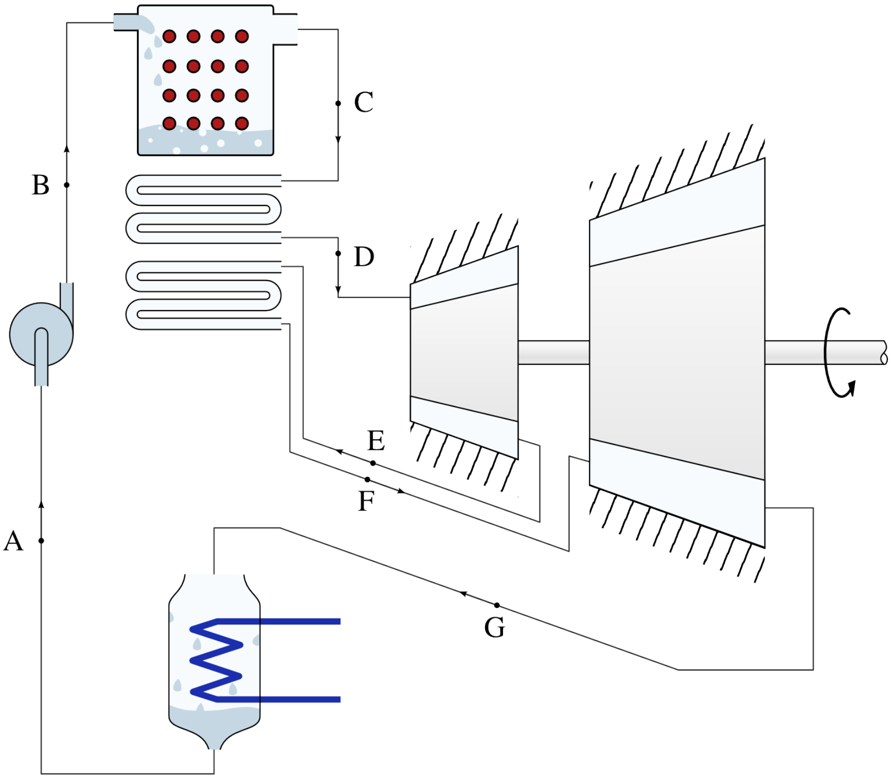


Figure 9.16: Diagram of a steam power plant operating on a reheated Rankine cycle.

Diagram CC-BY-SA *Olivier Cleynen*

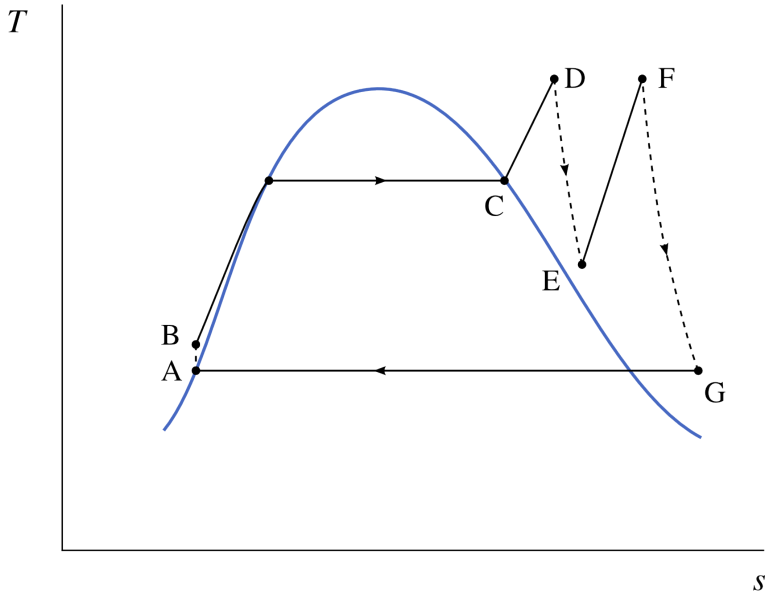


Figure 9.17: Temperature-entropy diagram of a steam power plant operating on a reheated Rankine cycle.

Diagram CC-0 *Olivier Cleynen*

With this modification, the expansion in the turbine is interrupted, and the steam is led into a new series of tubes to raise its temperature back to a high level (usually to the metallurgical limits of the turbine). The expansion is then finally completed down the condenser pressure.

Providing that the average heating temperature is increased, the overall efficiency of the power plant is increased too; therefore, the choice of the reheat pressure matters. The specific consumption is reduced in all cases, with the advantages described above.

9.4.5 Regeneration

When Rankine modified the Carnot cycle, he reduced the work required to compress the water and increased the heat necessary to bring it to the turbine inlet. However, the thermodynamic efficiency went down: indeed, the water temperature at the boiler entry was reduced. The reversibility of the heat transfer was thus reduced.

In order to increase the reversibility of the cycle (and therefore its efficiency), it is possible to gradually heat the water, using the heat from the turbine (where the steam temperature varies). This technique is called *regeneration*. One can imagine a cycle as described in figures 9.18 and 9.19 below, where the liquid water at the pump outlet is gradually heated by cooling the turbine.

In the limit case where all the heat used during regeneration is transferred with an infinitely small temperature difference, the cycle is reversible and the Carnot engine efficiency is reached even if one does not strictly follow the Carnot cycle.

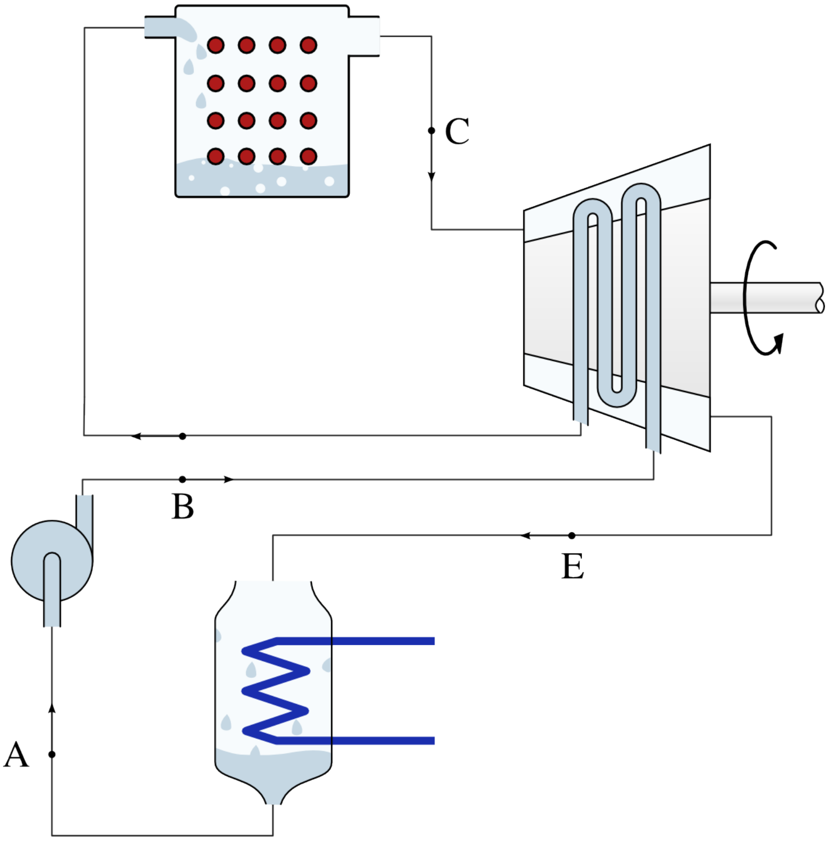


Figure 9.18: Diagram of a steam power plant with regeneration. Heat is extracted from the turbine to heat the liquid water before it enters the boiler. Ideally, heat transfer is reversible. *Diagram* CC-BY-SA *Olivier Cleynen*

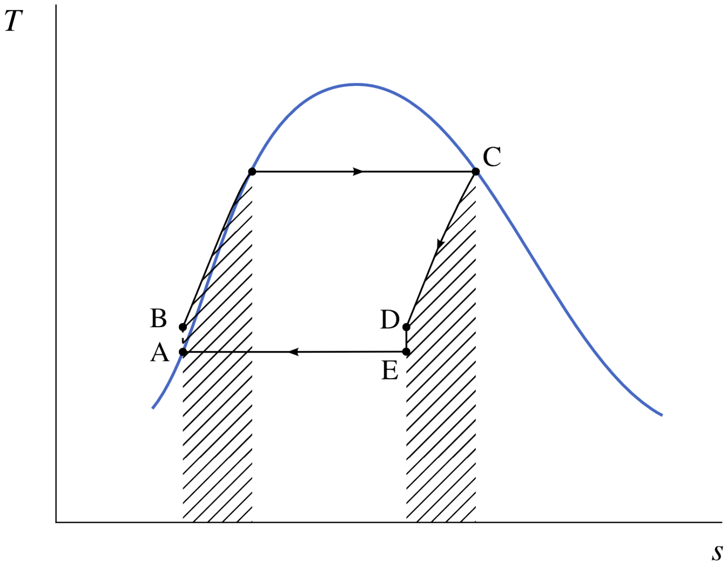


Figure 9.19: Temperature-entropy diagram of a steam power plant with regeneration. *Diagram* CC-0 *Olivier Cleynen*

In practice, such a device is difficult to implement, because it requires adding a non-uniform-temperature cooling system to the turbine, an element whose design and manufacturing are already very costly. Moreover, cooling the steam reduces its dryness fraction, increasing the amount of liquid water eroding the turbine components.

In order to implement regeneration, the *turbine bleed* technique is used. Steam is drawn off from the turbine and mixed with the liquid feedwater at the pump outlet (figures 9.20 and 9.21). This results in a heat transfer that is easier to implement.

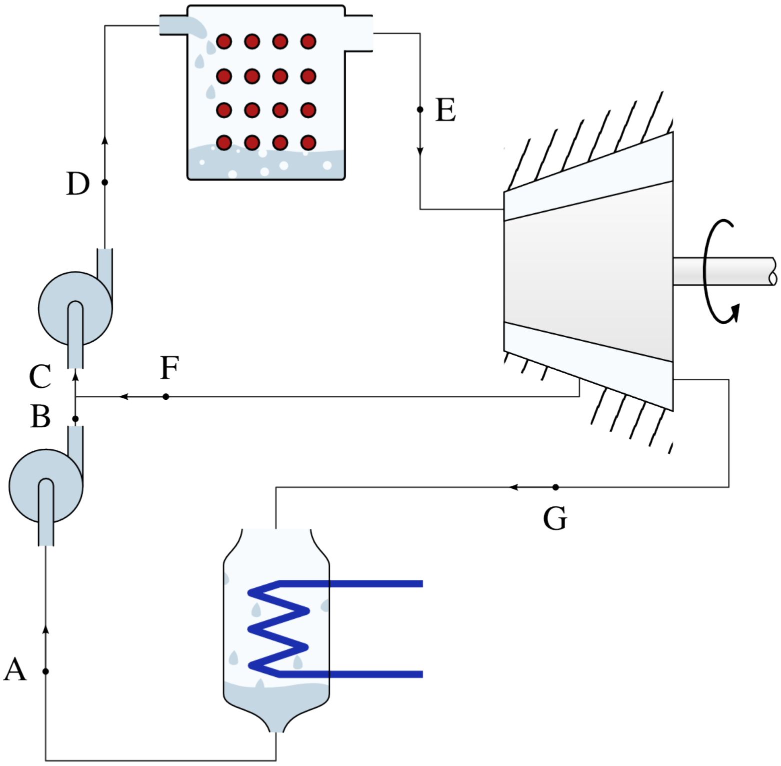


Figure 9.20: Diagram of a steam power plant with steam bleed. The steam prematurely extracted from the turbine is used to heat the liquid water during pumping.

Diagram CC-BY-SA *Olivier Cleynen*

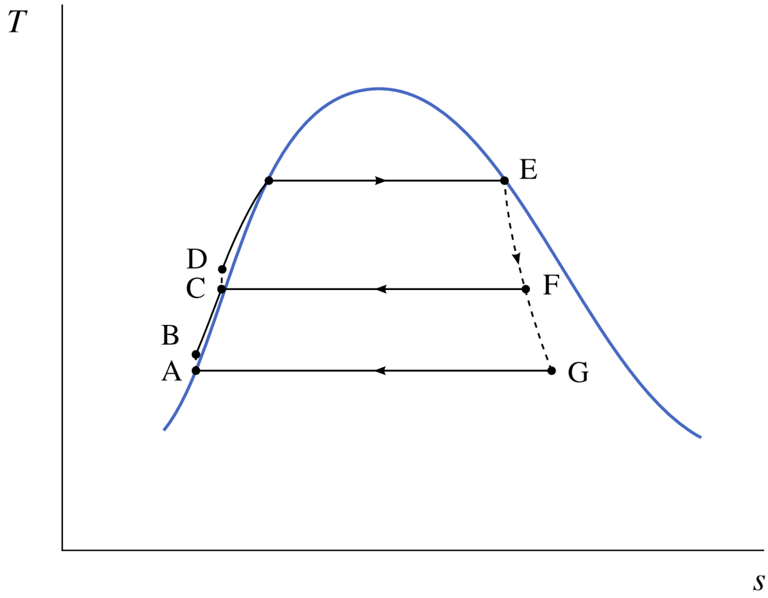


Figure 9.21: Temperature-entropy diagram of a power plant with steam bleed.

Diagram CC-0 *Olivier Cleynen*

In practice, many bleeds (sometimes called *steam extractions*) are performed in steam power plant circuits in order to control the heat flows (figure 9.22). They also allow, through discharge valves, to precisely regulate turbine mass flows and thus quickly adjust the power of the plant to the demand.

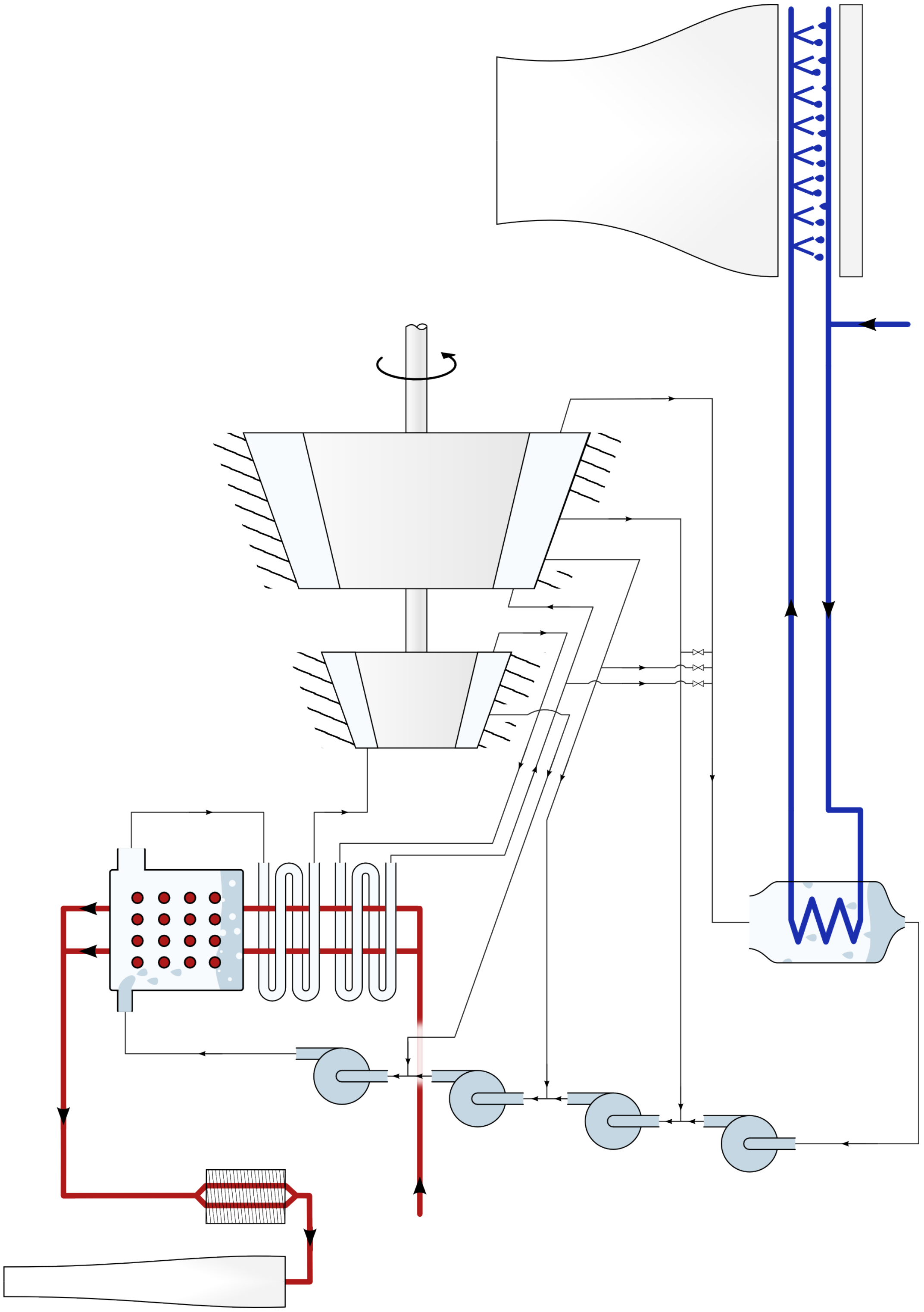


Figure 9.22: A steam power plant circuit combining superheating, reheat, regeneration, and discharge ducts. It is left to the curious student the pleasure of tracing the processes on a temperature-entropy diagram, and imagining themselves at the controls of the machine supplying their coffee maker with electricity.

Diagram CC-BY-SA *Olivier Cleynen*

A Bit of History: from the Steam Turbine to the Gas Turbine

*

At the beginning of the 20th century, the *turbine* replaced pistons and cylinders in all steam engines. A turbine has a complex geometry, sensitive to manufacturing imperfections, making its construction more delicate than that of cylindrical pistons. In return, it makes for an engine with simple arrangement, little vibration, and with easier assembly, maintenance, and lubrication, allowing for increased power or reduced volume. The Anglo-Irish engineer Charles Parsons dramatically demonstrated this in 1897 with the *Turbinia* (figure 9.23), the first ship of its kind, which was so fast that no military vessel could catch up with it. Ten years later, the entire Royal Navy had switched to turbines for ship propulsion.

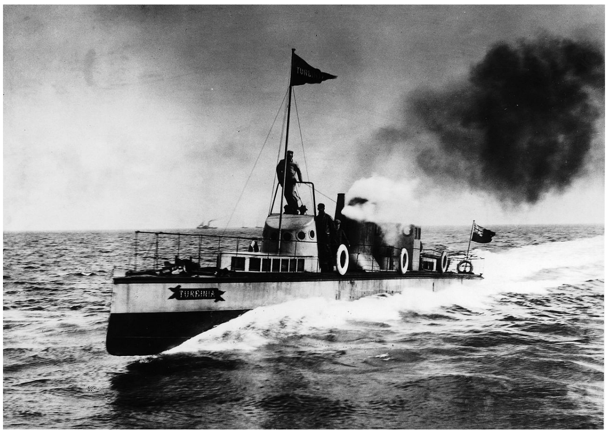


Figure 9.23: The *Turbinia*, Charles Parsons' yacht used as a demonstrator for his research in maritime propulsion. With its three steam turbines and nine propellers, it reached 60 km/h and allowed its owner to ridicule the Royal Navy during Queen Victoria's jubilee parade in 1897.

Photo by Alfred John West, 1897 (public domain)

In the realm of air-based engines, the situation was quite different: until the late 1930s, all engines were piston-cylinder based. Piston technology peaked in the aeronautical sector, where cylinders were arranged in a star pattern behind the propellers to reduce bulkiness and vibrations. In these machines, such as the *Twin Wasp* by Pratt & Whitney, the mechanical arrangement of cylinders, connecting rods, and crankshafts was absolutely phenomenal (figure 9.24), and the intake and exhaust systems going to and back from dozens of combustion chambers were labyrinthine. Two engineers, the German Hans von Ohain and the English Frank Whittle, independently dedicated themselves to the design of a gas turbine engine in order to break free from this complexity.

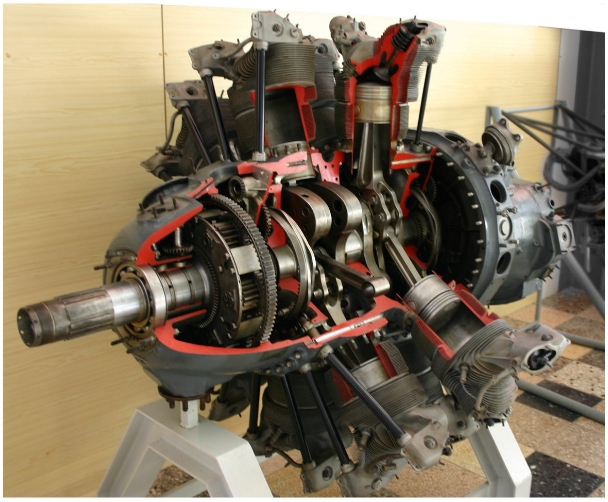


Figure 9.24: Cross-section of a *Pratt & Whitney Twin Wasp* engine (1932), showing the internal arrangement with connecting rods and crankshafts connecting the two rows of seven pistons arranged in a star pattern. The engine, with a displacement of 30 L, produced over 1000 hp and was produced in over 170 000 units.

Photo CC-by-sa *Olivier Cleynen*

Developing a gas turbine engine is much more challenging than for steam. Certainly, air (or burnt gases) and dry steam have very similar properties: thus a steam turbine works very well with compressed air. The difficulty lies at the other end of the engine. In steam engines, compressing water is done in the liquid state, which is very efficient. Compressing water at 10 °C from 1 to 10 bar, for example, only requires $w_{\mathrm{A}\to\mathrm{B}} \approx v_L(p_{\mathrm{B}} - p_{\mathrm{A}}) = 0.001(10-1)\times 10^5 = 900\,\mathrm{J\,kg^{-1}}$ (9/4). In contrast, doing the same with air requires a minimum specific power of $w_{\mathrm{A}\to\mathrm{B}} = c_p \Delta T = c_p \left(T_{\mathrm{A}} \left(p_{\mathrm{B}}/p_{\mathrm{A}}\right)^{\frac{\gamma-1}{\gamma}} - T_{\mathrm{A}} \right) = 1005 \left(283.15\,(10)^{\frac{0.4}{1.4}} - 283.15 \right) = 265\,\mathrm{kJ\,kg^{-1}}$ (4/34 & 4/37), almost three hundred times more!

We saw in §9.4.2 that using liquid compression is not without consequences – it must be compensated by greater power at the boiler and reduces the thermodynamic efficiency – but it greatly facilitates the development of the engine. Since almost all of the net power of the engine comes from the turbine, a highly irreversible or incomplete expansion only affects the power and efficiency of the engine. In a gas turbomachine, on the other hand, the turbine also powers the compressor: it plays a dual role. For as long as it does not supply enough power to match that of the compressor, the engine will not run at all. The isentropic efficiency of the turbine and compressor thus become paramount parameters (we will revisit this in §10.2.2 with the concept of *work ratio*) and it follows that the development of a gas turbomachine is an ambitious undertaking.

Both Whittle and von Ohain focused their efforts on an ingenious aeronautical engine called *turbojet*: it is the exhaust gases, in large quantities and with high residual pressure, that would provide the engine's thrust (§10.5.3). The operating principle is very simple (the air flows in a steady state and there is only one moving part) but the challenges were numerous. Like an aircraft wing, the compressor blades tend to stall at low power and during transient phases, causing abrupt and destructive flow changes. In the combustion chambers, it is necessary to prevent the flame from impinging on the walls (which would cause them to melt) or from extending, especially during ignition or reignition, into the turbine. Weight constraints require the use of lightweight materials which complicate manufacturing. The two engineers carried out their work in the heart of the Second World War, each funded by military budgets, and the first jet aircraft flew in 1940. The subsequent production aircraft were delicate to operate, unresponsive, and their service life barely reached 20 hours. They arrived too late and in insufficient numbers to affect the course of the conflict.

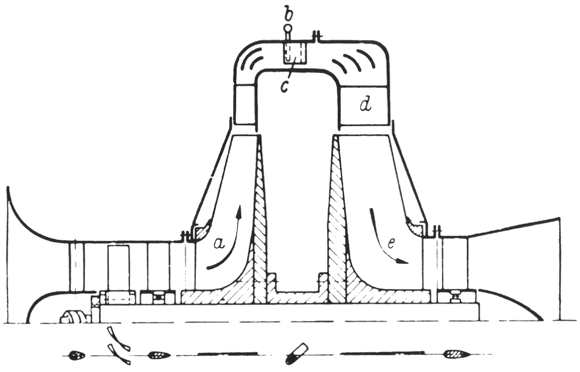


Figure 9.25: Cross-sectional diagram of the *Heinkel He S-1*, the first prototype tested by Hans von Ohain in 1937. The compressor consists of an axial stage and a centrifugal stage; the turbine is centripetal. There is only one moving part and its speed is invariant.

Diagram USAF (public domain)

At the end of the war, there was a surge of enthusiasm: aviation embraced the engine it had been waiting for over three decades. To understand why the jet engine became the Holy Grail of 20th century aeronautics, a bit of flight mechanics is needed. In subsonic flight, a well-designed aircraft has a *drag coefficient* $C_D \equiv F_D \div \left(\frac{1}{2} A_{\text{ref.}} \rho \, C_{\text{flight}}^2\right)$ that is almost constant. Thus, when reducing the reference area $A_{\text{ref.}}$ of the wing surface and the ambient density ρ (by gaining altitude), the flight speed C_{flight} can be increased *while keeping the drag F_D constant.* The energy cost of moving the aircraft remains constant – however, the required power $\dot{W}_{\text{engine}} = F_x C_{\text{flight}}$ increases proportionally to the speed. These characteristics make aircraft relatively energy-efficient machines, but very power-hungry, since they need to maintain the same thrust at very high speeds.

The jet engine had two advantages to address this issue. Firstly, it was compact, lightweight, and vibration-free, which is highly desirable for an application where drag (and thus the thrust to be provided) increases proportionally with the weight of the aircraft. Secondly, the propeller, which is very efficient at low speeds but with whose tips reach supersonic speeds early, thus limiting the speed of aircraft, was completely eliminated. Because of these qualities, the low efficiencies due to irreversible compressions and expansions, low pressure ratios, and excessively high gas speeds in the nozzles were acceptable.

Thus, the graceful Lockheed *Constellation*, the culmination of the era of propeller aviation, was instantly rendered obsolete by the arrival of the much faster De Havilland *Comet* in 1949, a remarkable quad-jet of the same size (figure 9.26). Even though it was initially unable to cover the same distance and it featured higher fuel consumption per kilometer, the *Comet* left no chance for its competitors. Its speed was an obvious quality for passengers, but also for the airlines, significantly increasing their productivity.

The *Comet*, after a serious design flaw was corrected, was itself surpassed by the Boeing *707* in 1957. Capable of flying further while carrying more payload, and even faster (at 900 km/h, the speed that all airliners have adopted since, the air on the wing's upper surface barely reaches the speed of sound), the *707* marked the entry into the *jet age*, where airliners were no longer built by dozens but

Figure 9.26: From top to bottom:
The 1943 Lockheed *Constellation*, the culmination of the propeller aircraft era: four Wright *Duplex-Cyclone* supercharged 18-cylinder engines, capable of covering 3700 km (2300 mi) at 500 km/h (310 mph).
The 1949 De Havilland *Comet*, the first jet airliner: four Halford *Ghost* turbojet engines, capable of covering 2400 km (1500 mi) at 740 km/h (460 mph).
The 1957 Boeing *707*, with a configuration and performances anticipating those of all its successors: four Pratt & Whitney *JT3C* turbojet engines, capable of covering 4300 km (2700 mi) at 900 km/h (560 mph).

Constellation Photo CC-by-sa *by Bill Larkins*
Comet Photo and 707 (edited) CC-by-sa *by Piergiuliano Chesi*

by thousands. Thus, in just twenty-five years, the gas turbine engine doubled the speed of aircraft and divided the price of tickets by four.

> “Ready?” “Takeoff, time!”
> The flight engineer pushes the throttle levers with me.
> NNggnniiiaavvrroooooooaaaaaaaarrrroooouuummmmm...
> “N1s green.” It’s pushing hard, but accelerates ever so gently, given the weight of the behemoth.
> “Eighty knots” “Thrust set.”
> I have the tips of my feet on the rudder pedals, a precision similar to a kickboxing move. I’m enjoying every bit of it.
> 120 knots. I’m in control, guys. 432 passengers and 15 crew members are strapped in the back, ears and senses alert.
> 140 knots. Two bursts of light beacons pass by on the sides. The rudder, precise.
> “V1.” Another 20 knots to reach before the machine can fly. I can see end of the runway coming up, over there ahead.
> “Rotate.” At 170 knots, I pull gently, then more firmly. Five degrees of pitch. Ten degrees. It’s no longer rolling, the needle is at 185 knots.
> Twelve-degree pitch. Come on, my dear, we must climb. “Positive climb.” “Landing gear up.” The truth lies tonight between twelve and thirteen degrees of pitch, where the airspeed indicator needle comes to a standstill. We pass the hill, and three hundred feet below, the *747* flying by must feel like an earthquake.
>
> Jacques Darolles, 1998
> *Le plus beau bureau du monde* [42]

Nearly sixty years after the first flight of the *707*, airliners still fly at the same speed, but jet engine technology has continued to advance [47]. With their carbon-epoxy or blown titanium fan blades, turbine stators printed in ceramics, their multiple laser-drilled pneumatic turbine cooling circuits, their electronic control, diagnostic and remote monitoring systems, they slowly but surely continue to increase in efficiency. Reliability is also remarkable: a modern engine on average only experiences an in-flight failure every 200 000 flight hours, and is separated from the aircraft for maintenance only every 20 000 hours or 10 000 flights. Will a new type of engine ever render the jet engine obsolete and propel aviation forward into a new era?

Problems

The properties of water are tabulated in Steam Tables 1, 2, and 3 (see Appendix A1 p. 307)

Air is considered an ideal gas.

$c_{v\,(\text{air})} = 718\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$ $\quad R_{\text{air}} = 287\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$

$c_{p\,(\text{air})} = 1005\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$ $\quad \gamma_{\text{air}} = 1.4$

9.1 Superheated Rankine cycle

The *Électricité de France* power plant in Porcheville (figure 9.27) received heat from the combustion of oil, and used a steam cycle to power an electric generator.

In the power plant, water cycled between the pressures of 0.1 and 140 bar (0.15 and 2031 psi). The steam reached 545 °C (1013 °F), and the turbines had an isentropic efficiency of 80 %.

For the purposes of the problem, we consider that the cycle was based on a superheated Rankine cycle.

1. Sketch the physical water circuit in the power plant; draw the cycle followed on a temperature-entropy diagram, qualitatively (that is, without showing numerical values), showing the saturation curve.
2. What is the enthalpy of the water at the outlet of the turbines?
3. What is the enthalpy of the water at the outlet of the pumps?
4. What is the thermal efficiency of the power plant?
5. What is the specific steam consumption of the power plant, namely, the mass of steam that must enter the turbine for the installation to supply 1 kWh of work?
6. What hourly steam flow rate is required in the circuit in order to achieve a net power of 60 MW?

Figure 9.27: The Porcheville power plant, running on coal until 1987, then on oil until 2017, when it closed down. It mainly served peak demands.

Photo CC-0 *Olivier Cleynen*

9.2 Implementation of a reheating process

The Porcheville power plant described in Problem 9.1 is modified to accommodate a series of reheating tubes. The water expansion is stopped at 18 bar in the turbines, and the steam is brought back to the maximum temperature of the cycle (that is, 545 °C).

The power plant is fueled by heavy fuel oil known as "ULSD", with a density of $1050\,\text{kg}\,\text{m}^{-3}$ and a heat of combustion of $40.2\,\text{MJ}\,\text{kg}^{-1}$.

The air used for combustion enters the boiler at a temperature of 15 °C and a pressure of 1 bar. It is heated to a temperature of 820 °C at constant pressure through combustion, before passing around the water pipes. When it leaves the boiler, its temperature is 180 °C.

1. What is the new thermal efficiency of the power plant?
2. What is its new specific steam consumption?
3. What air flow rate must be admitted to the boiler in order to maintain a net power of 60 MW?
4. What is the boiler efficiency?
5. What is the hourly volumetric flow rate of fuel?
6. An engineer proposes to pass the intake air duct through the exhaust gases (without mixing them) to increase its temperature before combustion. Do you think this is a good idea?

9.3 Cycle with regeneration

In a polar icebreaker ship (figure 9.28), the propellers are driven by a steam system, itself powered by a nuclear reactor.

The cycle is based on a superheated Rankine cycle between pressures of 30 and 0.5 bar. The steam, heated by a secondary pressurized water system that passes through the reactor, reaches 310 °C (590 °F).

For the sake of not overloading this problem, we consider that the turbine is isentropic and perfectly insulated.

1. What is the thermal efficiency of the power plant?
2. We define the specific steam consumption as the inverse of the net power of the power plant: it is the mass of steam that has passed through the turbine when the power plant has generated 1 kWh of work.
   What is the specific consumption of the power plant?

Figure 9.28: The *50 Let Podeby*, a 25,000-ton nuclear-turbo-electric powered icebreaker (two reactors of 171 $\mathrm{MW_{heat}}$, three engines of 17.6 $\mathrm{MW_{mech.}}$). Its construction started in 1989 but it only entered service in 2007.

Photo CC-by-sa *by Commons User:Kiselev d*

An engineer proposes to modify the cycle and make it regenerative by extracting steam from the turbine to insert it into the compression circuit.

S/he suggests separating the compression into two stages, one from 0.5 bar to 6 bar, and the second from 6 bar to 30 bar; and then inserting the extracted steam into the feedwater between the two pumps. The steam extraction rate is such that the water at the outlet of the mixer is exactly at the saturation point.

1. Make a sketch of the proposed setup (that is, the physical circuit followed by the steam).
2. Draw the cycle qualitatively (that is, without showing numerical values) on a temperature-entropy diagram, showing the saturation curve.
3. What percentage of the turbine steam flow rate would need to be extracted at 6 bar in order to heat the water to saturation between the two pumps?
4. Does the shaft power increase or decrease, and by how much?
5. Does the efficiency of the power plant increase or decrease, and by how much?

Answers

9.1 1) See figures 9.14 and 9.15 p. 258;

2) With $s_{\text{E}} = s_{\text{D}} = 6.5399\,\text{kJ}\,\text{kg}^{-1}$ and $\eta_{\text{T}} = 80\,\%$, we obtain $h_{\text{E}} = 2287.7\,\text{kJ}\,\text{kg}^{-1}$ as in example 9.2 p. 251;

3) Using equation 9/4 we obtain $h_{\text{B}} = 205.9\,\text{kJ}\,\text{kg}^{-1}$ as in example 9.1;

4) $\eta_{\text{thermal}} = \left|\frac{(h_{\text{E}}-h_{\text{D}})+(h_{\text{B}}-h_{\text{A}})}{(h_{\text{D}}-h_{\text{B}})}\right| = 35.29\,\%$ (6/4);

5) ssc = $3.15\,\text{kg}/(\text{kW}\,\text{h})$;

6) $\dot{m}_{\text{water}} = 52.5\,\text{kg}\,\text{s}^{-1} = 116\,\text{lb}/\text{s}$.

9.2 1) $h_{\text{D2}} = 2960.8\,\text{kJ}\,\text{kg}^{-1}$, $h_{\text{E2}} = 3570.3\,\text{kJ}\,\text{kg}^{-1}$, $h_{\text{F}} = 2642.7\,\text{kJ}\,\text{kg}^{-1}$, thus the efficiency reaches $\eta_{\text{thermal 2}} = 36.31\,\%$ (+1 point, already a significant improvement);

2) ssc = $2.576\,\text{kg}/(\text{kW}\,\text{h})$ ($-18\,\%$, good work);

3) In the boiler, the heat rejected by the air is gained by the water: $\dot{m}_{\text{air}} = \frac{-\dot{Q}_{\text{water}}}{c_p\,\Delta T} = \frac{\dot{W}_{\text{net}}}{\eta_{\text{thermal}}} \frac{1}{c_p\,(T_{\text{air 3}}-T_{\text{air 2}})} = 256.9\,\text{kg}\,\text{s}^{-1}$.

4) $\eta_{\text{boiler}} = \frac{\dot{Q}_{\text{water}}}{\dot{Q}_{\text{received by the air}}} = 79.5\,\%$;

5) $\dot{V}_{\text{fuel}} = \frac{\dot{Q}_{\text{received by the air}}}{\rho_{\text{fuel}} c_{\text{fuel}} \eta_{\text{boiler}}} = 17.7\,\text{m}^3\,\text{h}^{-1} = 3893\,\text{imp gal}/\text{h}$;

6) It is an excellent idea. In this way, we reduce the heat carried away by the exhaust gases at the boiler outlet, immediately increasing η_{boiler}.

9.3 The *50 Let Podeby* actually operates between 29 and 0.75 bar, but these values that are not tabulated in the steam tables for this book.

1) Using the diagram from figures 9.14 and 9.15 on page 258, $h_{\text{A}} = 340.5\,\text{kJ}\,\text{kg}^{-1}$, $h_{\text{B}} = 343.54\,\text{kJ}\,\text{kg}^{-1}$, $h_{\text{D}} = 3017.4\,\text{kJ}\,\text{kg}^{-1}$, $h_{\text{E}} = 2284.5\,\text{kJ}\,\text{kg}^{-1}$, thus $\eta_{\text{thermal}} = 27.294\,\%$;

2) ssc = $4.93\,\text{kg}/(\text{kW}\,\text{h})$;

3) See figure 9.20 p. 261;

4) See figure 9.21 p. 261;

5) $h_{\text{bleed}} = 2673.9\,\text{kJ}\,\text{kg}^{-1}$, $h_{\text{pre-mix}} = 341.1\,\text{kJ}\,\text{kg}^{-1}$, $h_{\text{post-mix}} = 670.4\,\text{kJ}\,\text{kg}^{-1}$: thus the proportion needed to saturate the water after mixing is $z = 14.1\,\%$;

6) $w_{\text{net 2}} = -674.87\,\text{kJ}\,\text{kg}^{-1}$ ($-9.2\,\%$: a tragedy!);

7) $q_{\text{boiler}} = 2344.4\,\text{kJ}\,\text{kg}^{-1}$, thus $\eta_{\text{thermal 2}} = 28.786\,\%$ (+1.49 pt: is it really desirable in this application?).

Chapter 10
Air-Based Power Cycles

or

Build Your Own Combustion Cycle: the Essential Starter Pack

Chapter 10 – Air-Based Power Cycles

Executive summary for chapter 10

Air-based engines are more compact and lighter than steam engines. Many modifications are made to ideal cycles to reduce bulk and adapt to the temperature limits of engine components.

Introduction

In this final chapter, we cover the cycles of engines that use air a as a working fluid. These are sometimes called *gas power cycles*, although because the term *gas engine* is understood differently in different contexts, we choose the term *air-based engine* in this book. This chapter 10 aims to answer the same two questions as its predecessor with liquids/vapors:

- Why and how are air-based engines used today?
- Why are we moving away from ideal cycles and how do we quantify these compromises?

10.1 Why Use an Air-Based Engine?

The use of air as a working fluid, rather than water, brings several advantages.

- First, it is possible to completely dispense with condensers and coolers. The cooling phase (§7.2.1) takes place directly in the atmosphere, which easily accommodates all the hot gases that are rejected, and which serves as a reservoir from which to draw fresh air to feed into the engine.

  For an equal power level, the mass, volume, and often the cost of air-based engines are therefore greatly reduced compared to their steam counterparts. This is particularly interesting when the engine must contribute to carrying its own weight.
- Second, the heat input is carried out without loss. It is now possible to have the combustion occur directly within the working fluid – this is why we speak of *internal combustion engines* – and thus to avoid the heat loss associated with a boiler (§9.3.3).

The main drawback of air-based engines is that internal combustion requires a high-quality fuel. Since the combustion residues must flow inside the thermodynamic part of the machine, we cannot use economical or otherwise advantageous heat sources such as the combustion of waste, wood, or coal.

Ultimately, the relatively lower weight of air engines compared to their steam counterparts means that they are systematically used when mass plays an important role, such as in air or road transport.

10.2 Criteria for Evaluating Air-Based Engines

10.2.1 Thermal efficiency

It now goes without saying that we always seek to achieve a high *thermal efficiency* $\eta_{\text{engine}} \equiv \left|\frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}}\right|$ (6/4), while keeping in mind that it cannot exceed its theoretical maximum $\eta_{\text{Carnot engine}} = 1 - \frac{T_{\text{min.}}}{T_{\text{max.}}}$ (7/6).

As we have already suggested in §9.2, thermal efficiency should not, however, be maximized at the expense of other important parameters, the most notable of which we present below for air-based engines.

10.2.2 Work ratio

In a running engine, the irreversibility of compressions and expansions is not independent of speed. When they operate outside their optimal operating range, engines thus see their specific power decrease. Irreversibilities can even reduce efficiency to zero, with the engine running without producing useful work (as would a disengaged car engine). The *work ratio* is a concept that assesses the robustness of a cycle to the increase of these irreversibilities.

In order to address this concept, let us first study the case of an engine whose arbitrarily-chosen energy flows are described in figure 10.1; in this engine, compressions and expansions are reversible. Now, if instead of being ideal, the turbine suddenly saw its isentropic efficiency drop down to 95 %, it would supply 95 W. The net power of the engine would then decrease from 10 to 5 W – a reduction of 50 %.

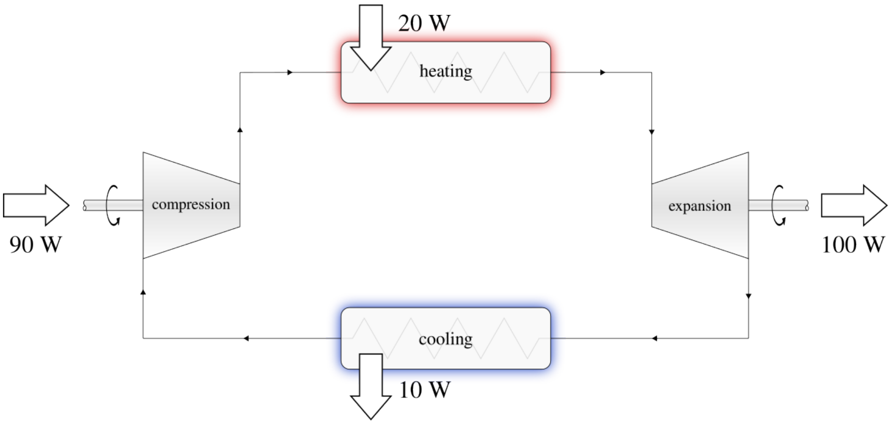


Figure 10.1: The cycle of a hypothetical engine with low work ratio. The delivered power is $\dot{W}_{\text{net}} = \dot{W}_{\text{compression}} + \dot{W}_{\text{expansion}} = +90 + (-100) = -10$ W, and the efficiency $\eta_{\text{engine}} = -\frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = -\frac{-10}{20} = 50$ %.

Diagram CC-0 *Olivier Cleynen*

Let us now compare this case with an engine of the same efficiency, same power, but with a different cycle, as shown in figure 10.2. In that new engine, if the isentropic efficiency of the turbine went from 100 % to 95 %, the net power would decrease from 10 to 9 W – a decrease of only 10 %.

We can see here that the larger the share of the turbine power in the net power delivered, and the less the cycle efficiency is affected by irreversibilities. We generalize and formalize this "turbine share" with the concept of *work ratio* M_w, defined as the ratio between the net power and the gross power of an engine:

$$M_w \equiv \frac{\dot{W}_{\text{net}}}{\dot{W}_{\text{gross}}} = \frac{\dot{W}_{\text{expansions}} + \dot{W}_{\text{compressions}}}{\dot{W}_{\text{expansions}}} \tag{10/1}$$

where M_w is the work ratio (dimensionless);
$\dot{W}_{\text{expansions}}$ is the shaft power released during expansions;
and $\dot{W}_{\text{compressions}}$ is the shaft power received during compressions.

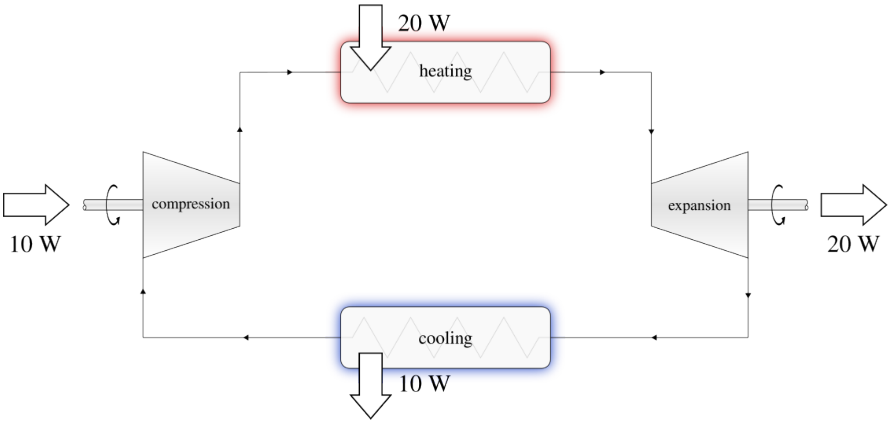


Figure 10.2: The cycle of a second (also hypothetical) engine, with a high work ratio. The delivered power $\dot{W}_{\text{net}} = +10 + (-20) = -10\,\text{W}$, and the efficiency $\eta_{\text{engine}} = -\frac{-10}{20} = 50\,\%$ are identical to those of the engine described in figure 10.1.

Diagram CC-0 *Olivier Cleynen*

A machine with a large work ratio loses less of its efficiency when operating outside its optimal speed range: it is therefore more flexible in use. The work ratio is one of the indicators of a engine's responsiveness, namely, its ability to change power and speed quickly. A parallel can be drawn with the concept of *net profit margin* in economics: all else being equal, it is more interesting to sell items bought at 2€ for 3€ than items bought at 100€ for 101€, notably because the profit of 1€ is then less sensitive to a change in price or cost imposed by the market.

The Carnot engine is the typical example of a thermodynamic cycle with high efficiency but low work ratio. By plotting the cycle on a pressure-volume diagram (figure 7.10 p. 190), this weakness is evident: the curves during the compression and expansion phases are very close to each other. Rankine, when he modified this cycle (§9.4.2), significantly increased the work ratio.

In general, obtaining high thermal efficiency requires a high compression ratio (so as to achieve a high temperature before heat transfer is initiated). Obtaining a high work ratio requires low compression work (so as to minimize the engine's sensitivity to irreversibilities). These two objectives are often contradictory, and it will be up to the engineer to find the best compromise.

10.2.3 Specific thrust and specific power

We use the concepts of *specific thrust* $\frac{F}{\dot{m}}$ and *specific power* w_{net}, in other words, the thrust and power of the engine divided by the mass flow rate of air passing through it, to compare engine cycles succinctly. Increasing these parameters is often desirable in applications where a high power-to-weight ratio is sought.

For example, a heavier aircraft must provide greater lift, which increases drag, and in turn the thrust, and finally and the power required to generate it – so in that case, an increase in efficiency is not always justified if it results in an increase in weight or size.

10.2.4 Other evaluation criteria

Many more criteria need to be considered in the design of an engine, which we will not explore in this book. They include:

- purchase cost, which is directly related to the complexity and size of the engine;
- ecological impact;
- ease of maintenance and reliability;
- responsiveness;
- the level of vibration generated.

Taking into account each of these factors can justify deliberately limiting the efficiency of the engine. Indeed, when the student acquires their first motor vehicle, they will likely attach more importance to the purchase cost than to the energy consumption – and in the same way, they may not choose to fit it with a racing engine requiring constant maintenance.

Truth be told, there is very little to add to what our eminent and favorite theorist already explained in 1824:

> One must not flatter oneself for ever exploiting, in practice, the entirety of the motive power contained in combustibles. Attempts that one would make to to approach this result would prove more harmful than beneficial, if they led to the neglect of other important considerations. The economy of fuel is but one of the conditions to be fulfilled by fire machines; in many circumstances, it is but secondary, and must often yield precedence to the safety, the solidity, the durability of the machine, to the small amount of space that it must be made to occupy, the low cost of its establishment, etc. Knowing how to appreciate, in each case, at their true value, the considerations of convenience and economy that may present themselves; knowing how to discern the most important from those that are merely accessory, weighing them all properly against one another, so as to attain by the easiest means the best possible result, such must be the principal talent of the man [or woman] called upon to lead, to coordinate between them the labors of his [or her] fellow beings, to make them converge toward a useful end, of whatever nature it may be.
>
> Sadi Carnot, 1824 [4]

10.3 Reciprocating Engines

Reciprocating engines, often called *piston engines*, take in a finite amount of air and perform their thermodynamic cycle on this mass. The cycle is repeated several times over time, and often several times in parallel, in order to provide continuous power. An automobile engine typically performs about fifty cycles per second.

10.3.1 Advantages of piston engines

From a thermodynamic perspective, the main advantage of these engines is that it is much easier to manipulate a fixed mass of air than a continuous flow. Manufacturing a cylindrical piston to fit tightly in a cylindrical bore is comparatively simple, and this allows the volume and pressure of the fluid to be precisely controlled. For example, it is possible to carry out combustion at a constant temperature (as prescribed by Carnot) by varying the volume during combustion. The same operation in a steady flow machine would require combustion to take place in a turbine (§4.4.4 p. 98), which is much more challenging.

Another advantage of piston engines is that the maximum temperature of the cycle is only reached sporadically (periodically, but always briefly). During combustion, it is thus possible to reach gas temperatures that exceed the metallurgical limits of the engine, which, as we saw in chapter 7 (*the second law*), improves efficiency.

On the other hand, the weight and complexity of mechanisms of piston engines (connecting rods, crankshaft, valves, and the various circuitry to and from the cylinders) become major disadvantages in applications very high powers and rotational speeds are required.

10.3.2 The Otto cycle

The German engineer Nicolaus Otto is credited with the development of the engine known today as the *petrol* or *gasoline engine* in 1864. The basic cycle of this engine, called the *Otto cycle*, consists of two isentropic phases framed by two isochoric phases; it is described in figure 10.3.

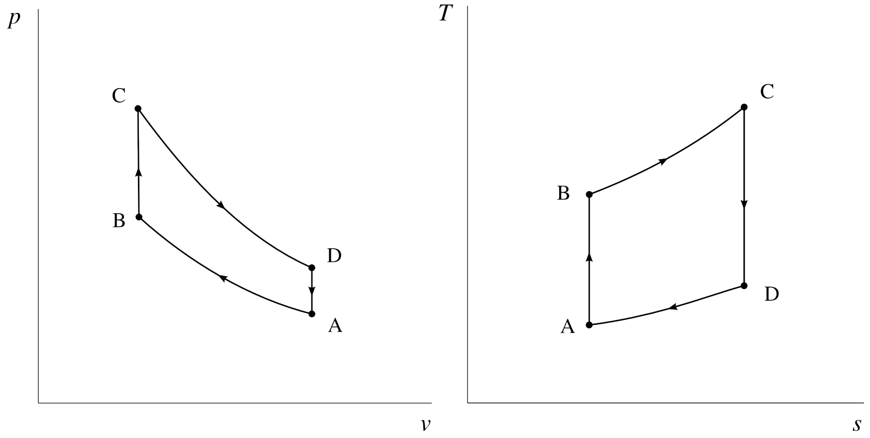


Figure 10.3: Otto's theoretical cycle represented on pressure-volume and temperature-entropy diagrams. These diagrams depict the ideal path, without irreversibilities in the compression or expansion. *Diagrams* CC-0 *Olivier Cleynen*

The Otto cycle is designed to allow for a simple implementation of the heat addition phase. The fuel is mixed with air before being inserted into the engine, and a very rapid combustion is triggered with a spark when the volume in the cylinder is minimal: this is known as *spark ignition*. Otto originally intended his engine for stationary applications, but its relative simplicity and responsiveness would ensure its success in transportation (notably through his son Gustav Otto, an aircraft manufacturer whose company would later become BMW).

The efficiency of the theoretical Otto cycle is easily calculable. The heat input $q_{\text{combustion}} = c_v(T_C - T_B)$ is performed at constant volume (4/22). The heat rejection, in practice, is done outside the engine, after exiting the exhaust pipe. From a thermodynamic point of view, the air continues its cycle in the atmosphere before re-entering the engine (§6.2.1), and thus the heat rejected is $q_{\text{cooling}} = c_v(T_A - T_D)$. Thus, since in theory no heat transfer takes place in the compression and expansion phases, and if we consider that the properties (c_v) of the gas do not change during combustion, the efficiency η_{Otto} of the theoretical cycle is simply given by:

$$\eta_{\text{Otto}} = \left| \frac{-q_{\text{combustion}} - q_{\text{cooling}}}{q_{\text{combustion}}} \right| = 1 + \frac{q_{\text{cooling}}}{q_{\text{combustion}}} = 1 + \left(\frac{T_A - T_D}{T_C - T_B} \right) \tag{10/2}$$

By defining the *compression ratio* ϵ as:

$$\epsilon \equiv \frac{v_A}{v_B} \tag{10/3}$$

it is possible to show that Equation 10/2 can be reformulated to express the efficiency as:

$$\eta_{\text{Otto}} = 1 - \frac{1}{\epsilon^{\gamma - 1}} \tag{10/4}$$

This equation indicates that the efficiency of the Otto engine depends solely on the compression ratio, and not on the amount of heat supplied during combustion. This may at first be surprising: why does T_C, the maximum temperature of the cycle, no longer appear here? The answer is that in this cycle, as the amount of burned fuel is increased, the increase in the average temperature during heat addition is exactly offset by the increase in the average temperature during heat rejection.

Equation 10/4 owes its simplicity to the the fact that we neglected the change in air properties during combustion, as well as the irreversibilities during compression and expansion. It should therefore be used with great caution; however, the trend it describes remains valid. Engineers are constantly seeking to increase the compression ratio of their engines to enhance efficiency. An immediate limit to this ratio is the temperature at which the air-fuel mixture spontaneously ignites, causing premature combustion.

10.3.3 The Diesel cycle

The *Diesel engine*, the child of his patient and hard-working inventor, German engineer Rudolf Diesel (§7.6), powers today the overwhelming majority of commercial road and marine transportation.

From a strictly thermodynamic point of view, the theoretical Diesel cycle differs from the Otto cycle only in its combustion mode: heat addition occurs at constant pressure and not at constant volume, as shown in figure 10.4.

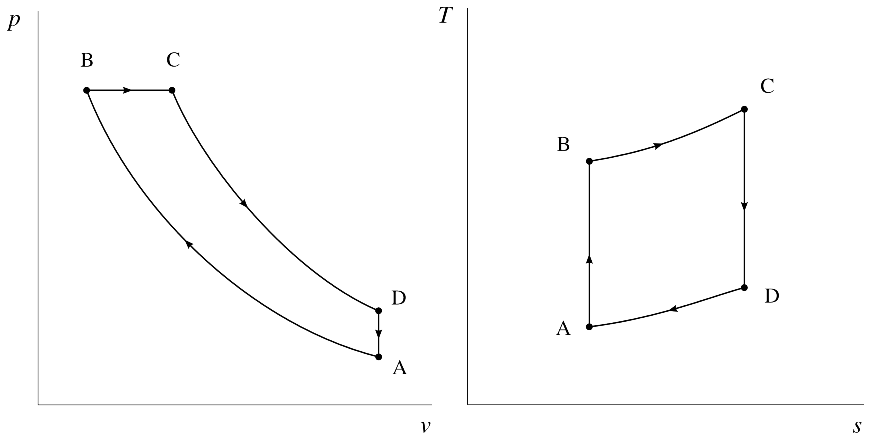


Figure 10.4: Theoretical Diesel cycle represented on pressure-volume and temperature-entropy diagrams. These diagrams depict the ideal path, without compression or expansion irreversibilities.

Diagrams CC-0 *Olivier Cleynen*

Since the heat addition $q_{\text{combustion}} = c_p(T_C - T_B)$ (4/17) is done simultaneously with the production of work, there is no simple expression for the efficiency η_{Diesel}, which no longer depends solely on the compression ratio. It will need to be calculated by studying the cycle step by step. It will then be observed that all else being equal (same compression ratio and same maximum temperature), the Diesel cycle has a lower efficiency than the Otto cycle.

> « [What is needed is the] production of a very high temperature (much above the ignition temperature of the combustible) not by combustion, but by compression of the air. … the combustible, therefore, must not be previously mixed with the air, but the latter must be compressed separately, otherwise, long before the required compression has been attained, ignition will be produced, and the cycle interrupted. »
>
> Rudolf Diesel, 1893 [23, 24]

In order to understand the interest of this cycle and the true difference between a Diesel engine and a gasoline engine, some historical context is needed. In 1892, Rudolf Diesel designed a “rational” engine to implement the Carnot cycle. He was looking for two characteristics:

- a high compression ratio, to increase the air temperature before combustion;
- combustion at a constant temperature.

To achieve this, Diesel had to wait until the end of compression to inject the fuel, in order to avoid premature ignition. The isothermal heat addition requires a progressive combustion. Thus, the original Diesel engine is inherently equipped with *direct fuel injection*, independent of air intake. The Diesel cycle is interesting because it *enables* a higher compression ratio and combustion quality compared to the Otto cycle.

Diesel’s engine evolved continuously from the impractical concept described in the 1893 *Theory and Construction of a Rational Heat Engine Intended to Replace the Steam Engines and Combustion Engines Known to Date* [23, 24] (400 bar and isothermal combustion of coal powder) to the first production models he developed at the manufacturer MAN (40 bar and isobaric combustion of petroleum). Like Otto, Diesel initially focused on stationary engines (his first prototypes were single-cylinder and over three meters high), but it was ultimately applications in commercial transportation, where its excellent efficiency and operating economics gave him the advantage over spark-ignition engines, that brought his work fame.

10.3.4 Implementation of the cycles

The two cycles described above are only idealizations – they serve as conceptual standards for comparing actual cycles. Their implementation in a real engine requires taking into account numerous factors, including:

- the need to purge the air and combustion products from inside the cylinder after the cycle, and the impossibility of doing it completely;
- the fact that the volume occupied by the gas is linked to the rotation of the engine shaft, and that it therefore is not possible to control it independently of the engine's operating speed;
- irreversibilities during compressions and expansions caused by fast piston movements;
- heat transfers to and from the cylinders during the cycle;
- gas leaks (blow-by) in the clearances between pistons and cylinders.

Once these factors, as well as the pursuit of objectives related to user comfort and control of air pollution, are taken into account, the cycle obtained inside a cylinder of a practical engine may for example resemble the one represented in figure 10.5. In the automotive sector in particular, the adoption of direct injection and the increase in compression ratios for gasoline engines to reduce fuel consumption and emissions has blurred the gasoline/Diesel distinction – gasoline engines are now closer to Rudolf Diesel's concept than to Nikolaus Otto's.

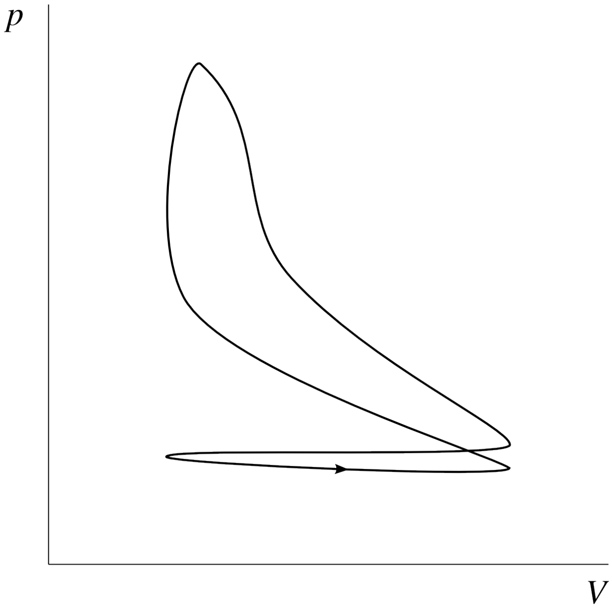


Figure 10.5: A realistic representation of the pressure and volume changes during a cycle in a practical gasoline engine.

Diagram CC-0 *Olivier Cleynen*

10.3.5 Number of cylinders and turbocharging

An important drawback of reciprocating engines is that the irreversibility of compressions and expansions increases significantly with the piston speed in the cylinders. The traditional approach to overcome this issue is to increase the number of cylinders operating simultaneously in the engine (figure 10.6). This way, the stroke traveled by each piston for a given displacement volume can be reduced. An advantage associated with this approach is that the motion of mechanical parts is better balanced (and the engine sounds more harmonious!).

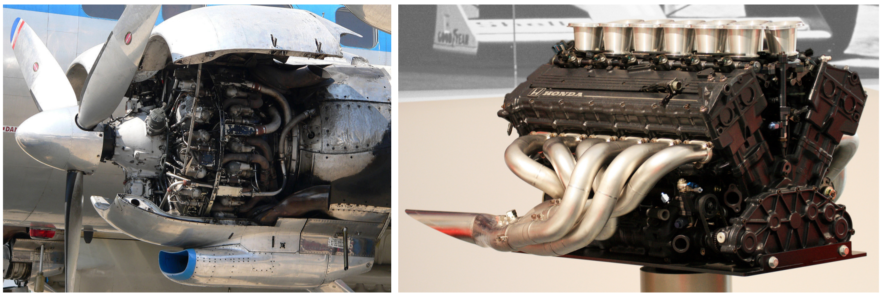


Figure 10.6: On the left, a 1950 Curtiss-Wright R-3350 *Duplex-Cyclone* of 3500 hp with 18 cylinders arranged in two successive rows. Four of these engines powered the long-range aircraft Lockheed *Super Constellation*.
On the right, a 1991 Honda RA121E V12 engine. It powered the McLaren MP4/6 Formula 1 car.

Photo of the Duplex-Cyclone CC-BY-SA *by Frank C. Müller*
Photo of the V12 RA121E CC-BY-SA *by Commons User:Morio*

Unfortunately, the mechanical complexity, size, and manufacturing and maintenance costs of engines increase rapidly with the number of cylinders; therefore, in applications where these factors are crucial (such as the majority of the automotive sector, for example), usually only four, or even three or two cylinders are used. It is nevertheless expected that these engines operate efficiently over a wide range of power outputs.

A commonly adopted solution for this is *turbocharging*. It involves delegating some of the compression and expansion work to a small device called a *turbocharger* or simply *turbo*, which is compact and lightweight (figure 10.7). The turbo's compressor is powered by its turbine, which operates with exhaust gases (we will study this system further in §10.5.2). Turbocharging helps increase the specific power (that is to say, it reduces the size and speed of an engine for a given power output).

Since the use of a turbocharger negatively affects the responsiveness of an engine, it is possible to allow the intake air to bypass it while the engine speed is changing. Furthermore, temperature changes in the turbo can be compensated for by cooling before insertion into the cylinders (this technique is studied further in §10.6.1). These processes make modern engines complex thermodynamic systems capable of performing a wide range of very different cycles depending on operating conditions.

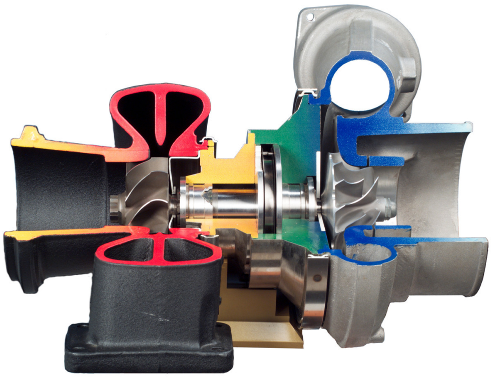


Figure 10.7: A cutaway view of a turbo to show its internal arrangement. Atmospheric air enters from the right and is compressed as it is propelled outward by the centrifugal compressor; it is then fed into the engine. Exhaust gases enter from the center left and exit to the left after spinning the centripetal turbine, which powers the compressor via the central rotating shaft. Since the only moving part is very compact (about 20 cm here), very high rotation speeds can be achieved, typically exceeding 200 000 rpm.

Photo NASA *(public domain)*

10.4 Components of Gas Turbomachinery

Before delving into the cycles of turbine engines, we will briefly review the operation of their main components. Since turbomachines operate with steady fluid flow, we will consistently refer to the concepts of chapter 3 (*open systems*) from now on.

10.4.1 Compressor

> « In order to grant air a great expansion in volume, in order to produce through this expansion a large change in temperature, it would be necessary to take it first under a sufficiently high pressure [...] This operation would require a special apparatus, an apparatus which does not exist in steam engines. In these, water is in the liquid state when it is made to enter the boiler; it requires, for its introduction, only a small force pump of small dimensions. »
>
> Sadi Carnot, 1824 [4]

The compression and expansion phases in engines are often adiabatic, and always irreversible. It is difficult to achieve high-quality flow in the compressor, moreso than in the turbine because the pressure gradient promotes boundary layer separation. It is a heavy, bulky component with complex geometry (Figures 10.8 and 10.9). Most compressors are *axial*, meaning that the air passes through them parallel to the axis of rotation, but sometimes *centrifugal* compressors are used, which sling the air radially; regardless of the method used, the thermodynamic process undergone by the air remains the same.

Figure 10.8: The stator casing which houses the rotor (not shown) in the an axial compressor of a turbojet engine. *Photo* CC-BY-SA *Olivier Cleynen*

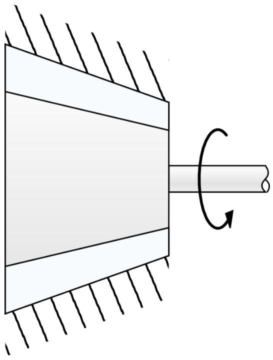


Figure 10.9: Schematic representation of an air compressor.

Diagram CC-BY-SA *Olivier Cleynen*

Just as we did for the turbine (9/6), we quantify the efficiency of a compressor by comparing its power with that of an ideal compressor (one that would be isentropic). We call this parameter the *isentropic efficiency* η_C of the compressor:

$$\eta_C \equiv \frac{\dot{W}_{\text{isentropic compressor}}}{\dot{W}_{\text{actual compressor}}} \tag{10/5}$$

where $\dot{W}_{\text{actual compressor}}$ is the actual shaft power received by the compressor,
and $\dot{W}_{\text{isentropic compressor}}$ is the power of an isentropic compressor that would operate with the same mass flow rate and between the same pressures.

Like that of a turbine, the isentropic efficiency of a compressor is always less than 1. If this efficiency is known, we can compare the actual properties of the air at the inlet and outlet of the compressor with those that would be measured in the ideal case:

$$w_{\text{compressor}} = c_p\,(T_{\text{actual}} - T_A) = \frac{1}{\eta_C} c_p\,(T_{\text{ideal}} - T_A) \tag{10/6}$$

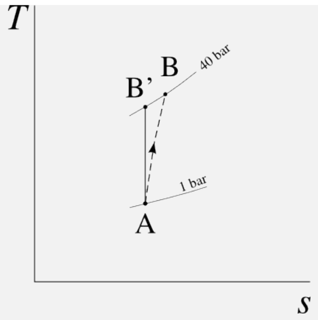
where $w_{\text{compressor}}$ is the specific power of the compressor ($\mathrm{J\,kg^{-1}}$),
T_{ideal} is the ideal outlet temperature (isentropic compressor) (K),
and T_{actual} is the actual outlet temperature (K).

Example 10.1

For air, we have $c_{p\,(\text{air})} = 1005\,\mathrm{J\,kg^{-1}\,K^{-1}}$, $c_{v\,(\text{air})} = 718\,\mathrm{J\,kg^{-1}\,K^{-1}}$, $R_{\text{air}} = 287\,\mathrm{J\,kg^{-1}\,K^{-1}}$, and $\gamma_{\text{air}} = 1.4$.
The compressor of a turbofan jet engine has an isentropic efficiency of 85 %; it takes in air at a rate of $38\,\mathrm{kg\,s^{-1}}$ at 1 bar and 5 °C. The outlet pressure is 40 bar. What is the required power?

The process can be qualitatively represented on a $T - s$ diagram as shown below.

We start by calculating the power of an ideal (isentropic) compressor: the outlet temperature in that case would be (4/37): $T_{B'} = T_A \left(\frac{p_B}{p_A}\right)^{\frac{\gamma-1}{\gamma}} = (5 + 273.15)\,(40)^{\frac{0.4}{1.4}} = 798\,\mathrm{K} = 524.9\,°\mathrm{C} = 976.7\,°\mathrm{F}$. The ideal compressor would then receive $w_{\text{isentropic compressor}} = c_{p(\text{air})}\,(T_{B'} - T_A) = 1005\,(798 - 278.15) = +5.225 \times 10^5\,\mathrm{J\,kg^{-1}} = +522.5\,\mathrm{kJ\,kg^{-1}}$.



With equation 10/5, the power of the compressor naturally comes as: $\dot{W}_{\text{compressor}} = \dot{m}\, \frac{1}{\eta_C}\, w_{\text{isentropic compressor}} = 38 \times \frac{1}{0.85} \times 5.225 \times 10^5 = 2.336 \times 10^7\ \text{W} = 23.36\ \text{MW}$.

☞ Note that unlike for turbines, the actual power is *greater* than the theoretical power, and we must divide by the efficiency in the final calculation.

☞ Equation 10/6 would allow us to calculate the actual outlet temperature: $T_{\text{B real}} = \frac{1}{\eta_C} c_p\, (T_{\text{B}'} - T_{\text{A}}) + T_{\text{A}} = \frac{1}{0.85}(798 - 278.15) + 278.15 = 889.7\ \text{K} = 616.6\ ^\circ\text{C} = 1141.8\ ^\circ\text{F}$. Here, the 92 °C (165 °F) difference from the isentropic case are the result of converting work into heat due to friction in the compressor, an unnecessary expense representing $\dot{m}\, c_p\, (T_{\text{B}} - T_{\text{B}'}) = +3.5\ \text{MW}$.

In practice, several air bleeds can be made within the compressor to feed other equipment and to cool the turbine (10.6.3). During transitional phases, the compressor can also be relieved of part of the mass flow by allowing air to leak through discharge valves.

10.4.2 Combustion chamber

The heat input of turbomachines takes place in one or more combustion chambers (figures 10.10 and 10.11). The air is heated at constant pressure by combustion; its temperature and specific volume greatly increase.

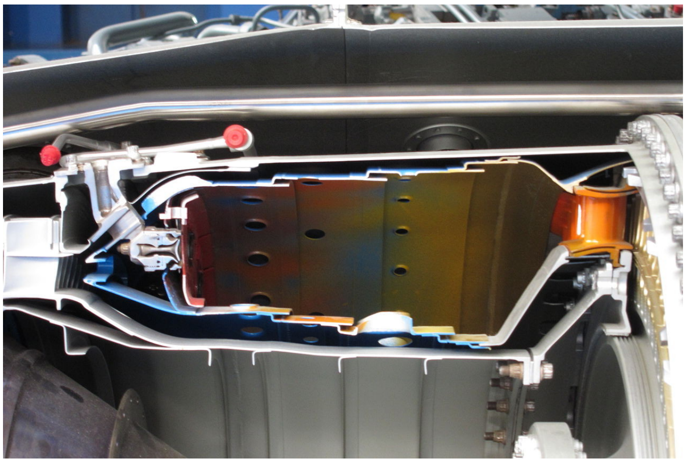


Figure 10.10: Section of an annular combustion chamber in which the flow was from left to right. The photo shows a section of a Rolls-Royce Turboméca *Adour*, a small turbofan engine designed in 1968.

Photo CC-BY-SA *Olivier Cleynen*

Figure 10.11: Schematic representation of a combustion chamber.

Diagram CC-0 Olivier Cleynen

No work is done in the combustion chamber, and the pressure remains approximately constant. Since the heat input occurs within the gas itself, the maximum temperature of the cycle is not limited by heat transfer through a solid wall. The maximum temperature of the air can even exceed that of the melting point of the chamber walls, which are insulated with several layers of compressed air. Compared to steam power plants, this allows for a temperature increase of about 200 K.

The power delivered in the combustion chamber is quantified rather easily by modifying equation 4/19 to account for the change in air properties during combustion, which increases the value of c_p by about 10 %:

$$q_{\text{chamber}} = h_\text{B} - h_\text{A} = c_{p(\text{gases})} T_\text{B} \ - \ c_{p(\text{air})} T_\text{A} \tag{10/7}$$

Fluid flow within the combustion chamber depends in a correlated manner on combustion chemistry and the spatial distribution of velocities and pressure: it is therefore difficult to model. In practice, a slight pressure drop is generated between the inlet and outlet of the chambers. The influence on the turbine power of the fuel mass flow rate $\dot{m}_{\text{fuel}}$, always much lower than that of air, can be safely neglected.

10.4.3 Turbine

The primary role of the turbine (figures 10.12 and 10.13) is to power the compressor: it must therefore extract enough power from the air to operate the latter and compensate for any transmission losses. Depending on the configuration of the turbomachine, the turbine may then be designed to further extract energy from the gases in order to power other components, as we will see in §10.5 below.

Just like for liquids/vapors (eq. 9/6 p. 249), we measure the performance of a turbine by quantifying its *isentropic efficiency* η_T:

$$\eta_T \equiv \frac{\dot{W}_{\text{actual turbine}}}{\dot{W}_{\text{isentropic turbine}}} \tag{10/8}$$

The power extracted by the turbine is thus easily expressed in terms of the actual $T_{2 \text{ real}}$ and ideal $T_{2'}$ temperatures at its outlet:

$$w_{\text{turbine}} = c_{p(\text{gases})}(T_{2 \text{ actual}} - T_1) = \eta_T \ c_{p(\text{gases})} \ (T_{2'} - T_1) \tag{10/9}$$

As the gases flow downstream through the turbine, they expand and their specific volume increases. The size of the blades (hence their weight and cost) must also increase, while the power they can extract decreases. Gases are often rejected at the outlet of a turbomachine with residual pressure because it is not economically viable to extract any more work from them.

Figure 10.12: Turbine of a gas generator. The photographed turbine, a Siemens SGT5, can accept an air and water flow rate of 690 kg s^{-1} (1521 lb/s). It delivers approximately 500 MW of shaft power.

Photo CC-BY-SA *Siemens Pressebild*

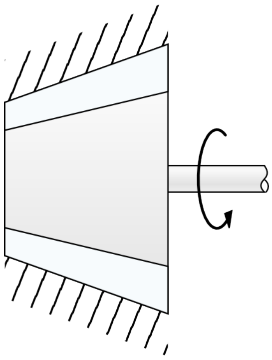


Figure 10.13: Schematic representation of a gas turbine.

Diagram CC-BY-SA *Olivier Cleynen*

10.4.4 Nozzle

> « When indeed two equally compressed fluids escape through two small equal orifices, their velocities are in inverse proportion to the square root of their densities. »
>
> Louis Joseph Gay-Lussac, 1807 [3]

The *nozzle* is a simple conduit with no moving part (figures 10.14 and 10.15). It allows the gas to expand, thereby accelerating towards the rear of the engine. It is this increase in gas velocity (difference between inlet and outlet velocities) that is the source of the thrust provided by an engine.

There is no heat or work input in the nozzle: the energy of the gas is conserved. The nozzle is the only element of the gas turbine engine for which the change in kinetic energy may not be neglected.

A quick return to equation 3/15 allows us to quantify the final speed of the gases as a function of the available pressure difference:

$$q_{\text{A}\to\text{B}} + w_{\text{A}\to\text{B}} = \Delta h + \Delta e_{\text{mech.}} \tag{3/15}$$

$$h_{\text{A}} + \frac{1}{2}C_{\text{A}}^2 = h_{\text{B}} + \frac{1}{2}C_{\text{B}}^2 \tag{10/10}$$

Figure 10.14: The nozzles of two *General Electric* F404 engines equipping a fighter aircraft. The geometry of the nozzle (not covered in this book) is programmed to adapt to the engine mass flow and whether or not afterburning is used.

Photo CC-BY-SA *by Peng Chen*

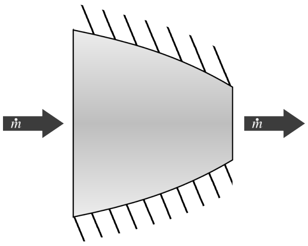


Figure 10.15: Schematic representation of a nozzle.

Diagram CC-0 *Olivier Cleynen*

In the case of an ideal nozzle, the expansion is isentropic, and we can relate the temperatures T_A and T_B just like in a turbine or a compressor, using the dreadful relations 4/36 to 4/38. Thus, knowing the inlet conditions h_A and p_A, for a given outlet pressure p_B (atmospheric pressure), we can quantify the change in gas velocity:

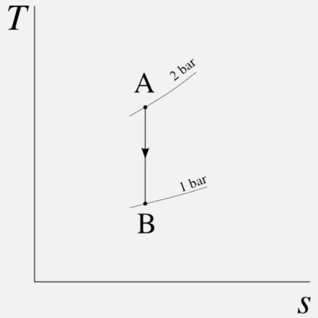
$$C_\text{B}^2 - C_\text{A}^2 = -2\ c_{p(\text{gas})}\ (T_\text{B} - T_\text{A}) \tag{10/11}$$

Ideally, the nozzle expands the gases to ambient pressure and converts all the change in enthalpy of the gases into kinetic energy. In practice, of course, some of this energy is converted into heat due to friction. The efficiency of nozzles is quantified in a similar way to that of compressors and turbines, and is not studied in this book.

Example 10.2

For burnt gases, we have $c_{p\,(\text{gases})} = 1150\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$, $c_{v\,(\text{gases})} = 823\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$, $R_\text{gases} = 327\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$, and $\gamma_\text{gases} = 1.333$.
A nozzle is fed with a steady flow of combustion gases at 2 bar, 10 m s^{-1}, and 400 °C (29 psi, 33 ft/s, 752 °F). At what speed can it accelerate these gases when they are rejected at 1 bar, if we neglect irreversibilities?

The process which would allow the highest ejection speed is an isentropic expansion; therefore, $T_B = T_A \left(\frac{p_B}{p_A}\right)^{\frac{\gamma-1}{\gamma}} = (400 + 273.15)\left(\frac{1}{2}\right)^{\frac{0.333}{1.333}} =$ $566.1\,\mathrm{K} = 293\,^\circ\mathrm{C} = 559\,^\circ\mathrm{F}$. Such a process can be qualitatively represented on a $T - s$ diagram as follows:



With equation 10/11, the outlet speed would therefore be:

$$C_B = \left[-2\, c_{p(\text{gases})}\, (T_B - T_A) + C_A^2\right]^{\frac{1}{2}}$$
$$= \left[-2 \times 1150 \times (293 - 400) + 10^2\right]^{0.5}$$
$$= 496.2\,\mathrm{m\,s^{-1}} = 1786\,\mathrm{km/h} = 1110\,\mathrm{mph}.$$

☞ In practice, the gases would never reach this speed. Indeed, a large part of the expansion takes place *downstream* of the nozzle, where it is very turbulent and therefore highly irreversible. This nevertheless does not influence the thrust generated by the nozzle, whose outlet orifice pressure is in fact higher than atmospheric pressure. The speed calculation performed here remains a good thermodynamic “indicator” of the phenomena at play. A full description of the fluid dynamics of the nozzle is outside of the scope of this book.

☞ In most cases, it is reasonable to consider that the kinetic energy of the gases at the outlet of the turbine (and therefore at the inlet of the nozzle) is negligible. The $10\,\mathrm{m\,s^{-1}}$ at A have no significant influence here.

Finally, we note that the *air intake* of aeronautical engines often serves as a diffuser: the mirror opposite of a nozzle in function. It thus slows down the air and increases its pressure. On supersonic aircraft, a well-designed inlet can generate a compression ratio of 2, with a corresponding temperature increase.

10.5 Gas Turbine Configurations

10.5.1 Advantages of gas turbines

Within the realm of *turbomachinery* (machines which transfer power between a fluid and a rotating shaft), we call *gas turbine* the complete internal-combustion, turbine-powered machine, and not merely the component of the same name. The word “gas” here refers to the working fluid, not to the fuel, which may be any kind of combustible liquid or gas. Gas turbines have two major advantages over piston engines:

- The power-to-weight ratio of turbomachines is approximately three times higher, since the number of moving parts is reduced, and their movement is very simple, allowing them to be lighter;
- In air propulsion, the working fluid can be used as a medium of propulsion itself. It is sufficient to let the air exit the turbine with a residual pressure and let it expand in a nozzle. This generates a thrust by reaction (equal to the mass flow rate multiplied by its speed): this is the working principle of the jet engine.

Thus, gas turbines are used in applications where high power is required with significant weight or space constraints.

The major drawback of gas turbines is that their efficiency and responsiveness drop very quickly at low power levels. At partial load, the compression ratio and isentropic efficiency of turbines and compressors collapse, because it is hard to control fluid flow when the velocity of the rotor blades relative to stator blades is suboptimal. Gas turbines are therefore only useful in applications where high powers are required continuously. A gas turbine would be for example very poorly suited for road transport, because changes in power there are frequent and must be instantaneously acted on.

> « As the steam turbine, without bringing an actual betterment of steam economy, has entered into the industry because of its constructive simplicity, so will it be with a gas turbine, which is constructively simpler than the gas motor, provided it will only exceed the steam motors in efficiency. »
>
> Aurel Stodola, 1904, [26, 27]

10.5.2 The gas generator

The heart of any gas turbine engine is called the *gas generator*. It contains only one shaft and one turbine (figure 10.16). This machine section has no use in itself, but the gases at its outlet, whose pressure is higher than at the inlet, can be used in a multitude of applications.

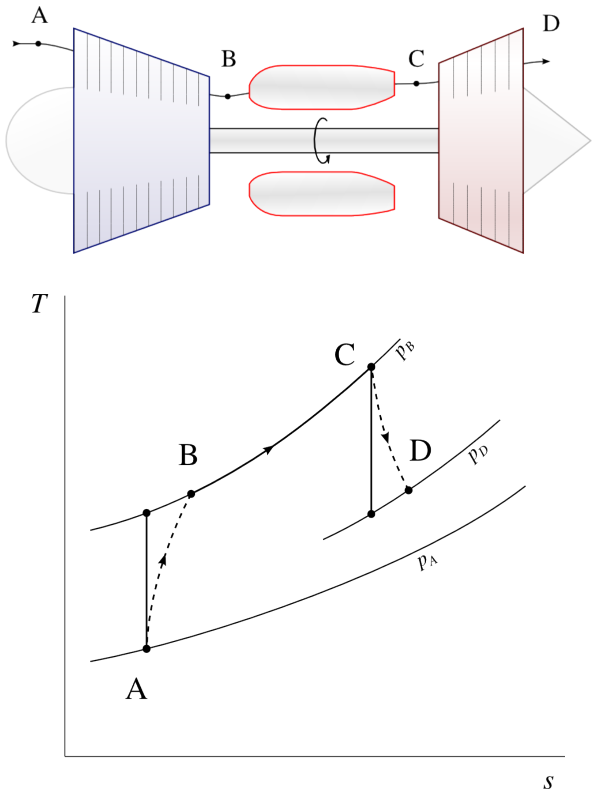


Figure 10.16: A "gas generator" (schematic drawing and temperature-entropy diagram). This machine has no interest in itself but has many derived applications. One of them is the *turbocharger*, for which a piston engine acts as the combustion chamber, as described in §10.3.5.

Schematic CC-BY-SA *Olivier Cleynen*
Diagram CC-0 *Olivier Cleynen*

In this configuration, the turbine extracts exactly enough power to power the compressor. At its outlet, the air is still compressed and can be used in a multitude of ways, as explored below.

10.5.3 Turbojet

The *turbojet* engine (figure 10.17) is the first application that has been made of gas generator. At the outlet of the turbine, the air is expanded in a nozzle, which accelerates it and provides net thrust. It is the working fluid itself that is used to generate thrust.

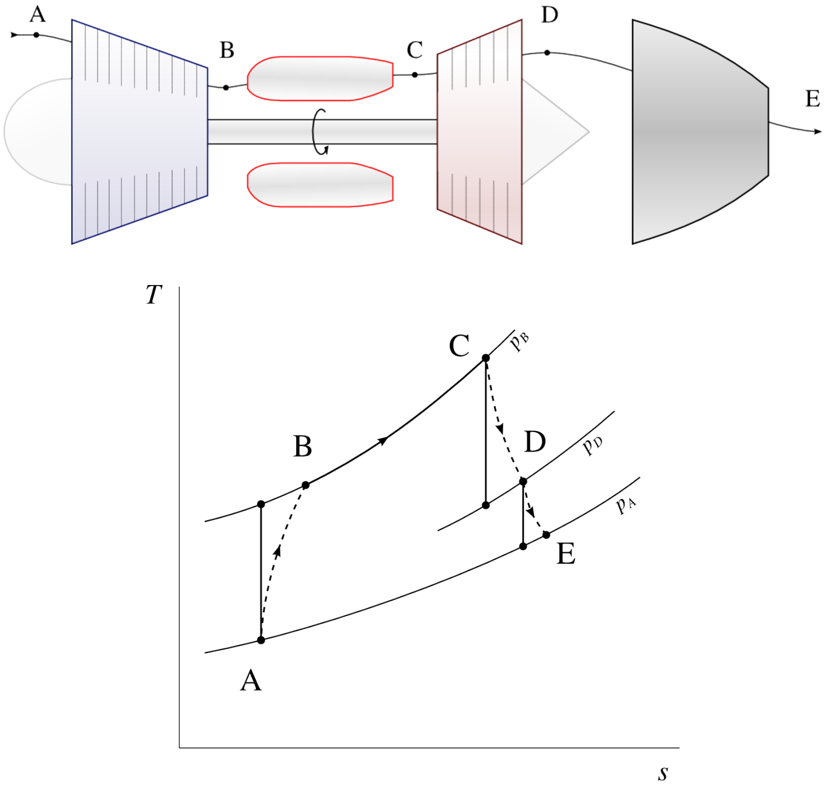


Figure 10.17: Turbojet (schematic and temperature-entropy diagram). At the outlet of the turbine, the air is still pressurized; it is expanded in a nozzle in order to be accelerated.

Diagram CC-by-sa *Olivier Cleynen*
Diagram CC-0 *Olivier Cleynen*

Turbojet engines are extremely compact and mainly used in military aircraft.

10.5.4 Turboprop and turboshaft

Instead of using a nozzle as in a turbojet, it is possible to continue the expansion in the turbine until the gases reach atmospheric pressure. The power supplied by the turbine is then *greater* than the power supplied to the compressor.

This surplus work in the the engine shaft can then be used to power a propeller (in the case of a *turboprop*) or an external element such as a generator or a pump (in the case of a *turboshaft*), as shown in figure 10.18. The cycle of these machines is sometimes called *Brayton cycle*.

For a given engine mass flow, powering a propeller or the fan of a turbofan engine instead of merely expanding the gases in a nozzle increases thrust

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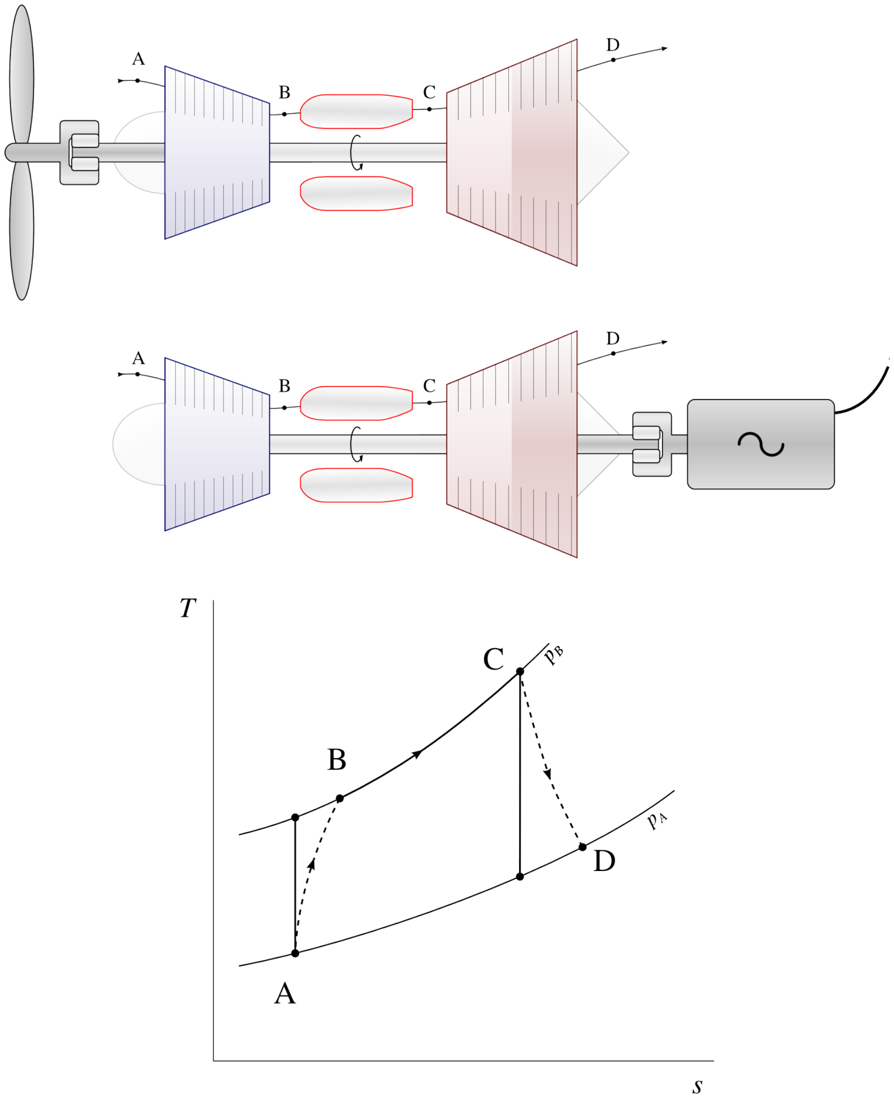

Figure 10.18: Schematics and temperature-entropy diagram of a turboprop (top) and a turboshaft engine (bottom). The power extracted by the turbine exceeds that absorbed by the compressor and is used to power the propeller or a generator.

Schematics CC-BY-SA *Olivier Cleynen*
Diagram CC-0 *Olivier Cleynen*

(this is quantified as the *propulsive efficiency*). The associated disadvantages are, of course, bulk and weight: the diameter of propellers and fans of modern engines often exceeds three meters, which translates into significant structural and mechanical constraints on the engine.

As for turboshaft engines, they find applications in helicopters, military ships, auxiliary electric generators, and gas turbine power plants. They are most often configured using the cycle modifications described in the following sections.

10.5.5 Turbofan

From a thermodynamic point of view, a *turbofan* (figure 10.19), is equivalent to a turboprop with a nacelle placed around it.

There are two separate air flows within a turbofan:

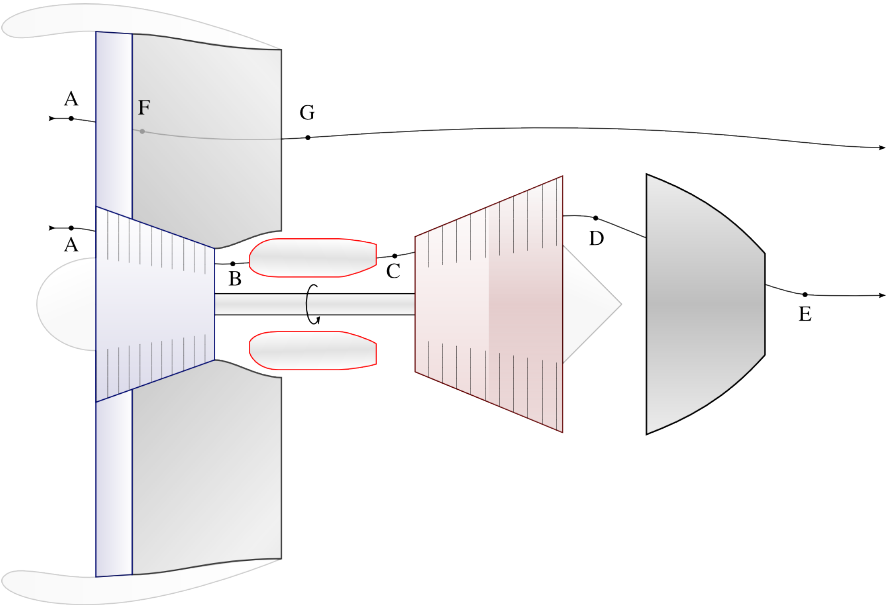

Figure 10.19: Schematic diagram of a turbofan. The engine's thermodynamic core (A → E) mechanically powers the fan, which allows the bypass flow (A → G) to provide the majority of the thrust.

Diagram CC-by-sa *Olivier Cleynen*

- the core flow is the flow through the thermodynamic engine. After combustion, it passes through a turbine whose power far exceeds that of the compressor. This excess power is transferred to the fan;
- the bypass flow is lightly compressed by the fan and directly expanded in the nozzle surrounding the hot core of the engine. It is never heated.
  It is this "cold" air which makes for the majority of the thrust. It can be shown that the greater the ratio of bypass to core air flow (the *bypass ratio*), and the more efficient the engine. The bypass ratio of modern engines is around 12.

10.5.6 Free turbine and multiple turbines

Depending on the applications, various arrangements of turbines and compressors can be used.

In a *free turbine* (also called *free spool*) configuration, the mechanical power supplied by the engine is transmitted by a dedicated turbine (figure 10.20). This allows each of the two shafts to be maintained at different speeds.

Since the speed of the turbine/compressor shaft is not constrained by the load imposed on the free shaft, it can operate at speeds closer to its optimum point and accelerate more easily. This advantage compensates for the increased mechanical complexity in applications such as helicopter engines, where significant power changes are sometimes required.

In a configuration with *multiple spools*, the compressor and turbine are each divided into several parts, thus forming two co-axial systems incorporated one into the other (figure 10.21).

The high-pressure turbine drives the high-pressure compressor (high-speed spool), and the low-pressure turbine drives the low-pressure compressor (low-speed spool).

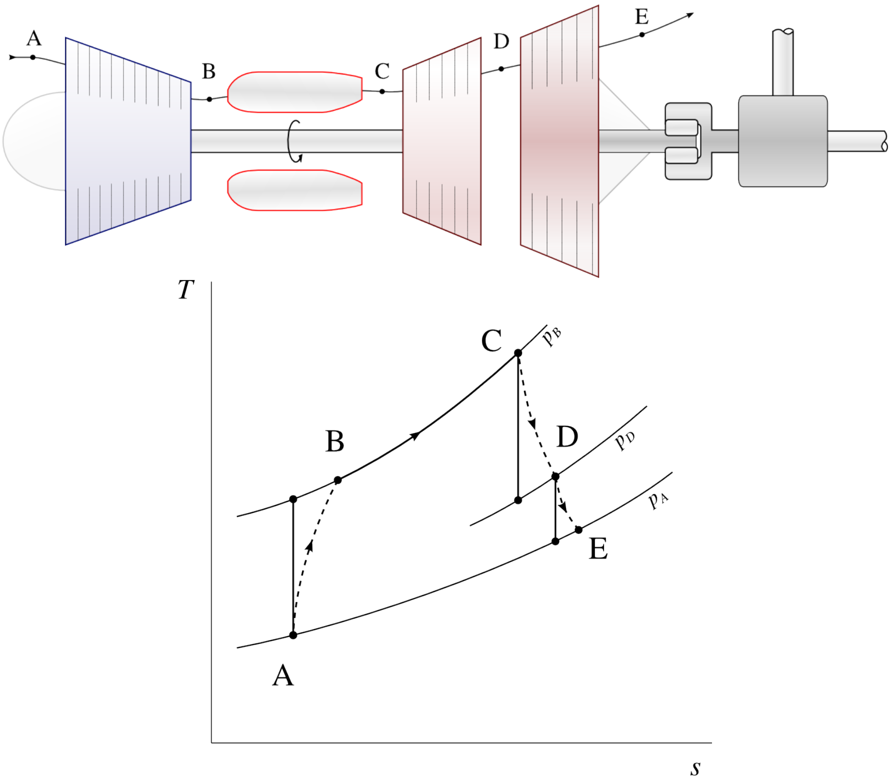

Figure 10.20: Turboshaft engine with a free turbine (schematic and temperature-entropy diagram). The power supplied by the engine comes exclusively from the free turbine.

Schematic CC-BY-SA *Olivier Cleynen*
Diagram CC-0 *Olivier Cleynen*

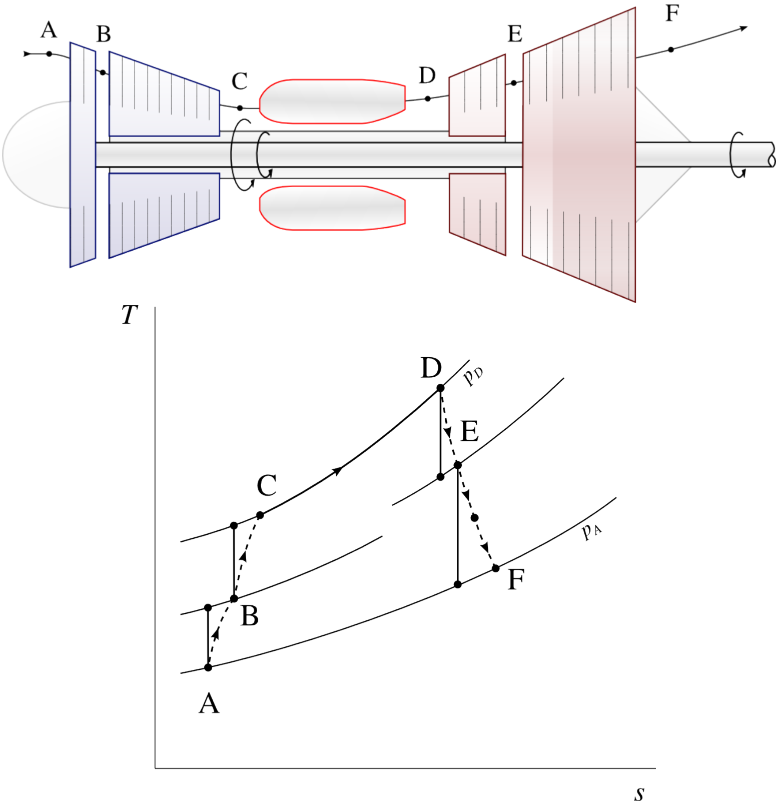

Figure 10.21: Twin-spool turboshaft engine (schematic and temperature-entropy diagram). The two shafts rotate at different speeds.

Schematic CC-BY-SA *Olivier Cleynen*
Diagram CC-0 *Olivier Cleynen*

Just like for the free turbine, this arrangement allows each spool to operate at its own speed. Indeed, as the air pressure increases in the compressor, its density and temperature also increase. This configuration allows the blades to be operated at higher speeds, thus reducing their size.

10.6 Modification of Gas Turbine Cycles

10.6.1 Intercooling and reheat

It is sometimes desirable to increase the work ratio and specific power, even at the cost of a decrease in total efficiency, as mentioned in §10.2 above.

In order reduce the power absorbed by the compressor, *intercooling* is sometimes used. The compression is interrupted and the air is cooled before the compression process is completed (figure 10.22).

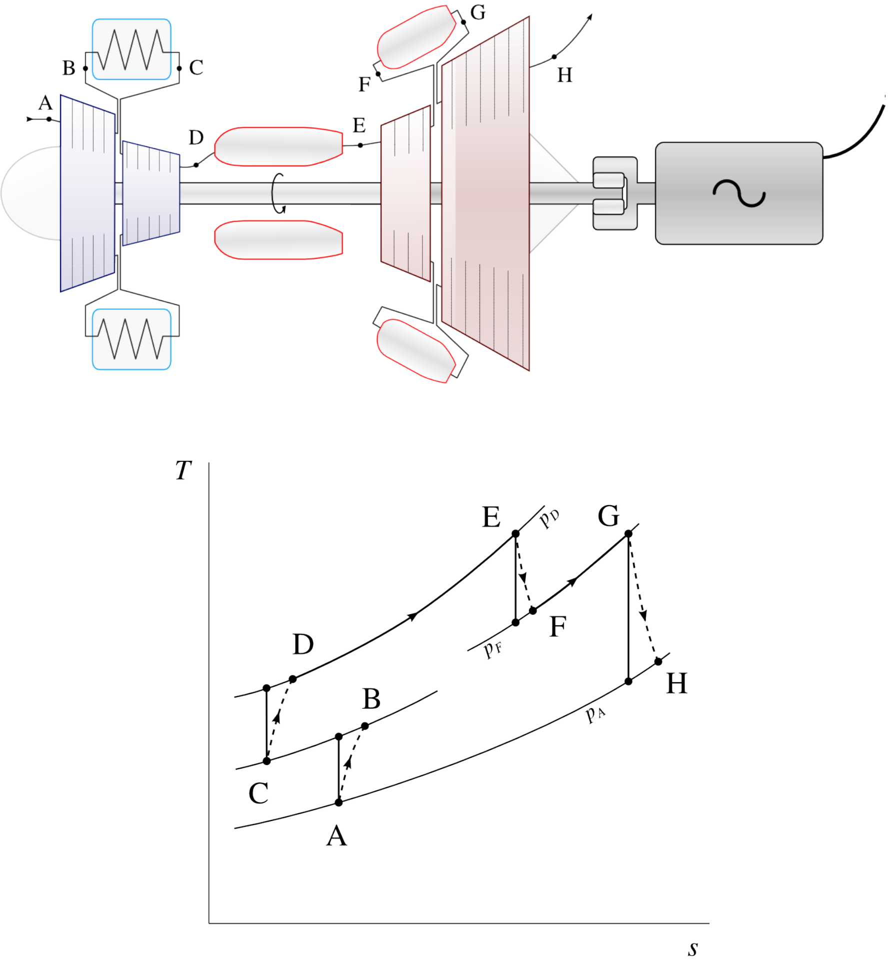

Figure 10.22: A turboshaft generator with intercooler and reheat system (schematic and temperature-entropy diagram).
The intercooler cools the air in the midst of compression; while the second combustion chamber reheats the gases it in the midst of the expansion. The two modifications are independent of each other and each can be installed alone.

Schematic CC-by-sa *Olivier Cleynen*
Diagram CC-0 *Olivier Cleynen*

The compression of a gas between two given pressures imposes a *ratio* between the initial and final temperatures (4/36). On the other hand, the power required to compress a gas between these two pressures depends on the *difference* between these two temperatures (10/6). Therefore, the lower the initial temperature, and the lower the power required to reach a given pressure.

In the same vein, we can increase the specific power supplied by the turbine by carrying out a second combustion with the gases before the end of the expansion: this is called *reheat*. The process is similar to the reheating of steam in steam power plants (§9.4.4 p. 258).

It will not have escaped the student that the efficiency is inevitably reduced by the use of intercooling. Indeed, the combustion chamber must supply more heat, at a lower average temperature. This reduction in efficiency will be balanced against the reduction in the size of the compressor (usually the largest component of an engine) and the increase in specific power. Intercooling and reheat are typical of machines where the power-to-size ratio must be maximized.

In order to partially offset the loss of efficiency in stationary engines, it is sometimes possible to recover heat from the exhaust gases and use it to heat the air at the compressor outlet, thus relieving the combustion chamber. The heat exchanger is sometimes called *economizer* (figure 10.23); it is left to the student to trace the cycle followed on a temperature-entropy diagram and to find the conditions required for its operation.

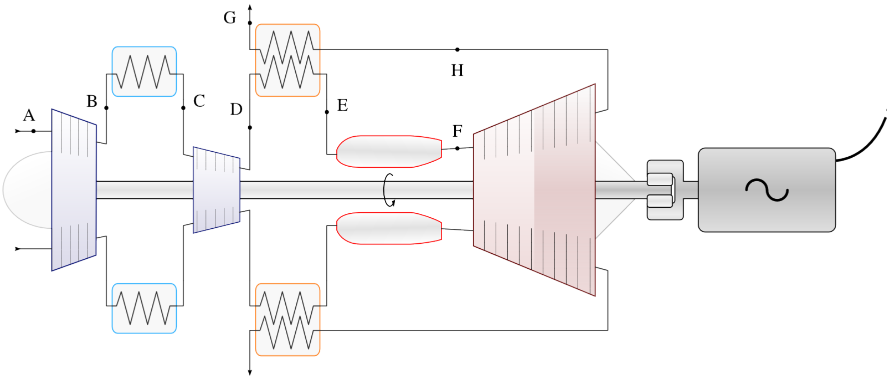

Figure 10.23: Turboshaft generator with an intercooler and an economizer heat exchanger. The exhaust gases are redirected inside the engine to supply heat to the gases at the entrance of the combustion chamber. It is left to the student to determine the limits of the process.

Diagram CC-BY-SA *Olivier Cleynen*

10.6.2 Afterburning

Afterburning (or *reheat* in British English) is the addition of a second combustion phase in a jet engine, downstream of the turbine and upstream of the nozzle (figure 10.24). The principle is exactly the same as that of reheat: increase the specific thrust of the machine (at the expense of its efficiency).

Like reheat, afterburning alters the properties (specific volume in particular) of the gases and requires resizing of downstream components. The geometry of the nozzle is adapted according to whether the post-combustion is active or not. Adding an afterburner system to a turbojet engine merely requires the installation of burners and a system for varying the geometry of the

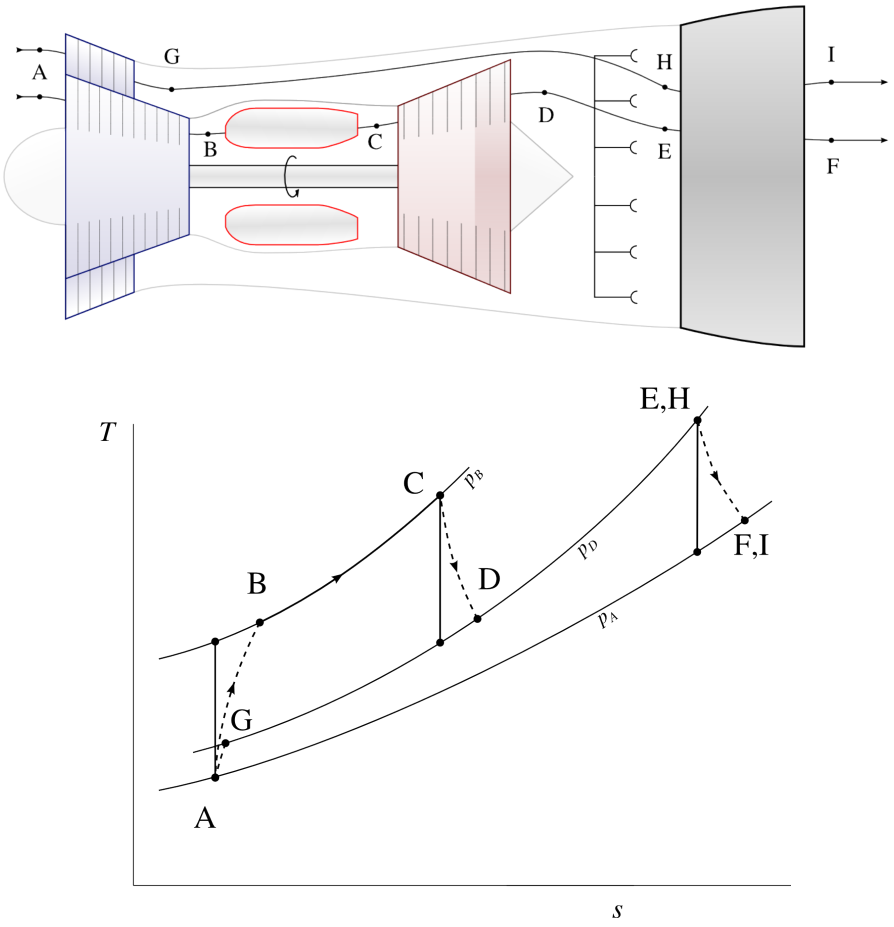

Figure 10.24: Afterburner on a dual-flow turbojet engine (schematic and temperature-entropy diagram). States E and H are not necessarily merged in practice.

Diagram CC-BY-SA *Olivier Cleynen*
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nozzle. The increase in weight is low compared to the increase in available power.

The outrageous loss of efficiency caused by the use of afterburners, as well as the deafening levels of noise and pollution they generate, limit their use to the military sector (especially on combat aircraft).

10.6.3 Turbine cooling

Because an increase in combustion temperature increases efficiency and specific power, engine designers are driven to develop technologies to maximize the temperature at the outlet of the combustion chamber (TET, for *turbine entry temperature*).

One of the used strategies is to cool the turbine with bleed air from the compressor (figure 10.25). The bleed air is passed through the turbine blades themselves, allowing for an increase in combustion temperature without risking damage to the blades. The most efficient and advanced cooling systems wrap the turbine blades with this cooler air. This allows, in modern engines, the TET temperature exceeds the melting temperature of the blades by more than 100 °C, or 180 °F!

Such turbine cooling comes at a significant cost. First, in a real engine, less work is recovered from the expansion of this bleed air than was required

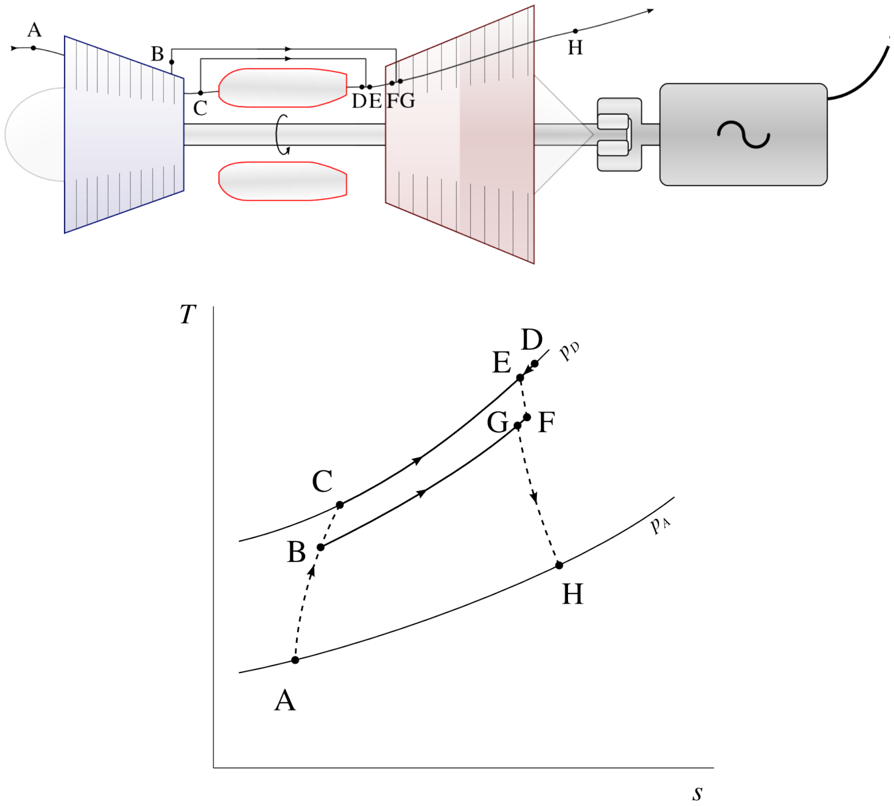

Figure 10.25: Turbine cooling using air taken from the compressor (schematic and temperature-entropy diagram). This air, at moderate temperature, bypasses the combustion chamber and never comes into contact with the fuel. Here the represented engine is a turboshaft, but turbine cooling can be used in any configuration.

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Diagram CC-0 *Olivier Cleynen*

for its compression (in the limiting case where compression and expansion are isentropic, this energy cost is zero). The circulation of this air therefore represents a burden that must be offset by the increase in efficiency it generates. Secondly, the compressor and the turbine must be oversized to accommodate a larger air flow.

Turbine cooling is a major research area in aeronautical propulsion. Techniques from a handful of fields (materials, fluid mechanics, mechanical design, combustion chemistry) are combined there in order to improve the thermodynamics of the engines.

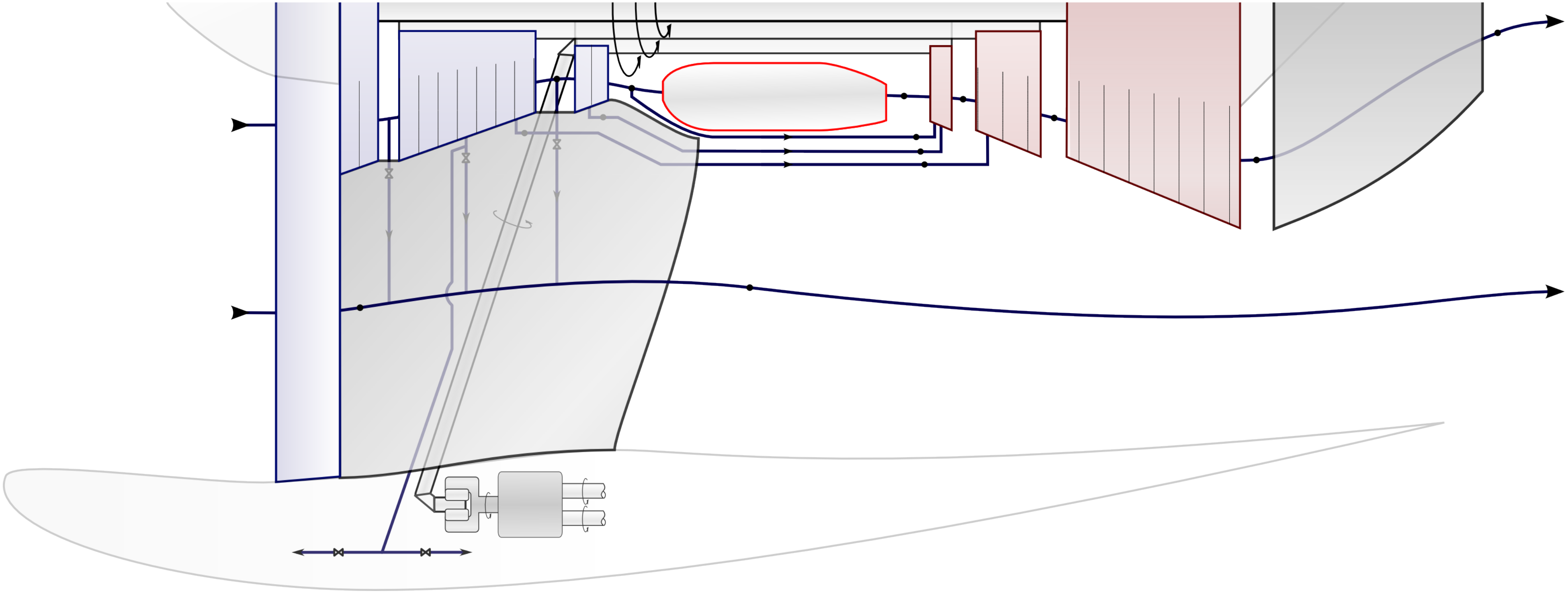


Figure 10.26: Thermodynamic circuit of a modern turbofan engine. The machine combines multiple spools, mechanical and pneumatic power extractions, compressor bleeds for turbine cooling, and two main air flows. It is left to the student to trace the cycle on a temperature-entropy diagram. On twin-engine aircraft qualified for ETOPS flights, each engine must be able to alone maintain the aircraft flying while supplying many systems (pressurization, de-icing, heating, electrical and pneumatic generation) for several hours with demonstrated reliability.

Diagram CC-BY-SA *Olivier Cleynen*

A Bit of History: the Napier Nomad

*

At the end of the Second World War, the British government issued a call for tenders for the development of a highly efficient, 6000 hp aeronautical engine, in order to foster the development of military and civilian aircraft. The British engine manufacturer Napier & Son then carried out research that led to the development of a curious and remarkable device: the *Napier Nomad*.

Based on a twelve-cylinder two-stroke Diesel engine with direct injection, the *Nomad* also featured all the elements of a turboprop engine. In order to increase the pressure and temperature at which heat was supplied, the two units were mounted *in series*; however, in order to allow for a high efficiency at all speeds, each drove one of the two contra-rotating propellers (Figures 10.27 and 10.28). Finally, in order to achieve high powers and increase responsiveness throughout the flight envelope, an intercooler and reheat system were added. The result: an astonishing and extravagant mechanical-thermal assembly seemingly produced by the out-of-control fantasy of thermodynamic engineers on a quest for efficiency.

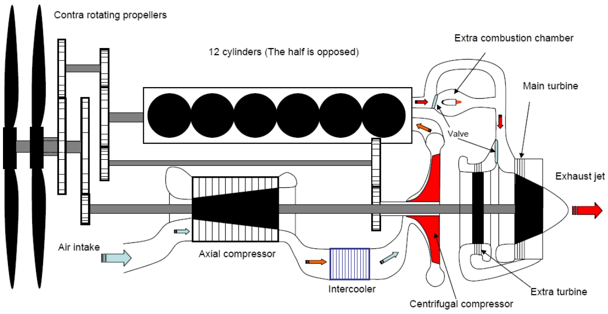


Figure 10.27: Schematic diagram of the thermodynamic circuit of the Napier *Nomad I*. The shafts of the turboshaft and of the Diesel engine each drove one propeller. However, the two units were mounted in series: the air first passed through the compressors, then through the cylinders, and finally through the turbine(s). The engine power, as was customary in 1950, was controlled using a single mechanical control lever!

Diagram by users · Commons Tataroko-common, Aaa3-other & Nimbus227 (public domain)

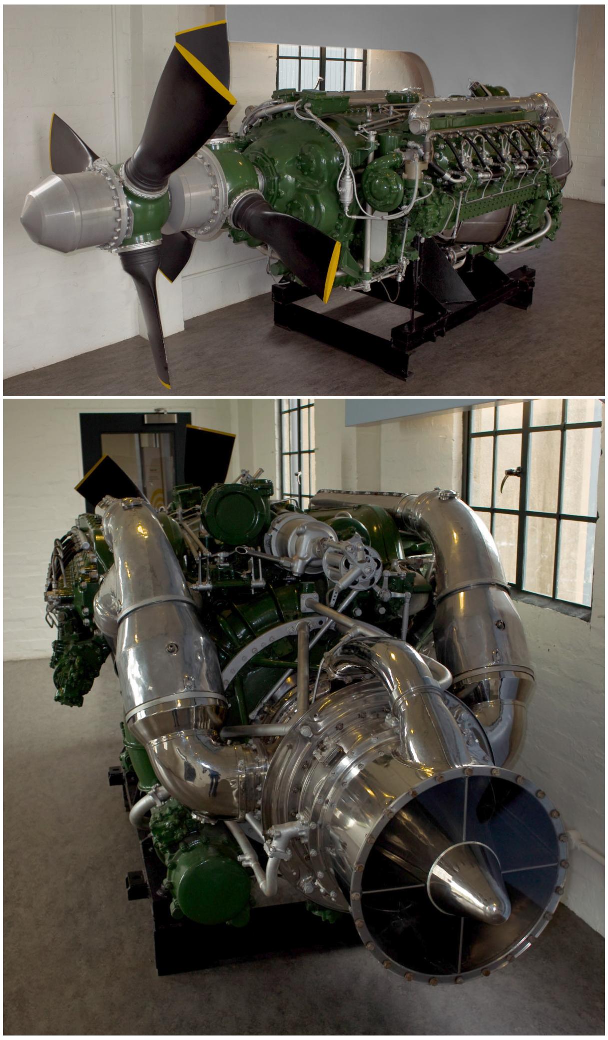


Figure 10.28: The prototype of the *Napier Nomad I*. The displacement was 40 L and the weight exceeded 2 tons.

Images edited from photos (1 and 2) CC-BY-SA *by Nigel Ish*

Napier & Son rapidly corrected course: the second prototype of the engine, the *Nomad II*, was greatly simplified. The intercooling, reheat, and centrifugal supercharger were all abandoned (figure 10.29). The two large mechanical units, one with pistons and the other with a turbine, were now connected to the same propeller. Both elements were connected with an ingenious but complex continuously-variable mechanical and hydraulic reducer that allowed each unit to operate at its optimal speed.

In an enlightening paper from 1954 [28], the engine designers showed a very clear vision and design approach. According to them, a simple turbocharged Diesel engine could only benefit from turbocharging over a very narrow power range — outside of this range, the turbine power would be either in surplus (and therefore lost) or insufficient to power the compressor. A different arrangement, in which the propeller would be driven solely by the turbine (with the Diesel engine then only providing supercharging and heat supply) would be far too inefficient at low power and unnecessarily strain the Diesel engine at high power. The simple turboprop, unable to reach the high pressures and

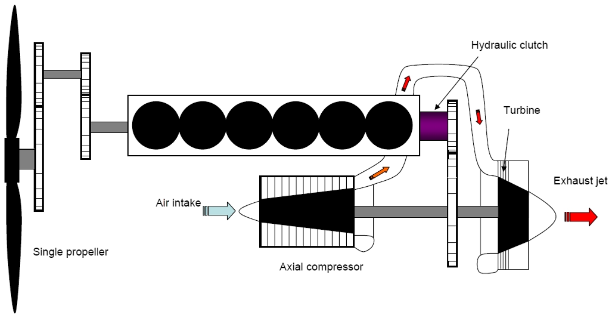


Figure 10.29: Schematic diagram of the thermodynamic circuit of the *Napier Nomad II.* A variable-ratio mechanical-hydraulic reducer connected the two units, which now drove the same propeller.

Diagram by users · Commons Tataroko-common, Aaa3-other & Nimbus227 (public domain)

temperatures of a Diesel engine, would be too inefficient. Only in the chosen arrangement, called *Diesel turbo-compound*, could the cylinder engine and the turboprop unit both contribute at all power levels, each always running at its optimal speed.

The performance of the *Nomad II* was indeed impressive —with its efficiency of 40 %, it used a third less fuel than its contemporaries— but its commercial failure was brutal: the project was abandoned in 1955 without a single sale. The engine was terribly heavy (with over 1600 kg for 2 MW, its power-to-weight ratio was three times lower than that of a turboprop), which erased a large part of the fuel savings it could have generated. Also, it was both too complex for regional aircraft and far too slow for jet airliners, and aircraft manufacturers were never interested.

The curious arrangement conceived by Napier & Son fell into obscurity but, sixty years later, it made a thunderous comeback in racing cars. In 2014, the International Automobile Federation, organizer of the Formula One races, sought to make it easier for new teams to join the sport, by limiting development expenses, increasing technological spin-offs applicable to the industry, and finding itself a (new-found) ecological conscience. The regulations were thus modified: turbocharging would be allowed, but the cars' fuel consumption was limited to 100 L/h. Most importantly, engine manufacturers would be allowed to use the turbocharger to recover energy in the form of electricity, as well as, conversely, accelerate the turbo by reinvesting this electrical energy into it (figure 10.30). Thus, the engine efficiency (and therefore, given the regulatory consumption limit, its power) can be increased at all speeds without sacrificing responsiveness. The system is poetically named MGU-H, but one could say that it is the unexpected revenge of the Anglo-Saxon *turbo-compounding*!

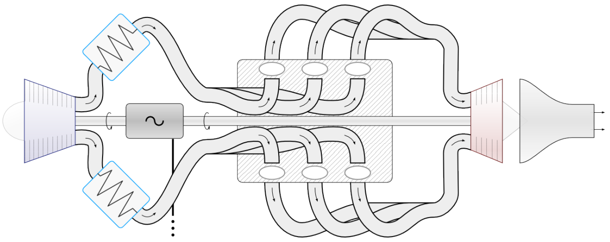


Figure 10.30: Thermodynamic circuit of the air in a 2014 Formula 1 engine. The shaft of the compressor (left) is not connected to the six-cylinder engine (center block) or the car's wheels. However, an electric motor/generator (named MGU-H) allows for extraction or addition of electrical energy. During high power phases, the power of the turbine (right) is in surplus and can be used to charge on-board batteries or drive the wheels with an electric motor. During low power phases, the turbine power is in deficit and the turbo can be driven by the generator to maintain the compression ratio and increase responsiveness.

Diagram CC-BY-SA *Olivier Cleynen*

Problems

Air is considered an ideal gas.

$c_{v\,(\text{air})} = 718\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$ $\quad R_\text{air} = 287\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$

$c_{p\,(\text{air})} = 1005\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$ $\quad \gamma_\text{air} = 1.4$

$c_{v\,(\text{gases})} = 823\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$ $\quad R_\text{gases} = 327\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$

$c_{p\,(\text{gases})} = 1150\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$ $\quad \gamma_\text{gases} = 1.333$

We assume that for a reversible adiabatic process (without heat transfer and infinitely slow), the properties of air are linked according to the following three relationships:

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{v_2}{v_1}\right)^{\gamma-1} \tag{4/36}$$

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} \tag{4/37}$$

$$\left(\frac{p_1}{p_2}\right) = \left(\frac{v_2}{v_1}\right)^{\gamma} \tag{4/38}$$

10.1 Knowledge Recap Questions

1. Why do piston engines allow for higher combustion temperatures than gas turbines?
2. What advantage does the Diesel cycle have over the Otto cycle?
3. Draw the cycle undergone by the air in a single-turbine turbojet on a pressure-volume diagram and on a temperature-entropy diagram, qualitatively (that is, without showing numerical values).
4. Why are two concentric spools (two shafts each connecting a compressor and a turbine) used in some gas turbines?
5. Draw the cycle undergone by the air in an intercooled turboshaft engine with an economizer heat exchanger (figure 10.23) on a temperature-entropy diagram, qualitatively.

10.2 Gasoline Engine

We propose to study the basic operation of the piston/cylinder engine of a private plane (figure 10.31). The engine is referred to as a “gasoline engine” and is based on the theoretical Otto cycle.

- At the beginning of the cycle, the air is at 21 °C and 1 bar (70 °F and 14.5 psi);
- The specific heat supplied during each cycle in cruise flight is 500 kJ kg^{-1};
- The compression ratio $\epsilon \equiv \frac{V_\text{max}}{V_\text{min}}$ is 7.

In this study, we consider that compression and expansion are isentropic and that heat addition and rejection occur at constant volume.

Figure 10.31: The 300 hp six-cylinder gasoline fuel injection Continental IO-550, in production since 1983. It equips among others the Cirrus SR22 aircraft.
This photo shows the turbocharged version of the engine, with the intercooler visible in the top left corner.

Engine photo CC-BY-SA *by Commons User:FlugKerl2; Aircraft photo* CC-BY-SA *by Commons User:Airman7474.*

1. Draw the cycle followed on a pressure-volume or temperature-entropy diagram, qualitatively, indicating all heat and work transfers.
2. What are the air temperatures at the beginning and at the end of combustion?
3. What is the amount of heat rejected during cooling?
4. What is the efficiency of this theoretical engine cycle?
5. In practice, the process undergone by the air is very different from the cycle described by Otto. Propose two reasons explaining this.
6. It is observed that when the aircraft gains altitude, the power that the engine can provide decreases significantly. What modification can be made to the engine to compensate for this?

10.3 Diesel Engine

A piston-cylinder engine used to propel a ship (figure 10.32) is turbocharged by a turbo that increases the pressure and temperature of the intake air using energy extracted from the exhaust gases (the turbocharger is a component that does not require any external input of energy in the form of work or heat, see §10.3.5 p. 278). The engine has the following operating characteristics:

- the air admitted to the cylinders is at 115 °C and 3 bar (239 °F and 45.3 psi;
- the specific heat supplied each cycle is 1250 kJ kg^{-1};
- the compression ratio $\epsilon \equiv \frac{V_{\text{max}}}{V_{\text{min}}}$ is 17.

We consider the optimal operating case, that is, following the Diesel cycle, according to the following characteristics:

- isentropic compression and expansion;
- combustion at constant pressure;
- heat rejection at constant volume.

1. Draw the thermodynamic cycle undergone by the air on a pressure-volume or temperature-entropy diagram, qualitatively, indicating all heat and work transfers.
2. What is the air temperature at the end of the compression?
3. What is the gas temperature at the end of the combustion?
4. What is the maximum pressure reached in the engine?
5. What is the temperature at the end of the expansion?
6. What is the engine cycle efficiency?
7. It is easy to show that at the same compression ratio, a Diesel cycle is less efficient than a so-called "gasoline" cycle (Otto cycle). Why is it used nevertheless?

Figure 10.32: The two Diesel engines of a 290.000 t oil tanker: a six-cylinder 1100 kW generator (top) and a seven-cylinder 25 MW propulsion engine (bottom).

Photos 1 and 2 CC-BY-SA *by Hervé Cozanet*

10.4 Turboprop Engine

A regional airliner is powered by two turboprop engines (figure 10.34). In each of them, a single turbine drives an axial compressor, as well as the propeller through a gearbox (figure 10.33).

During cruise, the air flow within the engine is 12.3 lb/s (5.6 kg s^{-1}), and the circuit is as follows:

- Air at ambient pressure and temperature (7.98 psi and 23 °F, or 0.55 bar and −5 °C) is admitted into the compressor;
- The compressor raises the air pressure to 110.2 psi (7.6 bar) with an isentropic efficiency of 80 %;
- The air is then heated in the combustion chamber to 2400 °F (1315 °C);
- The combustion gases are then expanded in the turbine and ejected into the atmosphere; the turbine has an isentropic efficiency of 80 %.

The turbine drives the compressor (through a shaft with negligible losses) and the propeller (through a transmission box with an efficiency of 83 %).

We want to quantify the shaft power actually received by the propeller during flight.

1. Draw the cycle undergone by the air on a temperature-entropy diagram, qualitatively.
2. What is the compressor outlet temperature?
3. What is the turbine outlet temperature?
4. What is the power supplied to the propeller?

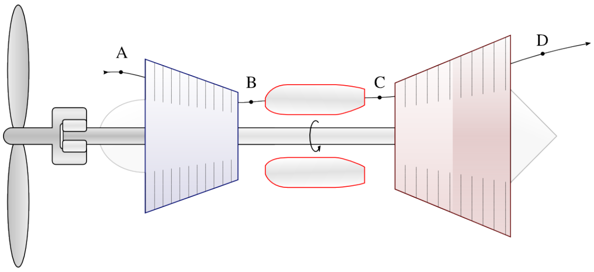


Figure 10.33: Internal arrangement of a turboprop.

Diagram CC-BY-SA *by Olivier Cleynen*

In order to de-ice the wings, a small gas bleed is carried out in the compressor. The extraction flow rate is 0.22 lb/s (0.1 $\mathrm{kg\,s^{-1}}$), and the air temperature is 392 °F (200 °C).

5. Propose and quantify a modification to the engine operation so that it may supply the propeller with the same power.

Figure 10.34: A Pratt & Whitney Canada PWC123 turboprop engine powering a Bombardier *Dash 8*. The PWC123 is configured with three concentric rotating assemblies, with the engine shaft powered by a free turbine, but its operation principle remains similar to that described in figure 10.33.

Engine photo derived from a photo CC-BY *by Flickr User:cliff1066; Aircraft photo* CC-BY-SA *by Flickr User:Björn*

10.5 Modification of a Turbojet

A turbojet operates with a single engine spool (single compressor and single turbine). Its operating characteristics are as follows:

Air flow rate:	4 $\mathrm{kg\,s^{-1}}$
Atmospheric temperature:	283 K
Atmospheric pressure:	0.969 $\mathrm{kg_f/cm^2}$ (0.95 bar)
Pressure ratio $\frac{p_{\text{max.}}}{p_{\text{min.}}}$:	25
Maximum temperature:	1300 K
Isentropic efficiencies of compressor and turbine:	85 %

We are looking to quantify its performance before it is modified.

1. Represent the components of the turbojet and the thermodynamic cycle undergone by the air on a temperature-entropy or pressure-volume diagram.
2. What is the pressure available at the turbine outlet?
3. What speed would the gases reach at the nozzle outlet if the expansion were isentropic?

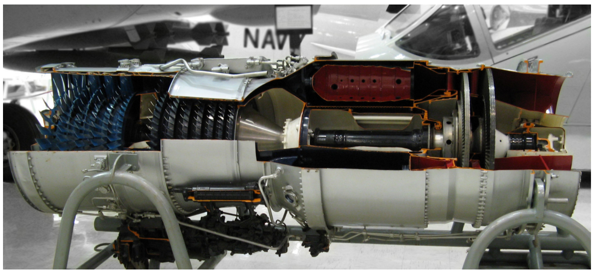


Figure 10.35: A turbojet with twin spools Pratt & Whitney J52 (or JT8A), built in 4500 units. It still equips the EA-6B *Prowler*.

Photo derived from a photo CC-BY *by Greg Goebel*

The team of engineers in charge of designing the components proposes to modify the engine by using two spools instead of one (figure 10.35). The rotating assembly closest to the center of the engine can operate at higher speeds, increasing the isentropic efficiency of the components:

- Isentropic efficiency of the low-pressure compressor and turbine (LP spool): 85 % (pressure ratio: 2)
- Isentropic efficiency of the high-pressure compressor and turbine (HP spool): 90 % (pressure ratio: 12.5)

All other operating characteristics of the engine remain unchanged.

4. What is the new available pressure at the turbine outlet?
5. What is the new theoretical exhaust gas velocity?

10.6 Intercooled Turboshaft

You are tasked by a small company to develop an engine that will be used to generate electricity in a factory. It is decided to base the engine on a turbofan jet engine from a retired commercial aircraft: it is a venerable General Electric CF6 (figures 10.36 and 10.37).

The turbofan engine has two concentric spools:

- The low-pressure spool connects the fan, a compressor section called the *booster*, and the low-pressure turbine;
- The high-pressure spool connects the rest of the compressor to the high-pressure turbine.

The turbofan engine has the following properties:

Maximum pressure ratio:	29.3
Booster pressure ratio:	1.2
Fan pressure ratio:	1.2
Maximum temperature:	1300 °C
Isentropic efficiency of compressors:	85 %

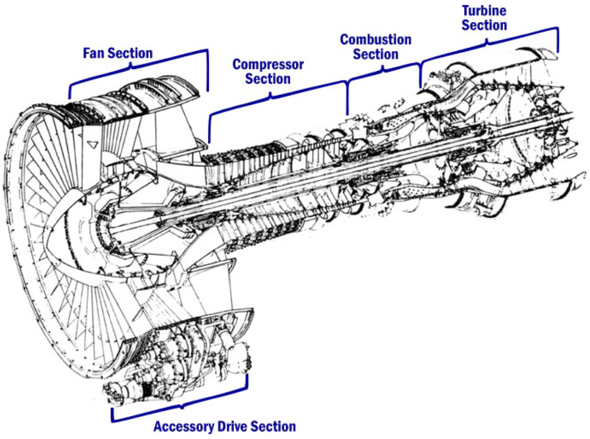


Figure 10.36: Cutaway diagram of a *General Electric* CF6-6. The engine propelled all major long-haul aircraft families of the 1970s and 1980s.

Diagram public domain U.S. FAA

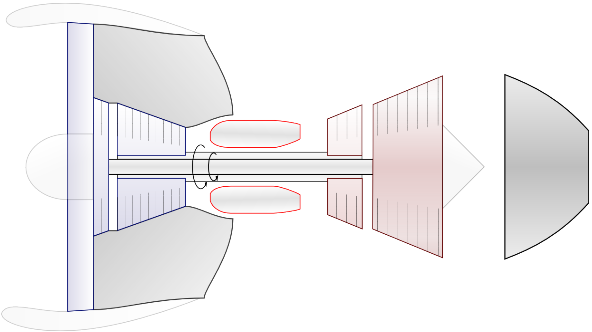


Figure 10.37: Schematic diagram of the arrangement of the *General Electric* CF6.

Diagram CC-BY-SA *Olivier Cleynen*

Isentropic efficiency of turbines:	85 %
Exhaust gas discharge pressure:	1.1 bar

In order to convert the turbofan into a turboshaft engine, you have the nacelle and the fan removed, and mechanically connect the low-pressure spool to the generator (figure 10.38). The turboshaft engine is started at atmospheric conditions of 1 bar and 18 °C. At full throttle, it uses an air flow rate of $80\ \mathrm{kg\,s^{-1}}$.

1. Draw the thermodynamic cycle undergone by the air on a pressure-volume diagram, qualitatively.
2. What is the net power delivered by the machine?
3. What is its work ratio?
4. What is its efficiency?

The client company receives your engine but wishes to increase its power. Since the engine is already operating at full capacity, you are unable to increase either the air mass flow rate or the combustion temperature.

In order to increase the power, you install an intercooling system (figure 10.38). The air compression is interrupted at a pressure of 7 bar; the air is led into a large heat exchanger where it is cooled at constant pressure. Once its temperature has dropped back to 40 °C, compression resumes in the compressor, which has not been modified.

5. Draw the new thermodynamic cycle on the pressure-volume diagram above, qualitatively.
6. What is the increase in net power?
7. What is the new work ratio?
8. What is the new efficiency?

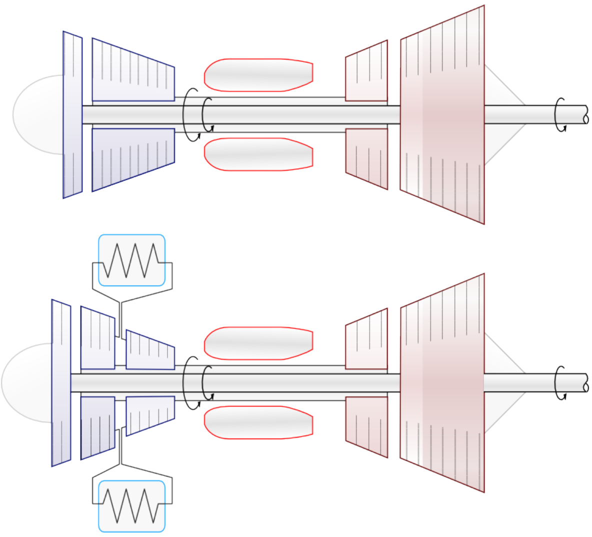


Figure 10.38: Top: schematic diagram of a turboshaft based on the CF6 from which the fan has been removed. Bottom: the same turboshaft engine modified by the addition of an intercooling system.

Diagrams CC-BY-SA *Olivier Cleynen*

Answers

10.2

1) See figure 10.3 p. 275;

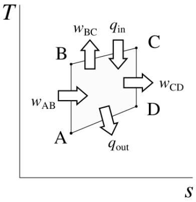
2) $T_\text{B} = T_\text{A} \left(\frac{v_\text{A}}{v_\text{B}}\right)^{\gamma_\text{air}-1} = 640.6\,\text{K} = 367.5\,^\circ\text{C}$ (4/36) and $T_\text{C} = \frac{q_\text{combustion}}{c_{v\,(\text{gaz})}} + \frac{c_{v\,(\text{air})}}{c_{v\,(\text{gaz})}} T_\text{B} = 1166.3\,\text{K} = 893.3\,^\circ\text{C} = 1640\,^\circ\text{F}$;

3) $T_\text{D} = 610.16\,\text{K} = 337\,^\circ\text{C} = 638.6\,^\circ\text{F}$ (4/36) so $q_{\text{D}\to\text{A}} = c_{v\,(\text{gaz})}(T_\text{D} - T_\text{A}) = -260.1\,\text{kJ}\,\text{kg}^{-1}$;

4) $\eta_\text{engine} = \frac{q_\text{net}}{q_\text{in}} = 47.99\,\%$ (purely theoretical value, since compressions and expansions are reversible: in practice, expect around $35\,\%$);

5) See §10.3.4 p. 278;

6) Power decreases since the air density of the atmosphere decreases with altitude. In order to increase $\dot{m}_\text{air}$, one can, for example, install a turbocharging system (see §10.3.5) as shown in figure 10.31.



10.3

2) $T_\text{B} = 1205.5\,\text{K} = 932.4\,^\circ\text{C} = 1710.2\,^\circ\text{F}$ (4/36);

3) $T_\text{C} = \frac{q_\text{combustion}}{c_{p(\text{gases})}} + \frac{c_{p(\text{air})}}{c_{p(\text{gases})}} T_\text{B} = 2140.5\,\text{K} = 1867.3\,^\circ\text{C} = 3393.2\,^\circ\text{F}$;

4) $p_\text{C} = p_\text{B} = 158.4\,\text{bar} = 2297.4\,\text{psi}$;

5) Using equation 4/36, $\frac{T_\text{D}}{T_\text{C}} = \left(\frac{v_\text{D}}{v_\text{C}}\right)^{\gamma_\text{gases}-1} = \left(\frac{v_\text{A}}{v_\text{B}}\frac{v_\text{B}}{v_\text{C}}\right)^{\gamma_\text{gases}-1} = \left[\epsilon\frac{R_\text{air}}{R_\text{gases}}\frac{T_\text{B}}{T_\text{C}}\right]^{\gamma_\text{gases}-1}$ Therefore $T_\text{D} = 1053.6\,\text{K} = 780.4\,^\circ\text{C} = 1436.8\,^\circ\text{F}$ (note that these gases will still need to power the turbocharger's turbine before being ejected into the atmosphere);

6) $\eta_\text{engine} = \frac{q_\text{net}}{q_\text{in}} = 56.19\,\%$ (close to reality since these engines are very slow);

7) See sections §10.3.3 p. 276 and §10.3.4 p. 278.

10.4

1) See figure 3.17 p. 78;

2) $T_\text{B} = 642.8\,\text{K} = 369.6\,^\circ\text{C} = 697.4\,^\circ\text{F}$ (4/37 & 10/6);

3) $T_\text{D} = 976.9\,\text{K} = 703.8\,^\circ\text{C} = 1298.8\,^\circ\text{F}$ (4/37 & 10/9);

4) $\dot{W}_\text{propellers} = -\dot{m}\ \eta_\text{transmission}\ (w_\text{turbine} + w_\text{compressor}) = +1.517\,\text{MW}$ (with a thermal efficiency before transmission of $27.6\,\%$, a value slightly lower than reality);

5) One possibility: increase $\dot{m}_\text{engine air}$ without modifying the temperatures. Then, $\dot{m}_\text{inlet engine 2} = 12.729\,\text{lb/s} = 5.774\,\text{kg}\,\text{s}^{-1}$ ($+0.383\,\text{lb/s}$ or $+0.174\,\text{kg}\,\text{s}^{-1}$).

10.5

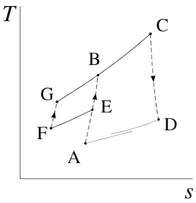
1) See figure 10.17 p. 288;

2) $T_\text{B} = 785.2\,\text{K}$, thus $T_\text{D} = 861.1\,\text{K}$ and $T_{\text{D}'} = 783.6\,\text{K}$: $p_\text{D} = 3.91\,\text{kg}_\text{f}/\text{cm}^2 = 3.13\,\text{bar}$;

3) Neglecting C_D, and with complete and reversible expansion, $C_\text{E} = 714.3\,\text{m}\,\text{s}^{-1}$ (the same remarks as in example 10.2 p. 285 apply here);

4) The temperature at the start of combustion drops to $T_3 = 774.2\,\text{K}$, the temperature at the turbine outlet is $T_6 = 870.7\,\text{K}$, and thus the pressure at the nozzle inlet rises to $p_6 = 4.776\,\text{kg}_\text{f}/\text{cm}^2 = 4.684\,\text{bar}$;

5) Cover your ears: $C_7 = 811.3\,\text{m}\,\text{s}^{-1}$ (the same remarks apply here as well).



10.6

2) $\dot{W}_\text{net} = \dot{m}\,c_{p(\text{gases})}\,(T_\text{D} - T_\text{C}) + c_{p(\text{air})}\,(T_\text{B} - T_\text{A}) = -3.536\,\text{MW}$ (approximately 3400 hp, not too bad for a machine first run in 1971… even after 15 years of service hung under a wing, a CF6 still sells for several million euros);

3) $M_{w1} = 38.2\,\%$;

4) $\eta_1 = 31.49\,\%$;

6) $\dot{W}_\text{net2} = -3.325\,\text{MW}$, that is a remarkable increase of $31\,\%$;

7) $M_{w1} = 50\,\%$, an increase of $+11.8\,\text{pt}$;

8) Power increases by $98.8\,\text{kJ}\,\text{kg}^{-1}$, while consumption increases by $332.6\,\text{kJ}\,\text{kg}^{-1}$, resulting in a marginal efficiency of $29.7\,\%$. The overall efficiency decreases to $\eta_2 = 31.04\,\%$, only $-0.5\,\text{pt}$… an interesting compromise!

Appendix

A1 Steam Tables

The following tables quantify the thermodynamic state of pure water across a large range of properties, as calculated according to the NIST-IAPWS 1995 model [44].

The notation used is detailed on page 321. The values of u and s are arbitrarily set to zero at the triple point of water, so that all values of u, h and s elsewhere are expressed relative to this point.

Data in these tables were produced by the US National Institute of Standards and Technology and are in the public domain. These tables were prepared by Olivier Cleynen and are released under a Creative Commons Attribution (CC-BY) license.

These tables are available for download in various formats and with different notation conventions at the URL:

https://freesteamtables.com/

Steam Table 1: Properties of pure water (compressed liquid and dry steam)

$\frac{m^3}{kg}$	$\frac{kJ}{kg}$	$\frac{kJ}{kg}$	$\frac{kJ}{K\,kg}$	°C	$\frac{m^3}{kg}$	$\frac{kJ}{kg}$	$\frac{kJ}{kg}$	$\frac{kJ}{K\,kg}$	°C	$\frac{m^3}{kg}$	$\frac{kJ}{kg}$	$\frac{kJ}{kg}$	$\frac{kJ}{K\,kg}$
v	u	h	s	T	v	u	h	s	T	v	u	h	s
p = 0.01 MPa ($T_{sat.}$ = 45.806 °C)					p = 0.05 MPa ($T_{sat.}$ = 81.317 °C)					p = 0.10 MPa ($T_{sat.}$ = 99.606 °C)			
0.001	42	42	0.1511	10	0.001	42	42.1	0.1511	10	0.001	42	42.1	0.1511
0.001002	83.9	83.9	0.2965	20	0.001002	83.9	84	0.2965	20	0.001002	83.9	84	0.2965
14.867	2 443.3	2 592	8.1741	50	0.001012	209.3	209.4	0.7038	50	0.001012	209.3	209.4	0.7038
17.196	2 515.5	2 687.5	8.4489	100	3.4187	2 511.5	2 682.4	7.6953	100	1.6959	2 506.2	2 675.8	7.361
21.826	2 661.3	2 879.6	8.9049	200	4.3562	2 660	2 877.8	8.1592	200	2.1724	2 658.3	2 875.5	7.8356
26.446	2 812.2	3 076.7	9.2827	300	5.284	2 811.6	3 075.8	8.5386	300	2.6388	2 810.6	3 074.5	8.2172
35.68	3 132.9	3 489.7	9.8998	500	7.1338	3 132.6	3 489.3	9.1566	500	3.5655	3 132.2	3 488.7	8.8361
40.296	3 303.3	3 706.3	10.163	600	8.0576	3 303.1	3 706	9.4201	600	4.0279	3 302.8	3 705.6	9.0998
44.911	3 480.8	3 929.9	10.406	700	8.9812	3 480.6	3 929.7	9.6625	700	4.49	3 480.4	3 929.4	9.3424
49.527	3 665.3	4 160.6	10.631	800	9.9047	3 665.2	4 160.4	9.8882	800	4.9519	3 665	4 160.2	9.5681
54.142	3 856.9	4 398.3	10.843	900	10.828	3 856.8	4 398.2	10.1	900	5.4137	3 856.6	4 398	9.78
58.758	4 055.2	4 642.8	11.043	1 000	11.751	4 055.2	4 642.7	10.3	1 000	5.8754	4 055.1	4 642.6	9.98
63.373	4 260	4 893.7	11.233	1 100	12.674	4 260	4 893.7	10.49	1 100	6.3371	4 259.8	4 893.5	10.17
67.988	4 470.8	5 150.7	11.413	1 200	13.598	4 470.8	5 150.7	10.67	1 200	6.7988	4 470.7	5 150.6	10.35
81.834	5 135.7	5 954	11.909	1 500	16.367	5 135.6	5 953.9	11.166	1 500	8.1836	5 135.5	5 953.9	10.846
104,91	6 327.9	7 377	12.615	2 000	20.982	6 327.9	7 377	11.872	2 000	10.491	6 327.9	7 377	11.552
p = 0.20 MPa ($T_{sat.}$ = 120.210 °C)					p = 0.40 MPa ($T_{sat.}$ = 143.608 °C)					p = 0.60 MPa ($T_{sat.}$ = 158.826 °C)			
0.001	42	42.2	0.1511	10	0.001	42	42.4	0.1511	10	0.001	42	42.6	0.151
0.001002	83.9	84.1	0.2964	20	0.001002	83.9	84.3	0.2964	20	0.001002	83.9	84.5	0.2964
0.001012	209.3	209.5	0.7037	50	0.001012	209.3	209.7	0.7036	50	0.001012	209.2	209.9	0.7035
0.001043	419	419.2	1.3071	100	0.001043	419	419.4	1.307	100	0.001043	418.9	419.5	1.3068
1.0805	2 654.6	2 870.7	7.5081	200	0.53433	2 647.2	2 860.9	7.1723	200	0.35212	2 639.3	2 850.6	6.9683
1.3162	2 808.9	3 072.1	7.8941	300	0.65489	2 805.1	3 067.1	7.5677	300	0.43442	2 801.3	3 062	7.374
1.7814	3 131.4	3 487.7	8.5152	500	0.88936	3 129.8	3 485.5	8.1933	500	0.592	3 128.2	3 483.4	8.0041
2.013	3 302.2	3 704.8	8.7792	600	1.0056	3 301	3 703.2	8.458	600	0.66976	3 299.8	3 701.7	8.2695
2.2443	3 479.9	3 928.8	9.022	700	1.1215	3 479	3 927.6	8.7012	700	0.74725	3 478.1	3 926.4	8.5131
2.4755	3 664.7	4 159.8	9.2479	800	1.2373	3 663.9	4 158.8	8.9273	800	0.82457	3 663.2	4 157.9	8.7395
2.7066	3 856.3	4 397.6	9.4598	900	1.353	3 855.7	4 396.9	9.1394	900	0.90178	3 855.1	4 396.2	8.9518
2.9375	4 054.8	4 642.3	9.6599	1 000	1.4686	4 054.3	4 641.7	9.3396	1 000	0.97893	4 053.7	4 641.1	9.1521
3.1685	4 259.6	4 893.3	9.8497	1 100	1.5841	4 259.2	4 892.8	9.5295	1 100	1.056	4 258.8	4 892.4	9.342
3.3994	4 470.5	5 150.4	10.03	1 200	1.6997	4 470.1	5 150	9.7102	1 200	1.1331	4 469.7	5 149.6	9.5228
4.0919	5 135.4	5 953.8	10.526	1 500	2.0461	5 135.2	5 953.6	10.206	1 500	1.3641	5 134.9	5 953.4	10.019
5.246	6 327.7	7 376.9	11.232	2 000	2.6232	6 327.6	7 376.9	10.912	2 000	1.749	6 327.4	7 376.8	10.725
p = 0.80 MPa ($T_{sat.}$ = 170.406 °C)					p = 1.0 MPa ($T_{sat.}$ = 179.878 °C)					p = 1.2 MPa ($T_{sat.}$ = 187.957 °C)			
0.001	42	42.8	0.151	10	0.001	42	43	0.151	10	0.001	42	43.2	0.151
0.001001	83.9	84.7	0.2963	20	0.001001	83.8	84.9	0.2963	20	0.001001	83.8	85	0.2962
0.001012	209.2	210	0.7034	50	0.001012	209.2	210.2	0.7034	50	0.001012	209.2	210.4	0.7033
0.001043	418.9	419.7	1.3067	100	0.001043	418.8	419.8	1.3065	100	0.001043	418.7	420	1.3064
0.26088	2 631	2 839.7	6.8176	200	0.20602	2 622.3	2 828.3	6.6955	200	0.16934	2 612.9	2 816.1	6.5909
0.32416	2 797.6	3 056.9	7.2345	300	0.25799	2 793.6	3 051.6	7.1246	300	0.21386	2 789.7	3 046.3	7.0335
0.44332	3 126.6	3 481.3	7.8692	500	0.35411	3 125	3 479.1	7.7641	500	0.29464	3 123.3	3 476.9	7.6779
0.50185	3 298.6	3 700.1	8.1354	600	0.40111	3 297.5	3 698.6	8.031	600	0.33394	3 296.3	3 697	7.9455
0.56011	3 477.2	3 925.3	8.3794	700	0.44783	3 476.3	3 924.1	8.2755	700	0.37297	3 475.3	3 922.9	8.1904
0.6182	3 662.4	4 157	8.6061	800	0.49438	3 661.7	4 156.1	8.5024	800	0.41184	3 661	4 155.2	8.4176
0.67619	3 854.5	4 395.5	8.8185	900	0.54083	3 854	4 394.8	8.715	900	0.45059	3 853.3	4 394	8.6303
0.73411	4 053.2	4 640.5	9.0189	1 000	0.58721	4 052.7	4 639.9	8.9155	1 000	0.48928	4 052.3	4 639.4	8.831
0.79197	4 258.3	4 891.9	9.2089	1 100	0.63354	4 257.9	4 891.4	9.1056	1 100	0.52792	4 257.5	4 891	9.0212
0.8498	4 469.4	5 149.2	9.3898	1 200	0.67983	4 469.1	5 148.9	9.2866	1 200	0.56652	4 468.7	5 148.5	9.2022
1.0232	5 134.6	5 953.2	9.8861	1 500	0.81857	5 134.4	5 953	9.783	1 500	0.68218	5 134.2	5 952.8	9.6987
1.3118	6 327.4	7 376.8	10.592	2 000	1.0496	6 327.2	7 376.8	10.489	2 000	0.87471	6 327	7 376.7	10.405

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Steam Table 1 (continued)

v	u	h	s	T	v	u	h	s	T	v	u	h	s
$p = 1.4$ MPa					$p = 1.6$ MPa					$p = 1.8$ MPa			
($T_{\text{sat.}} = 195.039\,^\circ\text{C}$)					($T_{\text{sat.}} = 201.370\,^\circ\text{C}$)					($T_{\text{sat.}} = 207.112\,^\circ\text{C}$)			
0.001	42	43.4	0.151	10	0.001	42	43.6	0.1509	10	0.000999	42	43.8	0.1509
0.001001	83.8	85.2	0.2962	20	0.001001	83.8	85.4	0.2962	20	0.001001	83.8	85.6	0.2961
0.001012	209.1	210.5	0.7032	50	0.001011	209.1	210.7	0.7031	50	0.001011	209.1	210.9	0.703
0.001043	418.7	420.1	1.3062	100	0.001043	418.6	420.3	1.306	100	0.001043	418.6	420.4	1.3059
0.14303	2 602.8	2 803	6.4975	200	0.001156	850.4	852.3	2.3305	200	0.001156	850.3	852.4	2.3301
0.18232	2 785.7	3 040.9	6.9552	300	0.15866	2 781.5	3 035.4	6.8863	300	0.14025	2 777.5	3 029.9	6.8246
0.25216	3 121.8	3 474.8	7.6047	500	0.22029	3 120.1	3 472.6	7.5409	500	0.19551	3 118.5	3 470.4	7.4845
0.28597	3 295	3 695.4	7.873	600	0.24999	3 293.9	3 693.9	7.81	600	0.222	3 292.7	3 692.3	7.7543
0.31951	3 474.4	3 921.7	8.1183	700	0.2794	3 473.5	3 920.5	8.0557	700	0.24821	3 472.6	3 919.4	8.0004
0.35287	3 660.3	4 154.3	8.3457	800	0.30865	3 659.5	4 153.3	8.2834	800	0.27426	3 658.7	4 152.4	8.2284
0.38614	3 852.7	4 393.3	8.5587	900	0.3378	3 852.1	4 392.6	8.4965	900	0.3002	3 851.5	4 391.9	8.4416
0.41933	4 051.7	4 638.8	8.7594	1 000	0.36687	4 051.2	4 638.2	8.6974	1 000	0.32606	4 050.7	4 637.6	8.6426
0.45247	4 257	4 890.5	8.9497	1 100	0.39589	4 256.6	4 890	8.8878	1 100	0.35188	4 256.1	4 889.5	8.8331
0.48558	4 468.3	5 148.1	9.1308	1 200	0.42487	4 467.9	5 147.7	9.0689	1 200	0.37766	4 467.5	5 147.3	9.0143
0.58476	5 133.9	5 952.6	9.6274	1 500	0.51169	5 133.7	5 952.4	9.5656	1 500	0.45486	5 133.4	5 952.1	9.5111
0.74982	6 327	7 376.7	10.334	2 000	0.65615	6 326.8	7 376.6	10.272	2 000	0.5833	6 326.7	7 376.6	10.218
$p = 2.0$ MPa					$p = 2.5$ MPa					$p = 3.0$ MPa			
($T_{\text{sat.}} = 212.377\,^\circ\text{C}$)					($T_{\text{sat.}} = 223.950\,^\circ\text{C}$)					($T_{\text{sat.}} = 233.853\,^\circ\text{C}$)			
0.000999	42	44	0.1509	10	0.000999	42	44.5	0.1509	10	0.000999	41.9	44.9	0.1508
0.001001	83.8	85.8	0.2961	20	0.001001	83.8	86.3	0.296	20	0.001	83.7	86.7	0.2959
0.001011	209	211.1	0.7029	50	0.001011	209	211.5	0.7027	50	0.001011	208.9	211.9	0.7024
0.001042	418.5	420.6	1.3057	100	0.001042	418.4	421	1.3053	100	0.001042	418.2	421.3	1.305
0.001156	850.1	852.5	2.3298	200	0.001156	849.8	852.7	2.329	200	0.001155	849.4	852.9	2.3282
0.12551	2 773.2	3 024.2	6.7684	300	0.098937	2 762.3	3 009.6	6.6459	300	0.081179	2 750.8	2 994.3	6.5412
0.17568	3 116.8	3 468.2	7.4337	500	0.13999	3 112.7	3 462.7	7.3254	500	0.1162	3 108.6	3 457.2	7.2359
0.19961	3 291.5	3 690.7	7.7043	600	0.15931	3 288.5	3 686.8	7.5979	600	0.13245	3 285.5	3 682.8	7.5103
0.22326	3 471.7	3 918.2	7.9509	700	0.17835	3 469.3	3 915.2	7.8455	700	0.14841	3 467	3 912.2	7.759
0.24674	3 658	4 151.5	8.179	800	0.19721	3 656.2	4 149.2	8.0743	800	0.1642	3 654.3	4 146.9	7.9885
0.27012	3 850.9	4 391.1	8.3925	900	0.21597	3 849.4	4 389.3	8.2882	900	0.17988	3 847.9	4 387.5	8.2028
0.29342	4 050.2	4 637	8.5936	1 000	0.23466	4 049	4 635.6	8.4896	1 000	0.19549	4 047.6	4 634.1	8.4045
0.31667	4 255.8	4 889.1	8.7842	1 100	0.2533	4 254.7	4 887.9	8.6804	1 100	0.21105	4 253.6	4 886.7	8.5955
0.33989	4 467.2	5 147	8.9654	1 200	0.2719	4 466.3	5 146	8.8618	1 200	0.22657	4 465.3	5 145	8.777
0.4094	5 133.1	5 951.9	9.4624	1 500	0.32757	5 132.5	5 951.4	9.359	1 500	0.27301	5 131.9	5 950.9	9.2745
0.52501	6 326.5	7 376.5	10.169	2 000	0.42011	6 326.1	7 376.4	10.066	2 000	0.35017	6 325.8	7 376.3	9.9818
$p = 3.5$ MPa					$p = 4.0$ MPa					$p = 4.5$ MPa			
($T_{\text{sat.}} = 242.557\,^\circ\text{C}$)					($T_{\text{sat.}} = 250.354\,^\circ\text{C}$)					($T_{\text{sat.}} = 257.437\,^\circ\text{C}$)			
0.000999	41.9	45.4	0.1508	10	0.000998	41.9	45.9	0.1507	10	0.000998	41.9	46.4	0.1507
0.001	83.7	87.2	0.2958	20	0.001	83.7	87.7	0.2956	20	0.001	83.6	88.1	0.2955
0.001011	208.8	212.4	0.7022	50	0.00101	208.7	212.8	0.702	50	0.00101	208.7	213.2	0.7017
0.001042	418.1	421.7	1.3046	100	0.001041	417.9	422.1	1.3042	100	0.001041	417.8	422.5	1.3038
0.001155	849	853.1	2.3275	200	0.001154	848.7	853.3	2.3267	200	0.001154	848.3	853.5	2.3259
0.068453	2 738.8	2 978.4	6.4484	300	0.05887	2 726.2	2 961.7	6.3639	300	0.051378	2 713	2 944.2	6.2854
0.099195	3 104.4	3 451.6	7.1593	500	0.086442	3 100.2	3 446	7.0922	500	0.076521	3 096.1	3 440.4	7.0323
0.11325	3 282.5	3 678.9	7.4356	600	0.098859	3 279.5	3 674.9	7.3705	600	0.087662	3 276.4	3 670.9	7.3127
0.12702	3 464.7	3 909.3	7.6854	700	0.11098	3 462.4	3 906.3	7.6214	700	0.0985	3 460.1	3 903.3	7.5646
0.14061	3 652.5	4 144.6	7.9156	800	0.12292	3 650.6	4 142.3	7.8523	800	0.10916	3 648.8	4 140	7.7962
0.1541	3 846.4	4 385.7	8.1303	900	0.13476	3 844.9	4 383.9	8.0674	900	0.11972	3 843.4	4 382.1	8.0118
0.16751	4 046.4	4 632.7	8.3324	1 000	0.14652	4 045.1	4 631.2	8.2697	1 000	0.1302	4 043.9	4 629.8	8.2144
0.18087	4 252.6	4 885.6	8.5235	1 100	0.15824	4 251.4	4 884.4	8.4611	1 100	0.14064	4 250.3	4 883.2	8.406
0.1942	4 464.4	5 144.1	8.7053	1 200	0.16992	4 463.4	5 143.1	8.643	1 200	0.15103	4 462.6	5 142.2	8.588
0.23404	5 131.3	5 950.4	9.203	1 500	0.20481	5 130.7	5 949.9	9.1411	1 500	0.18208	5 130	5 949.4	9.0863
0.30021	6 325.5	7 376.2	9.9105	2 000	0.26274	6 325	7 376	9.8487	2 000	0.2336	6 324.7	7 375.9	9.7942

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Steam Table 1 (continued)

v	u	h	s	T	v	u	h	s	T	v	u	h	s
p = 5.0 MPa					p = 5.5 MPa					p = 6.0 MPa			
($T_{\text{sat.}}$ = 263.941 °C)					($T_{\text{sat.}}$ = 269.965 °C)					($T_{\text{sat.}}$ = 275.585 °C)			
0.000998	41.9	46.9	0.1506	10	0.000998	41.9	47.4	0.1506	10	0.000998	41.9	47.9	0.1505
0.001	83.6	88.6	0.2954	20	0.000999	83.6	89.1	0.2953	20	0.000999	83.5	89.5	0.2952
0.00101	208.6	213.6	0.7015	50	0.00101	208.5	214.1	0.7013	50	0.00101	208.4	214.5	0.701
0.001041	417.6	422.9	1.3034	100	0.001041	417.5	423.2	1.303	100	0.00104	417.4	423.6	1.3026
0.001153	847.9	853.7	2.3251	200	0.001153	847.6	853.9	2.3243	200	0.001152	847.2	854.1	2.3235
0.045346	2 699	2 925.7	6.211	300	0.040373	2 684.1	2 906.2	6.1397	300	0.036189	2 668.4	2 885.5	6.0703
0.068583	3 091.8	3 434.7	6.9781	500	0.062086	3 087.4	3 428.9	6.9285	500	0.056671	3 083.1	3 423.1	6.8826
0.078704	3 273.3	3 666.8	7.2605	600	0.071374	3 270.2	3 662.8	7.213	600	0.065265	3 267.1	3 658.7	7.1693
0.088518	3 457.7	3 900.3	7.5136	700	0.080351	3 455.4	3 897.3	7.4672	700	0.073545	3 453	3 894.3	7.4246
0.098158	3 646.9	4 137.7	7.7458	800	0.089152	3 645.1	4 135.4	7.7001	800	0.081648	3 643.2	4 133.1	7.6582
0.10769	3 841.8	4 380.2	7.9618	900	0.097844	3 840.3	4 378.4	7.9166	900	0.089641	3 838.8	4 376.6	7.8751
0.11715	4 042.6	4 628.3	8.1648	1 000	0.10646	4 041.4	4 626.9	8.1198	1 000	0.09756	4 040	4 625.4	8.0786
0.12655	4 249.3	4 882	8.3566	1 100	0.11503	4 248.2	4 880.9	8.3118	1 100	0.10543	4 247.1	4 879.7	8.2709
0.13592	4 461.6	5 141.2	8.5388	1 200	0.12356	4 460.7	5 140.3	8.4941	1 200	0.11326	4 459.7	5 139.3	8.4534
0.1639	5 129.4	5 948.9	9.0374	1 500	0.14902	5 128.8	5 948.4	8.993	1 500	0.13662	5 128.2	5 947.9	8.9525
0.21029	6 324.4	7 375.8	9.7454	2 000	0.19121	6 324	7 375.7	9.7012	2 000	0.17532	6 323.7	7 375.6	9.6609
p = 6.5 MPa					p = 7.0 MPa					p = 7.5 MPa			
($T_{\text{sat.}}$ = 280.858 °C)					($T_{\text{sat.}}$ = 285.829 °C)					($T_{\text{sat.}}$ = 290.535 °C)			
0.000997	41.9	48.3	0.1505	10	0.000997	41.8	48.8	0.1504	10	0.000997	41.8	49.3	0.1504
0.000999	83.5	90	0.2951	20	0.000999	83.5	90.5	0.295	20	0.000998	83.5	91	0.2949
0.001009	208.4	214.9	0.7008	50	0.001009	208.3	215.4	0.7006	50	0.001009	208.2	215.8	0.7004
0.00104	417.2	424	1.3022	100	0.00104	417.1	424.4	1.3019	100	0.00104	416.9	424.7	1.3015
0.001152	846.8	854.3	2.3228	200	0.001151	846.5	854.5	2.322	200	0.001151	846.1	854.7	2.3212
0.032607	2 651.6	2 863.5	6.0019	300	0.029492	2 633.5	2 839.9	5.9337	300	0.026742	2 613.8	2 814.4	5.8646
0.052087	3 078.7	3 417.3	6.8399	500	0.048157	3 074.3	3 411.4	6.8	500	0.04475	3 069.9	3 405.5	6.7623
0.060096	3 264.1	3 654.7	7.1288	600	0.055665	3 260.9	3 650.6	7.091	600	0.051824	3 257.8	3 646.5	7.0555
0.067786	3 450.7	3 891.3	7.3853	700	0.06285	3 448.3	3 888.2	7.3486	700	0.058572	3 445.9	3 885.2	7.3144
0.075298	3 641.4	4 130.8	7.6195	800	0.069855	3 639.4	4 128.4	7.5836	800	0.065138	3 637.6	4 126.1	7.55
0.082699	3 837.3	4 374.8	7.8369	900	0.07675	3 835.8	4 373	7.8014	900	0.071593	3 834.2	4 371.1	7.7682
0.090027	4 038.8	4 624	8.0407	1 000	0.083571	4 037.5	4 622.5	8.0055	1 000	0.077975	4 036.3	4 621.1	7.9726
0.097305	4 246	4 878.5	8.2331	1 100	0.090341	4 244.9	4 877.3	8.1981	1 100	0.084306	4 243.9	4 876.2	8.1655
0.10455	4 458.8	5 138.4	8.4158	1 200	0.097074	4 457.9	5 137.4	8.381	1 200	0.0906	4 457	5 136.5	8.3485
0.12613	5 127.6	5 947.4	8.9152	1 500	0.11714	5 126.9	5 946.9	8.8807	1 500	0.10934	5 126.4	5 946.4	8.8485
0.16187	6 323.2	7 375.4	9.6238	2 000	0.15034	6 322.9	7 375.3	9.5895	2 000	0.14035	6 322.6	7 375.2	9.5575
p = 8 MPa					p = 9 MPa					p = 10 MPa			
($T_{\text{sat.}}$ = 295.008 °C)					($T_{\text{sat.}}$ = 303.345 °C)					($T_{\text{sat.}}$ = 310.997 °C)			
0.000997	41.8	49.8	0.1503	10	0.000996	41.8	50.8	0.1502	10	0.000996	41.8	51.7	0.1501
0.000998	83.4	91.4	0.2948	20	0.000998	83.4	92.4	0.2946	20	0.000997	83.3	93.3	0.2944
0.001009	208.2	216.2	0.7001	50	0.001008	208	217.1	0.6997	50	0.001008	207.9	217.9	0.6992
0.001039	416.8	425.1	1.3011	100	0.001039	416.5	425.9	1.3003	100	0.001038	416.2	426.6	1.2996
0.00115	845.7	854.9	2.3205	200	0.001149	845	855.4	2.3189	200	0.001148	844.3	855.8	2.3174
0.024279	2 592.3	2 786.5	5.7937	300	0.001402	1 331.9	1 344.5	3.2533	300	0.001398	1 329.3	1 343.3	3.2488
0.041767	3 065.4	3 399.5	6.7266	500	0.036793	3 056.3	3 387.4	6.6603	500	0.032811	3 047	3 375.1	6.5995
0.048463	3 254.7	3 642.4	7.0221	600	0.042861	3 248.4	3 634.1	6.9605	600	0.038378	3 242	3 625.8	6.9045
0.054828	3 443.6	3 882.2	7.2821	700	0.048589	3 438.8	3 876.1	7.2229	700	0.043597	3 434	3 870	7.1693
0.061011	3 635.7	4 123.8	7.5184	800	0.054132	3 631.9	4 119.1	7.4606	800	0.048629	3 628.2	4 114.5	7.4085
0.067082	3 832.6	4 369.3	7.7371	900	0.059562	3 829.6	4 365.7	7.6802	900	0.053547	3 826.5	4 362	7.629
0.073079	4 035	4 619.6	7.9419	1 000	0.064918	4 032.4	4 616.7	7.8855	1 000	0.05839	4 029.9	4 613.8	7.8349
0.079025	4 242.8	4 875	8.135	1 100	0.070224	4 240.7	4 872.7	8.079	1 100	0.063183	4 238.5	4 870.3	8.0288
0.084934	4 456	5 135.5	8.3181	1 200	0.075492	4 454.2	5 133.6	8.2625	1 200	0.067938	4 452.3	5 131.7	8.2126
0.10252	5 125.7	5 945.9	8.8184	1 500	0.091158	5 124.5	5 944.9	8.7633	1 500	0.082066	5 123.2	5 943.9	8.714
0.1316	6 322.3	7 375.1	9.5275	2 000	0.11703	6 321.6	7 374.9	9.4729	2 000	0.10538	6 320.8	7 374.6	9.4239

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Steam Table 1 (continued)

v	u	h	s	T	v	u	h	s	T	v	u	h	s
$p = 12$ MPa					$p = 14$ MPa					$p = 16$ MPa			
($T_{\text{sat.}} = 324.675\,^{\circ}$C)					($T_{\text{sat.}} = 336.666\,^{\circ}$C)					($T_{\text{sat.}} = 347.355\,^{\circ}$C)			
0.000995	41.7	53.6	0.1499	10	0.000994	41.7	55.6	0.1496	10	0.000993	41.6	57.5	0.1494
0.000996	83.2	95.1	0.2939	20	0.000996	83.1	97	0.2935	20	0.000995	82.9	98.9	0.293
0.001007	207.6	219.7	0.6983	50	0.001006	207.3	221.4	0.6974	50	0.001005	207	223.1	0.6964
0.001038	415.7	428.1	1.298	100	0.001037	415.1	429.6	1.2965	100	0.001036	414.6	431.1	1.295
0.001146	842.9	856.7	2.3144	200	0.001144	841.5	857.6	2.3114	200	0.001143	840.2	858.4	2.3085
0.00139	1 324.5	1 341.2	3.2401	300	0.001382	1 319.9	1 339.2	3.2319	300	0.001375	1 315.4	1 337.4	3.224
0.026828	3 028.1	3 350	6.4903	500	0.022544	3 008.5	3 324.1	6.3932	500	0.019323	2 988.1	3 297.3	6.3046
0.031651	3 229.1	3 608.9	6.8054	600	0.026845	3 216	3 591.8	6.7191	600	0.023238	3 202.6	3 574.4	6.6421
0.036109	3 424.4	3 857.7	7.0753	700	0.030761	3 414.6	3 845.3	6.9941	700	0.026749	3 404.9	3 832.9	6.9224
0.040375	3 620.6	4 105.1	7.3173	800	0.034479	3 613.1	4 095.8	7.2391	800	0.030058	3 605.4	4 086.3	7.1703
0.044524	3 820.4	4 354.7	7.5396	900	0.03808	3 814.3	4 347.4	7.4632	900	0.033247	3 808	4 340	7.3964
0.048599	4 024.8	4 608	7.7467	1 000	0.041605	4 019.6	4 602.1	7.6716	1 000	0.036361	4 014.5	4 596.3	7.606
0.052622	4 234.1	4 865.6	7.9416	1 100	0.045079	4 229.8	4 860.9	7.8673	1 100	0.039422	4 225.5	4 856.3	7.8025
0.056608	4 448.6	5 127.9	8.1259	1 200	0.048516	4 445	5 124.2	8.0523	1 200	0.042447	4 441.2	5 120.4	7.9882
0.068428	5 120.8	5 941.9	8.6284	1 500	0.058687	5 118.3	5 939.9	8.5559	1 500	0.051381	5 115.8	5 937.9	8.4929
0.087892	6 319.5	7 374.2	9.3392	2 000	0.075404	6 318	7 373.7	9.2674	2 000	0.066037	6 316.6	7 373.2	9.2052
$p = 18$ MPa					$p = 20$ MPa					$p = 30$ MPa			
($T_{\text{sat.}} = 356.992\,^{\circ}$C)					($T_{\text{sat.}} = 365.749\,^{\circ}$C)					($p > p_{cr.}$)			
0.000992	41.5	59.4	0.1491	10	0.000991	41.5	61.3	0.1489	10	0.000987	41.2	70.8	0.1475
0.000994	82.8	100.7	0.2925	20	0.000993	82.7	102.6	0.2921	20	0.000989	82.1	111.8	0.2897
0.001004	206.7	224.8	0.6955	50	0.001003	206.4	226.5	0.6946	50	0.000999	205.1	235.1	0.6901
0.001035	414	432.7	1.2935	100	0.001034	413.5	434.2	1.292	100	0.001029	410.9	441.7	1.2847
0.001141	838.8	859.4	2.3056	200	0.001139	837.5	860.3	2.3027	200	0.00113	831.1	865	2.2888
0.001368	1 311.2	1 335.8	3.2164	300	0.001361	1 307.2	1 334.4	3.2091	300	0.001332	1 288.9	1 328.9	3.176
0.01681	2 967.1	3 269.7	6.2223	500	0.014793	2 945.3	3 241.2	6.1446	500	0.00869	2 824	3 084.7	5.7956
0.020431	3 189	3 556.8	6.572	600	0.018185	3 175.3	3 539	6.5075	600	0.011445	3 103.4	3 446.7	6.2373
0.023629	3 395.1	3 820.4	6.8579	700	0.021133	3 385.1	3 807.8	6.799	700	0.013653	3 334.3	3 743.9	6.5598
0.026619	3 597.8	4 076.9	7.1089	800	0.023869	3 590.1	4 067.5	7.0531	800	0.015628	3 551.2	4 020	6.83
0.029489	3 801.9	4 332.7	7.3368	900	0.026483	3 795.7	4 325.4	7.2829	900	0.017473	3 764.6	4 288.8	7.0695
0.032282	4 009.4	4 590.5	7.5476	1 000	0.02902	4 004.3	4 584.7	7.495	1 000	0.01924	3 978.6	4 555.8	7.288
0.035023	4 221.2	4 851.6	7.745	1 100	0.031504	4 216.8	4 846.9	7.6933	1 100	0.020953	4 195.2	4 823.8	7.4906
0.037727	4 437.5	5 116.6	7.9313	1 200	0.033952	4 433.8	5 112.8	7.8802	1 200	0.02263	4 415.3	5 094.2	7.6807
0.045699	5 113.3	5 935.9	8.4372	1 500	0.041154	5 110.8	5 933.9	8.3871	1 500	0.027521	5 098.6	5 924.2	8.1932
0.058753	6 315.2	7 372.8	9.1502	2 000	0.052925	6 313.8	7 372.3	9.101	2 000	0.035443	6 306.8	7 370.1	8.9108
$p = 40$ MPa					$p = 50$ MPa					$p = 100$ MPa			
($p > p_{cr.}$)					($p > p_{cr.}$)					($p > p_{cr.}$)			
0.000982	40.9	80.2	0.1458	10	0.000978	40.6	89.5	0.144	10	0.000959	38.8	134.7	0.1326
0.000985	81.5	120.9	0.2872	20	0.00098	80.9	130	0.2845	20	0.000962	78	174.2	0.2699
0.000995	203.7	243.6	0.6855	50	0.000991	202.5	252	0.681	50	0.000973	196.6	293.9	0.6587
0.001024	408.4	449.3	1.2775	100	0.00102	405.9	456.9	1.2705	100	0.001	395.1	495.1	1.2375
0.001122	825.1	870	2.2755	200	0.001115	819.4	875.2	2.2628	200	0.001083	795.1	903.4	2.2064
0.001308	1 273.3	1 325.6	3.1473	300	0.001288	1 259.6	1 324	3.1218	300	0.001215	1 207.6	1 329.1	3.0219
0.005623	2 681.6	2 906.5	5.4744	500	0.00389	2 528.1	2 722.6	5.1762	500	0.001893	2 126.9	2 316.2	4.49
0.008089	3 026.8	3 350.4	6.017	600	0.006108	2 947.1	3 252.5	5.8245	600	0.002672	2 597.9	2 865.1	5.1581
0.00993	3 281.9	3 679.1	6.374	700	0.007717	3 228.8	3 614.6	6.2178	700	0.003546	2 976.1	3 330.7	5.6639
0.011521	3 511.8	3 972.6	6.6612	800	0.009072	3 472.2	3 925.8	6.5225	800	0.004336	3 281.7	3 715.3	6.0406
0.01298	3 733.3	4 252.5	6.9106	900	0.010296	3 702	4 216.8	6.7819	900	0.005042	3 551.4	4 055.6	6.344
0.01436	3 952.9	4 527.3	7.1355	1 000	0.011441	3 927.4	4 499.4	7.0131	1 000	0.00569	3 804	4 373	6.6038
0.015686	4 173.7	4 801.1	7.3425	1 100	0.012534	4 152.2	4 778.9	7.2244	1 100	0.006296	4 048.8	4 678.4	6.8347
0.016976	4 396.9	5 075.9	7.5357	1 200	0.01359	4 378.6	5 058.1	7.4207	1 200	0.006873	4 290.3	4 977.6	7.045
0.020709	5 086.2	5 914.6	8.0536	1 500	0.016626	5 074.1	5 905.4	7.944	1 500	0.008491	5 015.3	5 864.4	7.593
0.026705	6 299.9	7 368.1	8.775	2 000	0.021464	6 293	7 366.2	8.6691	2 000	0.010998	6 259.4	7 359.2	8.3352

Steam Table 2: Properties of pure water at its saturation points, sorted by temperature

°C	MPa	$kJ\,kg^{-1}$			$kJ\,kg^{-1}$			$kJ\,K^{-1}\,kg^{-1}$			$m^3\,kg^{-1}$	
$T_{sat.}$	$p_{sat.}$	u_L	u_V	$\Delta u_{L\to V}$	h_L	h_V	$\Delta h_{L\to V}$	s_L	s_V	$\Delta s_{L\to V}$	v_L	v_V
0.01	0.000612	[0]	2 374.9	2 374.9	small	2 500.9	2 500.9	[0]	9.1555	9.1555	0.001	205.991
5	0.000873	21	2 381.8	2 360.8	21	2 510.1	2 489	0.0763	9.0248	8.9486	0.001	147.011
10	0.001228	42	2 388.6	2 346.6	42	2 519.2	2 477.2	0.1511	8.8998	8.7487	0.001	106.303
15	0.001706	63	2 395.5	2 332.5	63	2 528.3	2 465.4	0.2245	8.7803	8.5558	0.001001	77.875
20	0.002339	83.9	2 402.3	2 318.4	83.9	2 537.4	2 453.5	0.2965	8.666	8.3695	0.001002	57.757
25	0.00317	104.8	2 409.1	2 304.3	104.8	2 546.5	2 441.7	0.3672	8.5566	8.1894	0.001003	43.337
30	0.004247	125.7	2 415.9	2 290.1	125.7	2 555.5	2 429.8	0.4368	8.452	8.0152	0.001004	32.878
35	0.005629	146.6	2 422.6	2 276	146.6	2 564.5	2 417.9	0.5051	8.3517	7.8466	0.001006	25.205
40	0.007385	167.5	2 429.4	2 261.9	167.5	2 573.5	2 406	0.5724	8.2555	7.6831	0.001008	19.515
45	0.009595	188.4	2 436.1	2 247.6	188.4	2 582.4	2 394	0.6386	8.1633	7.5247	0.00101	15.252
50	0.012352	209.3	2 442.7	2 233.4	209.3	2 591.3	2 381.9	0.7038	8.0748	7.371	0.001012	12.027
55	0.015762	230.2	2 449.3	2 219.1	230.3	2 600.1	2 369.8	0.768	7.9898	7.2218	0.001015	9.5643
60	0.019946	251.2	2 455.9	2 204.7	251.2	2 608.8	2 357.7	0.8313	7.9081	7.0769	0.001017	7.6672
65	0.025042	272.1	2 462.4	2 190.3	272.1	2 617.5	2 345.4	0.8937	7.8296	6.9359	0.00102	6.1935
70	0.031201	293	2 468.9	2 175.8	293.1	2 626.1	2 333	0.9551	7.754	6.7989	0.001023	5.0395
75	0.038595	314	2 475.2	2 161.3	314	2 634.6	2 320.6	1.0158	7.6812	6.6654	0.001026	4.1289
80	0.047414	335	2 481.5	2 146.6	335	2 643	2 308	1.0756	7.6111	6.5355	0.001029	3.4052
85	0.057867	356	2 487.8	2 131.8	356	2 651.3	2 295.3	1.1346	7.5434	6.4088	0.001032	2.8258
90	0.070182	377	2 493.9	2 117	377	2 659.5	2 282.5	1.1929	7.4781	6.2853	0.001036	2.3591
95	0.084608	398	2 500	2 102	398.1	2 667.6	2 269.5	1.2504	7.4151	6.1647	0.00104	1.9806
100	0.10142	419.1	2 506	2 087	419.2	2 675.6	2 256.4	1.3072	7.3541	6.0469	0.001043	1.6718
105	0.1209	440.1	2 511.9	2 071.8	440.3	2 683.4	2 243.1	1.3633	7.2952	5.9318	0.001047	1.4184
110	0.14338	461.3	2 517.7	2 056.4	461.4	2 691.1	2 229.6	1.4188	7.2381	5.8193	0.001052	1.2093
115	0.16918	482.4	2 523.4	2 041	482.6	2 698.6	2 216	1.4737	7.1828	5.7091	0.001056	1.0358
120	0.19867	503.6	2 528.8	2 025.2	503.8	2 705.9	2 202.1	1.5279	7.1291	5.6012	0.00106	0.89121
125	0.23224	524.8	2 534.3	2 009.4	525.1	2 713.1	2 188	1.5816	7.077	5.4955	0.001065	0.77003
130	0.27028	546.1	2 539.6	1 993.5	546.4	2 720.1	2 173.7	1.6346	7.0264	5.3918	0.00107	0.668
135	0.31323	567.4	2 544.7	1 977.3	567.7	2 726.9	2 159.1	1.6872	6.9772	5.29	0.001075	0.58173
140	0.36154	588.8	2 549.6	1 960.8	589.2	2 733.4	2 144.3	1.7392	6.9293	5.1901	0.00108	0.50845
145	0.41568	610.2	2 554.4	1 944.2	610.6	2 739.8	2 129.2	1.7907	6.8826	5.0919	0.001085	0.44596
150	0.47616	631.7	2 559	1 927.4	632.2	2 745.9	2 113.7	1.8418	6.8371	4.9953	0.001091	0.39245
155	0.5435	653.2	2 563.5	1 910.3	653.8	2 751.8	2 098	1.8924	6.7926	4.9002	0.001096	0.34646
160	0.61823	674.8	2 567.7	1 893	675.5	2 757.4	2 082	1.9426	6.7491	4.8066	0.001102	0.30678
165	0.70093	696.5	2 571.8	1 875.4	697.2	2 762.8	2 065.6	1.9923	6.7066	4.7143	0.001108	0.27243
170	0.79219	718.2	2 575.7	1 857.5	719.1	2 767.9	2 048.8	2.0417	6.665	4.6233	0.001114	0.24259
175	0.8926	740	2 579.4	1 839.4	741	2 772.7	2 031.7	2.0906	6.6241	4.5335	0.001121	0.21658
180	1.0028	761.9	2 582.8	1 820.9	763.1	2 777.2	2 014.2	2.1392	6.584	4.4448	0.001127	0.19384
185	1.1235	783.9	2 586	1 802.1	785.2	2 781.4	1 996.2	2.1875	6.5447	4.3571	0.001134	0.1739
190	1.2552	806	2 589	1 783	807.4	2 785.3	1 977.9	2.2355	6.5059	4.2704	0.001141	0.15636
195	1.3988	828.2	2 591.7	1 763.5	829.8	2 788.8	1 959	2.2832	6.4678	4.1846	0.001149	0.14089
200	1.5549	850.5	2 594.2	1 743.7	852.3	2 792	1 939.7	2.3305	6.4302	4.0996	0.001157	0.12721
205	1.7243	872.9	2 596.4	1 723.5	874.9	2 794.8	1 919.9	2.3777	6.393	4.0154	0.001164	0.11508
210	1.9077	895.4	2 598.3	1 703	897.6	2 797.3	1 899.6	2.4245	6.3563	3.9318	0.001173	0.10429
215	2.1058	918	2 599.9	1 681.9	920.5	2 799.3	1 878.8	2.4712	6.32	3.8488	0.001181	0.094679
220	2.3196	940.8	2 601.2	1 660.4	943.6	2 800.9	1 857.4	2.5177	6.284	3.7663	0.00119	0.086092
225	2.5497	963.7	2 602.2	1 638.5	966.8	2 802.1	1 835.4	2.564	6.2483	3.6843	0.001199	0.078403
230	2.7971	986.8	2 602.9	1 616.1	990.2	2 802.9	1 812.7	2.6101	6.2128	3.6027	0.001209	0.071503
235	3.0625	1 010.1	2 603.2	1 593.2	1 013.8	2 803.2	1 789.4	2.6561	6.1775	3.5214	0.001219	0.065298
240	3.3469	1 033.5	2 603.2	1 569.7	1 037.6	2 803	1 765.4	2.702	6.1423	3.4403	0.001229	0.059705
245	3.6512	1 057	2 602.6	1 545.7	1 061.5	2 802.2	1 740.7	2.7478	6.1072	3.3594	0.00124	0.054654
250	3.9762	1 080.8	2 601.8	1 520.9	1 085.8	2 800.9	1 715.2	2.7935	6.0721	3.2785	0.001252	0.050083

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Steam Table 2 (continued)

$T_{\text{sat.}}$	$p_{\text{sat.}}$	u_L	u_V	$\Delta u_{L\to V}$	h_L	h_V	$\Delta h_{L\to V}$	s_L	s_V	$\Delta s_{L\to V}$	v_L	v_V
255	4.3229	1 104.7	2 600.5	1 495.8	1 110.2	2 799.1	1 688.8	2.8392	6.0369	3.1977	0.001264	0.045938
260	4.6923	1 129	2 598.7	1 469.7	1 135	2 796.6	1 661.6	2.8849	6.0016	3.1167	0.001276	0.042173
265	5.0853	1 153.4	2 596.5	1 443	1 160	2 793.5	1 633.5	2.9307	5.9661	3.0354	0.001289	0.038746
270	5.503	1 178.1	2 593.7	1 415.5	1 185.3	2 789.7	1 604.4	2.9765	5.9304	2.9539	0.001303	0.035621
275	5.9464	1 203.1	2 590.4	1 387.3	1 210.9	2 785.2	1 574.3	3.0224	5.8944	2.872	0.001318	0.032766
280	6.4166	1 228.3	2 586.4	1 358.1	1 236.9	2 779.9	1 543	3.0685	5.8579	2.7894	0.001333	0.030153
285	6.9147	1 253.9	2 581.8	1 327.9	1 263.2	2 773.7	1 510.5	3.1147	5.8209	2.7062	0.001349	0.027756
290	7.4418	1 279.8	2 576.5	1 296.7	1 290	2 766.7	1 476.7	3.1612	5.7834	2.6222	0.001366	0.025555
295	7.9991	1 306.2	2 570.5	1 264.3	1 317.3	2 758.7	1 441.4	3.208	5.7451	2.5371	0.001385	0.023529
300	8.5879	1 332.9	2 563.6	1 230.6	1 345	2 749.6	1 404.6	3.2552	5.7059	2.4507	0.001404	0.02166
305	9.2094	1 360.2	2 555.8	1 195.7	1 373.3	2 739.4	1 366.1	3.3028	5.6657	2.3629	0.001425	0.019933
310	9.8651	1 387.9	2 547	1 159.1	1 402.2	2 727.9	1 325.7	3.351	5.6244	2.2734	0.001448	0.018335
315	10.556	1 416.3	2 537.2	1 121	1 431.8	2 715.1	1 283.2	3.3998	5.5816	2.1818	0.001472	0.016851
320	11.284	1 445.3	2 526	1 080.7	1 462.2	2 700.6	1 238.4	3.4494	5.5372	2.0878	0.001499	0.015471
325	12.051	1 475.1	2 513.4	1 038.3	1 493.5	2 684.3	1 190.8	3.5	5.4908	1.9908	0.001528	0.014183
330	12.858	1 505.8	2 499.1	993.3	1 525.9	2 666	1 140.2	3.5518	5.4422	1.8903	0.001561	0.012979
335	13.707	1 537.6	2 483	945.4	1 559.5	2 645.4	1 085.9	3.605	5.3906	1.7856	0.001597	0.011847
340	14.601	1 570.6	2 464.4	893.8	1 594.5	2 621.8	1 027.3	3.6601	5.3356	1.6755	0.001638	0.010781
345	15.541	1 605.3	2 443.1	837.8	1 631.5	2 594.9	963.4	3.7176	5.2762	1.5586	0.001685	0.009769
350	16.529	1 642.1	2 418.1	776	1 670.9	2 563.6	892.7	3.7784	5.211	1.4326	0.00174	0.008802
355	17.57	1 681.9	2 388.4	706.4	1 713.7	2 526.6	812.9	3.8439	5.138	1.2942	0.001808	0.007868
360	18.666	1 726.3	2 351.8	625.5	1 761.7	2 481.5	719.8	3.9167	5.0536	1.1369	0.001895	0.006949
365	19.821	1 777.8	2 303.7	525.9	1 817.8	2 422.9	605.2	4.0014	4.9497	0.9483	0.002017	0.006012
370	21.044	1 844.1	2 230.2	386.2	1 890.7	2 334.5	443.8	4.1112	4.8012	0.6901	0.002215	0.004954
373	21.814	1 915	2 141.6	226.6	1 969.7	2 229.8	260.1	4.2308	4.6334	0.4026	0.002508	0.004045
$T_{\text{cr.}}$	22.064	2 015.8	2 015.8	0	2 084.3	2 084.3	0	4.407	4.407	0	0.003106	0.003106

Values in brackets are arbitrary references. $T_{\text{cr.}} = 373.946\,^\circ\text{C}$

Steam Table 3: Properties of pure water at its saturation points, sorted by pressure

MPa	°C	$\mathrm{kJ\,kg^{-1}}$			$\mathrm{kJ\,kg^{-1}}$			$\mathrm{kJ\,K^{-1}\,kg^{-1}}$			$\mathrm{m^3\,kg^{-1}}$	
$p_{\text{sat.}}$	$T_{\text{sat.}}$	u_L	u_V	Δu_{LV}	h_L	h_V	Δh_{LV}	s_L	s_V	Δs_{LV}	v_L	v_V
611.657 Pa	0.01	[0]			small	2 500.9	2 500.9	[0]	9.1555	9.1555	0.001	205.991
0.001	6.97	29.3	2 384.5	2 355.2	29.3	2 513.7	2 484.4	0.1059	8.9749	8.869	0.001	129.178
0.002	17.5	73.4	2 398.9	2 325.5	73.4	2 532.9	2 459.4	0.2606	8.7226	8.462	0.001001	66.987
0.003	24.05	100.9	2 407.9	2 307.1	100.9	2 544.8	2 444	0.3539	8.5773	8.2234	0.001003	45.841
0.004	28.96	121.4	2 414.5	2 293.2	121.4	2 553.7	2 432.3	0.4224	8.4734	8.051	0.001004	34.791
0.005	32.87	137.7	2 419.8	2 282	137.8	2 560.7	2 423	0.4762	8.3938	7.9176	0.001005	28.185
0.006	36.16	151.5	2 424.2	2 272.7	151.5	2 566.6	2 415.2	0.5208	8.329	7.8082	0.001006	23.733
0.007	39	163.3	2 428	2 264.7	163.4	2 571.7	2 408.4	0.559	8.2745	7.7154	0.001008	20.524
0.008	41.51	173.8	2 431.4	2 257.6	173.8	2 576.2	2 402.4	0.5925	8.2273	7.6348	0.001008	18.099
0.009	43.76	183.2	2 434.4	2 251.2	183.3	2 580.2	2 397	0.6223	8.1858	7.5635	0.001009	16.199
0.01	45.81	191.8	2 437.2	2 245.4	191.8	2 583.9	2 392.1	0.6492	8.1488	7.4996	0.00101	14.67
0.012	49.42	206.9	2 442	2 235.1	206.9	2 590.3	2 383.4	0.6963	8.0849	7.3887	0.001012	12.358
0.014	52.55	220	2 446.1	2 226.2	220	2 595.8	2 375.8	0.7366	8.0311	7.2945	0.001013	10.691
0.016	55.31	231.6	2 449.7	2 218.2	231.6	2 600.6	2 369.1	0.772	7.9846	7.2126	0.001015	9.4306
0.018	57.8	241.9	2 453	2 211.1	242	2 605	2 363	0.8036	7.9437	7.1402	0.001016	8.4431
0.02	60.06	251.4	2 455.9	2 204.5	251.4	2 608.9	2 357.5	0.832	7.9072	7.0752	0.001017	7.648
0.04	75.86	317.6	2 476.4	2 158.8	317.6	2 636.1	2 318.4	1.0261	7.669	6.6429	0.001026	3.993
0.05	81.32	340.5	2 483.2	2 142.7	340.5	2 645.2	2 304.7	1.0912	7.593	6.5018	0.00103	3.24
0.06	85.93	359.8	2 489	2 129.1	359.9	2 652.9	2 292.9	1.1454	7.5311	6.3857	0.001033	2.7317
0.07	89.93	376.7	2 493.9	2 117.2	376.8	2 659.4	2 282.7	1.1921	7.479	6.2869	0.001036	2.3648
0.08	93.49	391.6	2 498.2	2 106.6	391.7	2 665.2	2 273.5	1.233	7.4339	6.2009	0.001039	2.0871
0.09	96.69	405.1	2 502.1	2 096.9	405.2	2 670.3	2 265.1	1.2696	7.3943	6.1246	0.001041	1.8694
0.1	99.61	417.4	2 505.5	2 088.1	417.5	2 674.9	2 257.4	1.3028	7.3588	6.0561	0.001043	1.6939
0.12	104.78	439.2	2 511.7	2 072.5	439.4	2 683.1	2 243.7	1.3609	7.2977	5.9367	0.001047	1.4284
0.14	109.29	458.3	2 516.9	2 058.6	458.4	2 690	2 231.6	1.411	7.2461	5.8351	0.001051	1.2366
0.16	113.3	475.2	2 521.4	2 046.2	475.4	2 696	2 220.7	1.4551	7.2014	5.7463	0.001054	1.0914
0.18	116.91	490.5	2 525.5	2 034.9	490.7	2 701.4	2 210.7	1.4945	7.1621	5.6676	0.001058	0.97747
0.2	120.21	504.5	2 529.1	2 024.6	504.7	2 706.2	2 201.5	1.5302	7.1269	5.5967	0.001061	0.88568
0.25	127.41	535.1	2 536.8	2 001.8	535.3	2 716.5	2 181.1	1.6072	7.0524	5.4452	0.001067	0.71866
0.3	133.52	561.1	2 543.2	1 982.1	561.4	2 724.9	2 163.5	1.6717	6.9916	5.3199	0.001073	0.60576
0.35	138.86	583.9	2 548.5	1 964.7	584.3	2 732	2 147.7	1.7274	6.9401	5.2128	0.001079	0.52418
0.4	143.61	604.2	2 553.1	1 948.9	604.7	2 738.1	2 133.4	1.7765	6.8955	5.119	0.001084	0.46238
0.5	151.83	639.5	2 560.7	1 921.2	640.1	2 748.1	2 108	1.8604	6.8207	4.9603	0.001093	0.37481
0.6	158.83	669.7	2 566.8	1 897	670.4	2 756.1	2 085.8	1.9308	6.7592	4.8284	0.001101	0.31558
0.7	164.95	696.2	2 571.9	1 875.6	697	2 762.8	2 065.8	1.9918	6.7071	4.7153	0.001108	0.27277
0.8	170.41	720	2 576	1 856.1	720.9	2 768.3	2 047.4	2.0457	6.6616	4.616	0.001115	0.24034
0.9	175.35	741.6	2 579.6	1 838	742.6	2 773	2 030.5	2.094	6.6213	4.5272	0.001121	0.21489
1	179.88	761.4	2 582.7	1 821.3	762.5	2 777.1	2 014.6	2.1381	6.585	4.447	0.001127	0.19436
1.1	184.06	779.8	2 585.4	1 805.6	781	2 780.6	1 999.6	2.1785	6.552	4.3735	0.001133	0.17745
1.2	187.96	797	2 587.8	1 790.8	798.3	2 783.7	1 985.4	2.2159	6.5217	4.3058	0.001139	0.16326
1.3	191.61	813.1	2 590	1 776.8	814.6	2 786.5	1 971.9	2.2508	6.4936	4.2428	0.001144	0.15119
1.4	195.04	828.4	2 591.7	1 763.3	830	2 788.8	1 958.9	2.2835	6.4675	4.1839	0.001149	0.14078
1.5	198.29	842.8	2 593.4	1 750.6	844.6	2 791	1 946.4	2.3143	6.443	4.1286	0.001154	0.13171
1.6	201.37	856.6	2 594.8	1 738.2	858.5	2 792.8	1 934.4	2.3435	6.4199	4.0765	0.001159	0.12374
1.7	204.31	869.8	2 596.2	1 726.4	871.7	2 794.5	1 922.7	2.3711	6.3981	4.027	0.001163	0.11667
1.8	207.11	882.4	2 597.2	1 714.9	884.5	2 795.9	1 911.4	2.3975	6.3775	3.98	0.001168	0.11037
1.9	209.8	894.5	2 598.3	1 703.8	896.7	2 797.2	1 900.5	2.4227	6.3578	3.9351	0.001172	0.1047
2	212.38	906.1	2 599.1	1 693	908.5	2 798.3	1 889.8	2.4468	6.339	3.8923	0.001177	0.099585
2.2	217.25	928.3	2 600.6	1 672.3	930.9	2 800.1	1 869.2	2.4921	6.3038	3.8116	0.001185	0.090698
2.4	221.79	949	2 601.6	1 652.6	951.9	2 801.4	1 849.6	2.5343	6.2712	3.7369	0.001193	0.083244
2.6	226.05	968.5	2 602.4	1 633.8	971.7	2 802.3	1 830.7	2.5736	6.2409	3.6672	0.001201	0.076899

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Steam Table 3 (continued)

$p_{sat.}$	$T_{sat.}$	u_L	u_V	$\Delta u_{L\to V}$	h_L	h_V	$\Delta h_{L\to V}$	s_L	s_V	$\Delta s_{L\to V}$	v_L	v_V
2.8	230.06	987.1	2 602.9	1 615.8	990.5	2 802.9	1 812.4	2.6106	6.2124	3.6018	0.001209	0.071429
3	233.85	1 004.6	2 603.2	1 598.6	1 008.3	2 803.2	1 794.8	2.6455	6.1856	3.54	0.001217	0.066664
3.2	237.46	1 021.5	2 603.2	1 581.7	1 025.4	2 803.1	1 777.7	2.6787	6.1602	3.4815	0.001224	0.062475
3.4	240.9	1 037.6	2 603.1	1 565.5	1 041.8	2 802.9	1 761	2.7102	6.136	3.4258	0.001231	0.058761
3.6	244.18	1 053.1	2 602.8	1 549.7	1 057.6	2 802.4	1 744.8	2.7403	6.1129	3.3726	0.001239	0.055446
3.8	247.33	1 068.1	2 602.3	1 534.3	1 072.8	2 801.7	1 728.9	2.7691	6.0908	3.3217	0.001246	0.052467
4	250.35	1 082.5	2 601.7	1 519.2	1 087.5	2 800.8	1 713.3	2.7968	6.0696	3.2728	0.001253	0.049776
4.2	253.26	1 096.4	2 601	1 504.6	1 101.7	2 799.8	1 698.1	2.8234	6.0491	3.2257	0.001259	0.047332
4.4	256.07	1 109.9	2 600.2	1 490.2	1 115.5	2 798.6	1 683.1	2.849	6.0293	3.1803	0.001266	0.045102
4.6	258.78	1 123	2 599.2	1 476.2	1 128.9	2 797.3	1 668.4	2.8738	6.0102	3.1364	0.001273	0.043059
4.8	261.4	1 135.8	2 598.1	1 462.4	1 141.9	2 795.8	1 653.9	2.8978	5.9917	3.0939	0.00128	0.04118
5	263.94	1 148.2	2 597	1 448.8	1 154.6	2 794.2	1 639.6	2.921	5.9737	3.0527	0.001286	0.039446
5.5	269.97	1 177.9	2 593.7	1 415.7	1 185.1	2 789.7	1 604.6	2.9762	5.9307	2.9545	0.001303	0.035642
6	275.59	1 206	2 589.9	1 383.9	1 213.9	2 784.6	1 570.7	3.0278	5.8901	2.8623	0.001319	0.032448
6.5	280.86	1 232.7	2 585.7	1 353	1 241.4	2 778.9	1 537.5	3.0764	5.8516	2.7752	0.001336	0.029727
7	285.83	1 258.2	2 581	1 322.7	1 267.7	2 772.6	1 505	3.1224	5.8148	2.6924	0.001352	0.027378
7.5	290.54	1 282.6	2 575.9	1 293.3	1 292.9	2 765.9	1 473	3.1662	5.7793	2.6131	0.001368	0.02533
8	295.01	1 306.2	2 570.5	1 264.3	1 317.3	2 758.7	1 441.4	3.2081	5.745	2.5369	0.001385	0.023526
8.5	299.27	1 329	2 564.7	1 235.7	1 340.9	2 751	1 410.1	3.2483	5.7117	2.4634	0.001401	0.021923
9	303.35	1 351.1	2 558.5	1 207.4	1 363.9	2 742.9	1 379.1	3.287	5.6791	2.3922	0.001418	0.02049
9.5	307.25	1 372.6	2 552	1 179.4	1 386.2	2 734.4	1 348.2	3.3244	5.6473	2.3229	0.001435	0.019199
10	311	1 393.6	2 545.2	1 151.6	1 408.1	2 725.5	1 317.4	3.3606	5.616	2.2553	0.001453	0.01803
11	318.08	1 434	2 530.4	1 096.4	1 450.4	2 706.3	1 255.9	3.4303	5.5545	2.1242	0.001489	0.01599
12	324.68	1 473.2	2 514.2	1 041	1 491.5	2 685.4	1 194	3.4967	5.4939	1.9972	0.001526	0.014264
13	330.85	1 511.1	2 496.6	985.4	1 531.5	2 662.7	1 131.2	3.5608	5.4336	1.8728	0.001566	0.01278
14	336.67	1 548.5	2 477.1	928.6	1 571	2 637.9	1 066.9	3.6232	5.3727	1.7495	0.00161	0.011485
15	342.16	1 585.3	2 455.6	870.3	1 610.2	2 610.7	1 000.5	3.6846	5.3106	1.626	0.001657	0.010338
16	347.36	1 622.3	2 431.9	809.5	1 649.7	2 580.8	931.1	3.7457	5.2463	1.5006	0.001709	0.009309
17	352.29	1 659.9	2 405.2	745.3	1 690	2 547.5	857.5	3.8077	5.1787	1.371	0.001769	0.008371
18	356.99	1 699	2 374.8	675.8	1 732.1	2 509.8	777.7	3.8718	5.1061	1.2342	0.00184	0.007502
19	361.47	1 740.6	2 339.1	598.5	1 777.2	2 466	688.9	3.9401	5.0256	1.0855	0.001927	0.006677
20	365.75	1 786.4	2 295	508.6	1 827.2	2 412.3	585.1	4.0156	4.9314	0.9158	0.00204	0.005865
21	369.83	1 841.3	2 233.7	392.4	1 887.6	2 338.6	451	4.1064	4.8079	0.7015	0.002206	0.004996
22	373.71	1 951.8	2 092.9	141.1	2 011.3	2 173.1	161.7	4.2945	4.5446	0.2501	0.002704	0.003648
$p_{cr.}$	373.95	2 015.8	2 015.8	0	2 084.3	2 084.3	0	4.407	4.407	0	0.003106	0.003106

Values in brackets are arbitrary references. $p_{cr.} = 22.064$ MPa

A2 Gauge Pressure and Real Pressure

It is necessary to distinguish the *real* pressure p_{real} or simply p of a fluid, which we use for our calculations throughout this book, from the *gauge* pressure often indicated by barometers and manometers.

Indeed, it is common for a pressure instrument to be calibrated to atmospheric pressure. For example, when inflating a car tire at a gas station, the dial reads 0 bar at ambient pressure – all of the values that it displays will be offset by the value of the atmospheric pressure at that time. The value indicated by such a manometer is called *gauge pressure*, denoted by p_g and defined as follows:

$$p_{\text{g}} \equiv p_{\text{real}} - p_{\text{atm.}} \tag{A2/1}$$

where p_{g} is gauge pressure, shown by the device (Pa),
p_{real} is the real pressure where the measurement is made, generally noted p (Pa),
and $p_{\text{atm.}}$ is the ambient atmospheric pressure (Pa).

A pressure gauge left exposed to the atmosphere will therefore read 0 bar regardless of ambient pressure. The pressure indicated during a measurement will depend on the ambient atmospheric pressure; it can be positive (for example, in a car tire) or sometimes negative (for example, in a water or oil pipeline).

Gauge pressure is interesting because it indicates the difference in pressure between each side of the reservoir walls (tire, pipeline); it is therefore indicative of the stresses they undergo.

However, it is the actual, real pressure that we need to predict the state of fluids. In our thermodynamic calculations, we always use real pressure, simply noted p.

A3 Additive Quantities

Quantities in thermodynamics can be classified into the following categories:

- A quantity is *additive* or *extensive* if its value depends on the size of the system. For example, if an object is split into two parts, its mass and volume (additive quantities) are also split. The additive quantities used in this book are E, H, m, S, U, and V (see the list of symbols on page 329).
- A quantity is *non-additive* or *intensive* if its value does not depend on the size of the system. For example, if an object is split into two parts, its temperature remains unchanged. Specific (or mass-related) quantities are all non-additive.
  The non-additive quantities used in this book are C, c, e, h, p, s, T, u, v, x, z, γ, and ρ (see the list of symbols on page 329).
- Some quantities are neither additive nor non-additive; so is, for example, the area of the surface surrounding a system.

A4 State Quantities and Process Quantities

Definition

Thermodynamic quantities can be classified into the following categories:

- A quantity is called a *state quantity* if its value depends only on the current state of the system. Temperature T is an example of a state quantity.
- A quantity is called a *path quantity* or if its value depends on the path taken. Heat transfer Q and work W are the only two path quantities used in this book.

Using the example provided by Rogers & Mayhew [37], one can illustrate the distinction as follows: a cyclist travels from A to B. Their altitude at the start z_{A} and at the end z_{B} are state quantities (z being a *state function*), and we can quantify $\Delta z \equiv z_{\mathrm{B}} - z_{\mathrm{A}}$ without knowing anything about the route. However, the work $W_{\mathrm{A}\to\mathrm{B}}$ expended to go from A to B depends on the process: it will be larger, for example, if the route is longer or if there is wind. Quantifying W (a *path function*) requires knowledge of all intermediate states between A and B.

Quantities are sometimes referred to as *variables*; state quantities are sometimes referred to as *physical properties*. Process quantities and functions are sometimes also called *transfer* or *path* quantities.

Notation

Infinitesimal changes in state quantities are denoted by the symbol d; these are *exact differentials* and can be integrated by only knowing their initial and final values. For example, for temperature T:

$$\int_{\mathrm{A}}^{\mathrm{B}} \mathrm{d}T = \Delta T = T_{\mathrm{B}} - T_{\mathrm{A}} \tag{A4/1}$$

Infinitesimal transfers of path quantities are denoted by the symbol δ; these are *inexact differentials* and their integral can only be quantified by knowing all states encountered along the path. For example, for work W, one cannot write “$W_{\mathrm{B}} - W_{\mathrm{A}}$”, or “$\Delta W$”, but only:

$$\int_{\mathrm{A}}^{\mathrm{B}} \delta W = W_{\mathrm{A}\to\mathrm{B}} \tag{A4/2}$$

This notation can be confusing, since it is d and not δ that becomes Δ upon integration. It may also appear as a complicated way to avoid using partial derivatives. Clifford Truesdell [36] mischievously remarks that because of this notation, equation 8/1 on page 213 might make us believe that some types of differentials are larger than others… Although this notation is widely used in French literature (which is why it is adopted in this book, which is translated from French [51]), it must be recognized that one can fully cover the field of engineering thermodynamics [37, 38] or physical thermodynamics and its history [36] without ever having to use the symbol δ in the above sense.

A5 Conversion of Units to SI

The following relationships can be used to convert customary physical units into SI units. The conversion factors here are rounded to seven significant digits.

- The inch:

$$l(\mathrm{m}) = 39.370\,08\; l(\mathrm{in}) \tag{A5/1}$$

$$l(\mathrm{in}) = 2.54 \times 10^{-2}\; l(\mathrm{m}) \tag{A5/2}$$

- The foot:

$$l(\mathrm{m}) = 3.280\,84\; l(\mathrm{ft}) \tag{A5/3}$$

$$l(\mathrm{ft}) = 0.3048\; l(\mathrm{m}) \tag{A5/4}$$

- The mile:

$$l(\mathrm{m}) = 6.213\,712 \times 10^{-4}\; l(\mathrm{mi}) \tag{A5/5}$$

$$l(\mathrm{mi}) = 1.609\,344 \times 10^{3}\; l(\mathrm{m}) \tag{A5/6}$$

- The mile per hour:

$$C(\mathrm{m\,s^{-1}}) = 0.447\,04\; C(\mathrm{mph}) \tag{A5/7}$$

$$C(\mathrm{km/h}) = 1.609\,344\; C(\mathrm{mph}) \tag{A5/8}$$

$$C(\mathrm{mph}) = 0.621\,371\; C(\mathrm{km/h}) \tag{A5/9}$$

$$C(\mathrm{mph}) = 2.236\,936\; C(\mathrm{m\,s^{-1}}) \tag{A5/10}$$

- The US gallon:

$$V(\mathrm{US\ gal}) = 2.641\,721 \times 10^{2}\; V(\mathrm{m^3}) \tag{A5/11}$$

$$V(\mathrm{m^3}) = 3.785\,412 \times 10^{-3}\; V(\mathrm{US\ gal}) \tag{A5/12}$$

- The imperial gallon:

$$V(\mathrm{imp\ gal}) = 2.199\,692 \times 10^{2}\; V(\mathrm{m^3}) \tag{A5/13}$$

$$V(\mathrm{m^3}) = 4.546\,09 \times 10^{-3}\; V(\mathrm{imp\ gal}) \tag{A5/14}$$

- The tonne or metric ton:

$$m(\mathrm{t}) = 10^{-3}\; m(\mathrm{kg}) \tag{A5/15}$$

$$m(\mathrm{kg}) = 10^{3}\; m(\mathrm{t}) \tag{A5/16}$$

- the pound:

$$m(\mathrm{lb}) = 2.204\,623\; m(\mathrm{kg}) \tag{A5/17}$$

$$m(\mathrm{kg}) = 0.453\,592\,37\; m(\mathrm{lb}) \tag{A5/18}$$

- the pound – force:

$$F(\text{lbf}) = 0.224\,808\,9\;F(\text{N}) \tag{A5/19}$$

$$F(\text{N}) = 4.448\,222\;F(\text{lbf}) \tag{A5/20}$$

- the inch of mercury:

$$p(\text{inHg}) = 2.067\,099 \times 10^{-6}\;p(\text{Pa}) \tag{A5/21}$$

$$p(\text{Pa}) = 3.386\,389 \times 10^{3}\;p(\text{inHg}) \tag{A5/22}$$

- the bar:

$$p(\text{bar}) = 10^{-5}\;p(\text{Pa}) \tag{A5/23}$$

$$p(\text{Pa}) = 10^{5}\;p(\text{bar}) \tag{A5/24}$$

- the pound per square inch:

$$p(\text{psi}) = 1.450\,377 \times 10^{-4}\;p(\text{Pa}) \tag{A5/25}$$

$$p(\text{psi}) = 14.503\,77\;p(\text{bar}) \tag{A5/26}$$

$$p(\text{Pa}) = 6.894\,757 \times 10^{3}\;p(\text{psi}) \tag{A5/27}$$

$$p(\text{bar}) = 6.894\,757 \times 10^{-2}\;p(\text{psi}) \tag{A5/28}$$

- the kilogram – force per square centimeter (or kilopond per square centimeter):

$$p(\text{kg}_\text{f}/\text{cm}^2) = 1.019\,716 \times 10^{-5}\;p(\text{Pa}) \tag{A5/29}$$

$$p(\text{kg}_\text{f}/\text{cm}^2) = 1.019\,716\;p(\text{bar}) \tag{A5/30}$$

$$p(\text{Pa}) = 9.806\,65 \times 10^{4}\;p(\text{kg}_\text{f}/\text{cm}^2) \tag{A5/31}$$

$$p(\text{bar}) = 0.980\,665\;p(\text{kg}_\text{f}/\text{cm}^2) \tag{A5/32}$$

- the degree Fahrenheit:

$$T(^\circ\text{F}) = 1.8 \times [T(\text{K}) - 273.15] + 32 \tag{A5/33}$$

$$T(\text{K}) = \frac{T(^\circ\text{F}) - 32}{1.8} + 273.15 \tag{A5/34}$$

- the imperial horsepower:

$$\dot{W}(\text{hp imp}) = 1.341\,022 \times 10^{-3}\;\dot{W}(\text{W}) \tag{A5/35}$$

$$\dot{W}(\text{W}) = 7.456\,999 \times 10^{2}\;\dot{W}(\text{hp imp}) \tag{A5/36}$$

- the metric horsepower (DIN 66036):

$$\dot{W}(\text{hp}) = 1.359\,622 \times 10^{-3}\;\dot{W}(\text{W}) \tag{A5/37}$$

$$\dot{W}(\text{W}) = 7.354\,988 \times 10^{2}\;\dot{W}(\text{hp}) \tag{A5/38}$$

A6 Notation

The graphical and sign conventions are described in sections §2.2.2 p. 34, §3.2.1 p. 61, and §6.1 p. 153.

$\equiv$	"By definition". The symbol $\equiv$ sets the definition of the term on its left (which does not depend on previous equations).
$\dot{}$	(dot above a symbol). Time rate: $\dot{} \equiv \frac{\mathrm{d}}{\mathrm{d}t}$. For example, $\dot{Q}$ is the power as heat (in watts) representing an amount Q (in joules) per second.
Δ	net difference between two state quantities: $(\Delta X)_{\mathrm{A}\to\mathrm{B}} = X_{\mathrm{B}} - X_{\mathrm{A}}$. It can be negative. See the Appendix A4 p. 318.
δ	inexact differential. See Appendix A4 p. 318.
d	exact differential. See Appendix A4 p. 318.

italics Physical quantities (mass m, temperature T). The symbols are all listed on page 329.

indices

- In upright characters: points in time or in space (temperature T_{A} at point A). The index "cst." denotes a quantity that remains constant, the index "rev." indicates that the calculation is carried out along a reversible process, "in" denotes "incoming" and "out" denotes "outgoing".
- In italics: T_H, T_L, and the indices TH and TL denote respectively a high and low temperature, as detailed in section §7.2.2 p. 181. The indices L and V indicate the saturation points of a liquid-vapor mixture, as detailed in section §5.3.2 p. 128.

lowercase Specific values (energy or power). See section §1.1.5 p. 14.

operators Exponential $\exp x \equiv e^x$, natural logarithm $\ln x \equiv \log_e x$;

units Units are in upright characters and colored in gray (1 kg). In sentences, units are written out in full and in lowercase (one hundred watts). The liter is noted as L to make it more readable ($1\,\mathrm{L} \equiv 10^{-3}\,\mathrm{m}^3$). The units in equations are from the International System of Units (SI) [50] unless otherwise indicated, and non-SI units are listed in Appendix A5 p. 319.

numbers Numbers are written according to the recommendations of the BIPM in the 9th edition of the *SI Brochure* [50]. The decimal separator is a dot, digits are grouped by three ($12.3456 \times 10^3 = 12\,345.6$). When only four digits are present, grouping is omitted. In tables, integers are always grouped, and decimals never grouped, to increase readability. Rounding is done as late as possible and never in series; leading and trailing zeros are not indicated.

A7 Errata & Change Log

Any future error corrections and improvements will be listed here.

The most recent version of this book, its full change log, and a contact form to report any remaining errors to the author can all be found on the book's website, https://thermodynamicsbook.com/. The author can also be reached by email at olivier@cleynen.fr.

A8 Contributors

Olivier Cleynen wrote the book, starting with the French original [51] first published in 2015, and created most diagrams. His deepest thanks go to the following individuals for their contributions:

- Philippe Depondt (Pierre and Marie Curie University, Paris) wrote four historical sections (§1.6, §3.6, §4.5, §8.6);
- Nicolas Horny (Grespi, Université de Reims) reviewed the technical and scientific aspects of the first edition of the book;
- Christophe Masutti and Mireille Bernex (Framabook/Framasoft) contributed significantly to the finalization of the first edition of the book;
- Many persons, by correcting errors or suggesting improvements, have kindly reduced the entropy of this book, among whom: Antoine L., Hamassala David Dicko, Kévin R., Florianne B., Julien D., Anthony Jouny, Thomas N., Amazigh.L.H, Victor D., Daniel C.-N., Pierrick Degardin, Arthur A., Ulrick M., Solène J., Florian Paupert, Gatien Bovyn, Mehdi Z., Jean-Bernard Marcon, Luc Benoit, Thibault Mattera, Arnaud Gallant, Olivier Kim Hak, Gwen Messé, Laurent Prost, Christophe Valerio, Bruno Turgeon, Saou Ibrahim, Jean Claude Cailton, Philippe Godard, Robert Tomczak, Dimitrios Anagnostou, Sallah Azzdine, Selim Fikri, and Frédéric Novel.
- Many creators, by publishing their photos and diagrams under a free license, have helped illustrate the book, although they are not associated with it.

Any remaining errors in this book are Olivier Cleynen's responsibility. You can send your comments, feedback, critiques, and other contributions, which are always highly appreciated, to the contact form on the book's website, https://thermodynamicsbook.com/, or by email at olivier@cleynen.fr.

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  author=     {Cleynen, Olivier},
  title=      {Engineering Thermodynamics},
  subtitle=   {International edition with SI units},
  language=   {english},
  publisher=  {Olivier Cleynen / Thermodynamicsbook.com},
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Bibliography

This book was originally written in French [51] and first published in 2015, so the bibliography naturally reflects this. The author's reading recommendations are:

To study engineering thermodynamics:

- Çengel, Boles & Lacroix 2007 [48, 49], provide accessible, comprehensive, and solid coverage of the topics addressed here;
- Eastop & McConkey 1993 [38], an essential reference (itself based on the very robust Rogers & Mayhew 1992 [37]);

To explore thermodynamics in physics:

- Depondt 2001 [43], a playful exploration of our favorite discipline (Philippe Depondt wrote the sections §1.6, §3.6, §4.5, and §8.6 of this manual) ;
- Feynman 1963 [30, 35] addresses thermodynamics several times in his reference introductory physics book, in particular with a beautiful exploration of the concept of irreversibility.

For a short time travel:

- Gay-Lussac 1807 [3] and Joule 1845 [8] for a glimpse of the meticulous work that laid the foundations of what is now called the *first law*;
- Carnot 1824 [4], the stunningly modern booklet written by the first thermodynamics engineer in history;
- Clausius 1854 [13] to witness the birth of the term "entropy" and see its father's self-confidence.

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List of Symbols

	Name (unit)	First appearance
A	Surface area ($\mathrm{m^2}$)	eq. 2/5 p. 37
C	Velocity ($\mathrm{m\,s^{-1}}$)	eq. 1/5 p. 15
COP	Coefficient of Performance: efficiency η of a heat pump or refrigeration system (unitless)	eq. 6/6 p. 161
c	Thermal capacity ($\mathrm{J\,kg^{-1}\,K^{-1}}$)	eq. 1/16 p. 21
c_p	Specific heat capacity at constant pressure ($\mathrm{J\,kg^{-1}\,K^{-1}}$)	§4.2.1 p. 87
c_v	Specific heat capacity at constant volume ($\mathrm{J\,kg^{-1}\,K^{-1}}$)	§4.2.1 p. 87
d	Exact differential (mathematical operator)	§A4 p. 318
E	Energy (J)	eq. 1/3 p. 14
$\dot{E}$	Power (W)	eq. 1/6 p. 15
E_k	Kinetic energy (J)	eq. 1/5 p. 15
E_m	Mechanical energy (J)	eq. 1/9 p. 15
E_p	Potential energy (J)	eq. 1/7 p. 15
e	Specific energy ($\mathrm{J\,kg^{-1}}$) and specific power ($\mathrm{J\,kg^{-1}}$)	eq. 1/3 p. 14
e_k	Specific kinetic energy ($\mathrm{J\,kg^{-1}}$)	eq. 1/6 p. 15
e_m	Specific mechanical energy ($\mathrm{J\,kg^{-1}}$)	eq. 1/9 p. 15
e_p	Specific potential energy ($\mathrm{J\,kg^{-1}}$)	eq. 1/7 p. 15
F	Force (N)	eq. 1/10 p. 16
g	Gravitational acceleration ($\mathrm{m\,s^{-2}}$)	eq. 1/8 p. 15
H	Enthalpy (J)	eq. 3/13 p. 65
h	Specific enthalpy ($\mathrm{J\,kg^{-1}}$)	eq. 3/12 p. 65
h_L	Specific enthalpy of saturated liquid ($\mathrm{J\,kg^{-1}}$)	tab. 5.3 p. 128
h_V	Specific enthalpy of saturated vapor ($\mathrm{J\,kg^{-1}}$)	tab. 5.3 p. 128
h_{LV}	Specific enthalpy of vaporization: $h_{LV} \equiv \Delta h_{L \rightarrow V} \equiv h_V - h_L$ ($\mathrm{J\,kg^{-1}}$)	tab. 5.3 p. 128
h_x	Specific enthalpy of a liquid-vapor mixture with dryness fraction x ($\mathrm{J\,kg^{-1}}$)	eq. 5/4 p. 130
k	Any constant (units in SI as needed)	
l	Displacement (m)	eq. 1/10 p. 16
M_w	Work ratio (unitless)	eq. 10/1 p. 272
m	Mass (kg)	eq. 1/3 p. 14
$\dot{m}$	Mass flow rate ($\mathrm{kg\,s^{-1}}$)	eq. 1/4 p. 14
p	Pressure (Pa)	eq. 2/5 p. 37
$p_{\text{cr.}}$	Critical pressure (Pa)	§5.2.1 p. 120
$p_{\text{sat.}}$	Saturation pressure (Pa)	§5.3.2 p. 128
Q	Heat (J)	§1.1.3 p. 12
$\dot{Q}$	Power as heat (W)	ex. 1.2 p. 14

q	Specific heat ($\mathrm{J\,kg^{-1}}$) and specific power as heat ($\mathrm{J\,kg^{-1}}$)	ex. 1.1 p. 14
R	Ideal gas constant ($\mathrm{J\,K^{-1}\,kg^{-1}}$)	eq. 4/1 p. 83
S	Entropy ($\mathrm{J\,K^{-1}}$)	eq. 8/1 p. 213 & eq. 8/2 p. 213
s	Specific entropy ($\mathrm{J\,K^{-1}\,kg^{-1}}$)	§5.3.1 p. 125
s_L	Specific entropy of saturated liquid ($\mathrm{J\,K^{-1}\,kg^{-1}}$)	eq. 8/14 p. 223
s_V	Specific entropy of saturated vapor ($\mathrm{J\,K^{-1}\,kg^{-1}}$)	eq. 8/14 p. 223
s_{LV}	Specific entropy of vaporization: $s_{LV} \equiv \Delta s_{L\to V} \equiv s_V - s_L$ ($\mathrm{J\,K^{-1}\,kg^{-1}}$)	eq. 8/14 p. 223
s_x	Specific entropy of a liquid-vapor mixture with dryness fraction x ($\mathrm{J\,K^{-1}\,kg^{-1}}$)	eq. 8/14 p. 223
ssc	Specific Steam Consumption ($\mathrm{kg\,J^{-1}}$)	eq. 9/2 p. 245
T	Temperature (K)	§1.4.1 p. 19
$T_{\text{cr.}}$	Critical temperature (K)	§5.2.1 p. 120
$T_{\text{sat.}}$	Saturation temperature (K)	§5.3.2 p. 128
t	Time (s)	ex. 1.7 p. 21
U	Internal energy (J)	§1.1.3 p. 12
$\dot{U}$	Rate of increase of internal energy (W)	eq. 3/6 p. 63
u	Specific internal energy ($\mathrm{J\,kg^{-1}}$)	eq. 2/2 p. 35
u_L	Specific internal energy of saturated liquid ($\mathrm{J\,kg^{-1}}$)	tab. 5.3 p. 128
u_V	Specific internal energy of saturated vapor ($\mathrm{J\,kg^{-1}}$)	tab. 5.3 p. 128
u_{LV}	Specific internal energy of vaporization: $u_{LV} \equiv \Delta u_{L\to V} \equiv u_V - u_L$ ($\mathrm{J\,kg^{-1}}$)	tab. 5.3 p. 128
u_x	Specific internal energy of a liquid-vapor mixture with dryness fraction x ($\mathrm{J\,kg^{-1}}$)	eq. 5/5 p. 131
V	Volume ($\mathrm{m^3}$)	eq. 2/10 p. 37
$\dot{V}$	Volumetric flow rate ($\mathrm{m^3\,s^{-1}}$)	eq. 3/3 p. 63
v	Specific volume ($\mathrm{m^3\,kg^{-1}}$)	eq. 2/15 p. 40
v_L	Specific volume of saturated liquid ($\mathrm{m^3\,kg^{-1}}$)	tab. 5.3 p. 128
v_V	Specific volume of saturated vapor ($\mathrm{m^3\,kg^{-1}}$)	tab. 5.3 p. 128
v_x	Specific volume of a liquid-vapor mixture with dryness fraction x ($\mathrm{m^3\,kg^{-1}}$)	eq. 5/6 p. 131
W	Work (J)	§1.1.3 p. 12
$\dot{W}$	Power as work (W)	§1.1.4 p. 13
w	Specific work ($\mathrm{J\,kg^{-1}}$) and specific power as work ($\mathrm{J\,kg^{-1}}$)	eq. 2/2 p. 35
x	Dryness fraction of a liquid-vapor mixture (unitless)	§5.3.3 p. 130
z	Altitude (m)	eq. 1/7 p. 15
γ	Ratio of heat capacities (unitless)	eq. 4/9 p. 89
Δ	Net difference between two values (mathematical operator)	§A4 p. 318
δ	Inexact differential (mathematical operator)	§A4 p. 318

ϵ	Compression ratio (unitless)	eq. 10/3 p. 276
η	Efficiency of a thermodynamic machine (unitless)	eq. 6/3 p. 160
η_{T}	Isentropic efficiency of a turbine (unitless)	eq. 9/6 p. 249
η_{C}	Isentropic efficiency of a compressor (unitless)	eq. 10/5 p. 281
λ	Number of configurations (unitless)	eq. 8/17 p. 232
ρ	Density, $\rho \equiv 1/v = m/V$ ($\mathrm{kg\,m^{-3}}$)	

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« We have studied this subject with all the interest, we might say, with all the ardor that it awakened in us. For what a marvelous tableau, indeed, is this triumph of human intelligence! What an imposing spectacle is a locomotive moving forward without apparent effort, drawing behind it a train of 40 or 50 loaded carriages, each weighing ten thousand pounds! What, henceforth, are the heaviest burdens, when machines exist that can set such prodigious weights in motion; what are distances, when engines can traverse daily an interval of 12 leagues in 1½ of an hour? The very ground seems as it were to vanish beneath your eyes; trees, houses, mountains are swept away behind you with the swiftness of a stroke; and when you cross another train, with a relative speed of 15 to 20 leagues per hour, you see it in an instant appear, grow, and reach you; and scarcely have you seen it flash by with a shudder, before it is already borne far from you, reduced to a point, and vanished once more into the distance. »

François-Marie Guyonneau de Pambour
Traité théorique et pratique des machines locomotives [6], 1835