Adult Basic Education
Advanced Level
MATHEMATICS

Financial Mathematics

BRITISH COLUMBIA
Ministry of Advanced Education

CENTRE FOR CURRICULUM,
TRANSFER & TECHNOLOGY

Adult Basic Education
Advanced Level Mathematics

Financial Mathematics

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Province of British Columbia
Ministry of Advanced Education
and the
Centre for Curriculum, Transfer and Technology
2001

Victoria, B.C.

Contents

Learning outcomes

We are often faced with questions concerning our finances: Am I saving enough to put my daughter through college? How much will my investment be worth five years from now? Should I pay down my mortgage with weekly or monthly payments? The mathematics of finance allows us to answer these and similar questions.

Upon completion of this Module, you should be able to:

- use the simple interest formula to calculate the interest, the principal, the annual interest rate, or the time
- explain the difference between simple and compound interest
- find the compound amount using the compound interest formula
- calculate the present value of a compound amount using a formula
- calculate the effective interest rate using a formula
- calculate ordinary annuities using a formula
- calculate annuity payments using a formula
- calculate loan payments and mortgage payments using a formula
- use tables to determine the annual interest rate on a loan

Procedure for independent study

1. Read each of the units in order and complete all of the exercises. If you need assistance, contact your instructor.

2. Study the terminology in the Glossary to become familiar with the definitions.

3. If recommended by your instructor, complete additional problem sets.

4. Complete the Review Questions.

Glossary

Annual interest rate

The interest rate over a one year period or interest rate per annum.

Annuity

A series of equal payments at regular intervals.

Annum

Referring to one year.

Balance

The amount remaining after a sequence of transactions.

Bond

A note to pay the holder both the principal and interest due on the maturity date.

Canada Savings Bond (abbreviated C.S.B.)

A bond issued by the government of Canada.

Capital gain

The amount by which an investment increases over a period of time.

Capital loss

The amount by which an investment decreases over a period of time.

Compounding

The process of periodically calculating the interest earned on the principal and adding this amount to the principal.

Compound interest

The amount of interest earned after calculating the interest on the previous principal plus interest earned.

Compounding period

The time period between successive calculations and conversions of interest to principal.

Dividend

A payment, usually quarterly, made to the owners of certain stocks.

Effective interest rate

The interest rate which reflects the total amount of interest earned on an investment in one year.

Finance charge

The total interest paid for a loan or mortgage. The sum of the periodic payments minus the amount of the loan or mortgage.

Gross income

The amount of money earned before deductions.

Interest

The fee lenders charge to borrowers for the temporary use of money.

Interest rate

The percentage charged or earned for the use of money per year.

Loan

Money lent for a fixed period of time.

Maturity date

The date on which the loan (principal plus interest) is due, or the date on which an investment (principal plus interest) is payable.

Mortgage

Loans made for the purchase of homes and other real estate property.

Mutual fund

An investment fund composed of a selection of various stocks and bonds.

Net income

The amount remaining after deductions are subtracted from gross income.

Nominal interest rate

The stated annual interest rate.

Present value

The starting principal or amount required to obtain a specific amount in the future.

Principal

The original amount of a loan or an investment.

Quarterly

Four times a year.

Semi-annually

Twice a year.

Share

An investment that is a part ownership in a corporation.

Simple interest

Interest calculated only on the original principal amount and paid only at the maturity date.

Term

The time period for which a loan or investment is made.

Variable

A symbol, usually a letter, used to represent a value in an equation or a formula.

Unit 1: Simple interest

Interest is the price paid for the use of money. If you borrow money from another person or a bank, eventually you must pay back this amount plus the interest owing. When you deposit money in a bank, you are lending them money and after some time they will pay you interest on the money you lent them.

The amount of money that you lend or borrow is called the **principal**. The amount of interest you will owe or receive is determined by the **interest rate**. This interest rate is given as a percent. **Interest rates are quoted for periods of 1 year**. For example, if you deposit $100 in a savings account paying 3% interest, then 1 year later you will have earned $3 in interest.

Simple interest is defined as the product of the principal (P), the rate (r), and the time (t).

The **simple interest formula** is,

$I = Prt$ where I = interest earned, P = principal, r = annual interest rate, t = time (in **years**)

Example 1

Jo deposits $200 in a savings account paying 5% per year. How much interest will she receive after one year?

Solution

Identify the P, r, and t.

Here,	P = $200	$I = Prt$	
	r = 5% or 0.05	$I = 200\ (5\%)\ (1)$	Replace P, r, and t with their values
	t = 1 year	$I = 200\ (0.05)\ (1)$	Change 5% to its decimal equivalent, 0.05
		$I = 10$	Multiply $200 \times 0.05 \times 1$

Jo will receive **$10** in interest.

Simple interest is calculated solely on the principal investment or loan. It is calculated only once, depending on when the investment or loan is due.

Example 2

Linda lends Ed $500. Ed says he will pay her back in 60 days at 9%. How much interest should Linda receive? How much must Ed pay Linda altogether?

Solution

$P = \$500$	$I = Prt$	
$r = 9\%$ or 0.09	$I = 500\,(0.09)\left(\frac{60}{365}\right)$	Replace $P, r,$ and t with their values
$t = 60$ days $= \frac{60}{365}$ years	$I = 7.39726$	Multiply
	$I = \$7.40$	Round to the nearest cent

Linda should receive **$7.40** in interest.
Ed owes Linda $500 + $7.40 = **$507.40** altogether.

Example 3

The amount of interest earned on a Canada Savings Bond is determined by the number of **months** between purchase and redemption date.

If Sid purchased an $800 C.S.B. at 3.75% on November 1, 2000 and redeemed it on May 31, 2001, how much interest should he receive?

Solution

Sid held the bond for 7 full months.

$P = \$800$	$I = Prt$	
$r = 3.75\% = 0.0375$	$I = 800(0.0375)\left(\frac{7}{12}\right)$	Replace variables with their values
$t = 7$ months $= \frac{7}{12}$ years	$I = 17.50$	Multiply

Sid will receive **$17.50** in interest.

Now complete Exercise 1 and check your answers.

Exercise 1

1. a. How many days in 1 year?

   b. How many weeks in 1 year?

   c. How many months in 1 year?

2. Calculate the interest earned in each of the following.

   a. \$1000 at 10% for 1 year

   b. \$150 at 5% for 1 year

   c. \$500 at 4.5% for 0.5 years

   d. \$200 at 11% for 0.25 years

   e. \$100 at 7% for 6 months

   f. \$50 at 6% for 8 months

   g. \$2500 at $6\frac{1}{2}\%$ for 100 days

   h. \$1000 at 5.25% for 30 weeks

3. Mike borrowed \$1500 from his mother. He agreed to pay her back in 9 months at 5%. How much interest will he owe her and how much will he owe altogether?

4. Mark bought some R.O.C.K. shares valued at \$4 per share. After 1 year, the share had a loss of 18% per share. How much did each share lose and what is one share now worth?

5. Barb invested $100. At the end of one year the investment had earned 16%. She then invested the whole amount (principal plus interest) and earned 12% in the second year.

   a. How much interest did Barb earn at the end of the first year?

   b. How much did she invest at the beginning of the second year?

   c. How much interest did she earn in the second year?

   d. Over the two year period, how much interest did Barb earn?

6. Larry loaned Mary $250 at 7%. Mary said she would pay Larry the $250 plus interest in 90 days. What is the total amount of money that Mary should pay Larry in 90 days?

7. A certain $5000, five-year bond earns 5.5%. The bond pays interest twice a year (semi-annually).

   a. How much interest will the bond earn after 6 months?

   b. What is the total amount of interest the bond will earn at the end of 5 years?

Answers are on page 65.

Unit 2: Variations on simple interest

Simple interest is used to determine the amount of interest earned on an investment or due on a loan.

Investors might ask:

1. How much **principal** do I need in order to earn so much interest over a certain period of time?

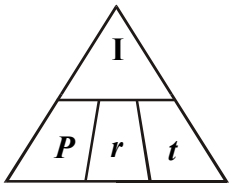
2. What **interest rate** do I need in order to earn so much interest over a given time period?

3. How much **time** will it take in order to earn so much interest at a given rate of return?

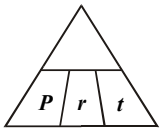
These questions can be answered by solving the **simple interest formula**,
I = *Prt*, for *P*, or *r* or *t*.

	To determine the **principal** use	$P = \frac{\mathrm{I}}{rt}$ where	P = principal I = interest
	To determine the **interest rate** use	$r = \frac{\mathrm{I}}{Pt}$	r = annual interest rate
	To determine the **time** use	$t = \frac{\mathrm{I}}{Pr}$	t = time (in years)

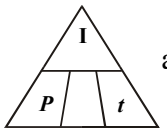
The following memory aid is often called the "Magic Triangle", because if you cover the variable you are trying to find, the formula will magically appear!



Example 1

Cover the I and  appears, or I = *Prt*.

Example 2

Cover the r and 

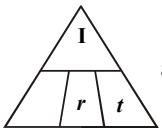
appears, or $r = \dfrac{\mathrm{I}}{Pt}$.

Example 3

A six month investment will earn 5.25%. How much would you need to invest if you want to earn \$100 in interest?

Solution

The principal is unknown. Cover P in the Magic Triangle.

P = ? or 

appears. Use the formula, $P = \dfrac{\mathrm{I}}{rt}$

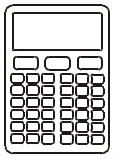
I = \$100
r = 5.25% = 0.0525
t = 6 months = $\dfrac{6}{12}$ or 0.5 years

$$P = \frac{100}{0.0525\,(0.5)}$$ Replace I, r, and t with their values

$$P = \frac{100}{0.02625}$$ Multiply 0.0525×0.5

P = 3809.52 Divide 100 by 0.02625 and round answer to the nearest cent

You would need to invest **\$3809.52**

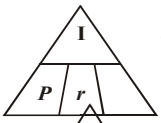
 Calculator Tip	To calculate $P = \dfrac{100}{0.0525\,(0.5)}$, press 100 [÷] [(] 0.0525 [×] 0.5 [)] [=] Did you get 3809.52381?

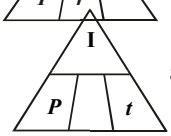
Example 4

Mariko had \$240 in the bank for the month of April. At the end of the month she had earned \$0.90 in interest. What interest rate was the bank paying?

Solution

The interest rate is unknown. Cover r in the Magic Triangle.



r = ? or 

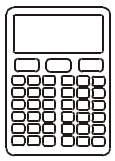
appears. Use $r = \dfrac{\mathrm{I}}{Pt}$

P = \$240
I = \$0.90
t = 1 month = $\dfrac{1}{12}$ year

$$r = \frac{0.90}{240\left(\frac{1}{12}\right)}$$

$$r = \frac{0.90}{20} = 0.045$$

The interest rate is **4.5%**



Calculator Tip

To calculate $\dfrac{0.90}{240\left(\frac{1}{12}\right)}$ press

0.9 [÷] [(] 240 [×] 1 [÷] 12 [)] [=]

Did you get 0.045?

Example 5

Carol invested \$500 paying 6%. How long will it take her to earn \$250 in interest?

Solution

The time is unknown. Cover t in the Magic Triangle.

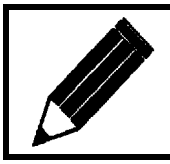
t = ? or appears. Use $t = \dfrac{\mathrm{I}}{Pr}$

I = \$250
P = \$500
r = 6% = 0.06

$$t = \frac{250}{500(0.06)}$$

$$t = \frac{250}{30} = 8.33 \text{ or } 8\frac{1}{3} \text{ years}$$

It will take **8.33 years**.



Now complete Exercise 2 and check your answers.

Exercise 2

1. Find the principal needed to earn,

   a. \$100 at 5% in 1 year

   b. \$10 at 20% in 1 year

   c. \$60 at $9\frac{1}{2}\%$ in 90 days

   d. \$1000 at 2.75% in 9 months

2. Find the interest rate when,

   a. \$1000 earns \$25 in 1 year

   b. \$100 earns \$3 in 0.5 years

   c. \$400 earns \$1.60 in 1 month

   d. \$550 earns \$4.80 in 73 days

3. Find the time needed to earn,

   a. \$5 interest on \$100 at 10%

   b. \$1 interest on \$10 000 at 12.5% (answer in days)

   c. \$4 interest on \$100 at 7.5%

   d. \$3 interest on \$100 at 10%

4. Fill in the missing values.

I	*P*	*r*	*t*
	\$100.00	3%	1 year
\$50.00		5%	6 months
\$3.42	\$631.38		1 month
\$38.00	\$800.00	4.75%	

5. At the beginning of the year, Bill invested \$500 in a special account. At the end of the year the account was worth a total of \$523.25. What interest rate did he earn on the \$500 investment?

6. Velma invests \$1200 at 6.5%. How long will it take to earn \$10 in interest on the investment? (Answer in days.)

7. A certain credit card company charged \$3.45 interest on a \$230 unpaid bill over a 30 day period. What interest rate was the credit card company charging?

8. Kim bought 200 shares of Ajax Fuelcells at $18 per share. Over a 9 month period she received quarterly dividends of $0.26, $0.24, and $0.31 **per share**. Also, over this time the shares have increased from $18 to $22.50.

a. How much money did Kim invest?

b. How much in dividends, **per share**, did Kim receive altogether?

c. What were her total earnings in dividends?

d. What percent did the dividends earn **per share**?

e. How much did Kim earn altogether when her stock went from $18 to $22.50? This is called capital gains. (Do not include the dividend earnings.)

f. What were her total earnings (dividends plus capital gains)?

g. What total percent gain has Kim realized on her investment?

Answers are on page 65.

Unit 3: Compound interest

Imagine you have a $100 investment that earns 10% interest per year. At the end of one year you will have earned $10 in interest. If the investment pays **simple interest** you take the $10 and let the original principal of $100 earn interest at 10% in the second year. If the investment pays **compound interest** and you leave the $10 in the investment you will earn interest on **$110** in the second year. In this case you would be earning interest not only on the original principal, but also on the previously earned interest.

When interest is earned on interest, we say the interest is **compounded** and the amount earned is called **compound interest**.

The following table shows how the value of a $100 investment, earning 10% **compounded annually**, changes over a 6-year period.

Year	Principal Amount	Earned Interest	Year End Total
1	$100	$10	$110
2	$110	$11	$121
3	$121	$12.10	$133.10
4	$133.10	$13.31	$146.41
5	$146.41	$14.64	$161.05
6	$161.05	$16.11	$177.16

Notice that the investment is worth $177.16 at the end of 6 years. It has earned $77.16 in interest. If the investment had earned simple interest as opposed to compound interest, it would have only earned (I = Prt = 100 × 0.10 × 6) $60 in interest.

The above method of calculating the compound interest is very time consuming. Fortunately, there is a mathematical formula that we can use to calculate compound interest.

The **compound interest formula** is

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where, A = total compound amount (includes principal and interest)
P = principal
r = annual interest rate
n = number of times in one year that interest is calculated
t = time (in years)

Since A includes the principal and interest, to find the interest amount calculate: I = A - P

Example 1

Find the compound amount and the interest earned on \$100 compounded annually at 10% for 6 years.

Solution

P = \$100
r = 10% = 0.1
n = 1 (since the interest is calculated once a year)
t = 6 years

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A = 100\left(1+\frac{0.1}{1}\right)^{1\times 6}$$

Replace the variables with their values

$$A = 100(1+0.1)^6$$

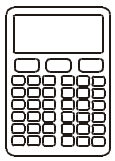
$\frac{0.1}{1} = 0.1$ and $1\times 6 = 6$

$$A = 100(1.1)^6$$

A = 100(1.771 561) = 177.1561 Raise $(1.1)^6 = 1.771561$

The compound amount is **\$177.16**
The interest earned is A – P = \$177.16 - \$100 = **\$77.16**

Notice that this answer agrees perfectly with the answer calculated by the table method on the previous page. To find $(1.1)^6$ use the power function $\boxed{y^x}$ or $\boxed{x^y}$ on your calculator.



Calculator Tip

To calculate $A = 100\left(1+\frac{0.01}{1}\right)^6$ press,

100 $\boxed{\times}$ $\boxed{(}$ 1 $\boxed{+}$ 0.1 $\boxed{\div}$ 1 $\boxed{)}$ $\boxed{x^y}$ 6 $\boxed{=}$

Did you get 177.1561?

NOTE If you did not get 177.1561 on your calculator, you may have to press a different sequence of buttons. Check with your instructor.

Example 2

Find the compound amount and the interest earned on \$500 compounded monthly at 6% for 3 years.

Solution

$P = \$500$
$r = 6\% = 0.06$
$n = 12$
(since the interest is calculated monthly, or 12 times per year)
$t = 3$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 500\left(1 + \frac{0.06}{12}\right)^{12\times 3}$$

$$A = 500\left(1 + \frac{0.06}{12}\right)^{36}$$

$$A = 500(1.005)^{36}$$
$$A = 500(1.19668)$$
$$A = 598.34$$

The compound amount is **$598.34** and the interest earned is **$598.34 - $500 = $98.34**

NOTE If you did not get 598.34 on your calculator, you may have to press a different sequence of buttons. Check with your instructor.

Interest can be compounded in a variety of ways. The variable n in the compound interest formula reflects the number of times in one year that interest is calculated.

If interest is compounded annually, then $n = 1$.
If interest is compounded semi-annually, then $n = 2$.
If interest is compounded quarterly, then $n = 4$.
If interest is compounded monthly, then $n = 12$.
If interest is compounded weekly, then $n = 52$.
If interest is compounded daily, then $n = 365$.

Now complete Exercise 3 and check your answers.

Exercise 3

1. Determine the value of n for each of the following.

   a. yearly, then $n =$ _____
   b. semi-annually, then $n =$ _____
   c. quarterly, then $n =$ _____
   d. monthly, then $n =$ _____
   e. weekly, then $n =$ _____
   f. daily, then $n =$ _____

2. Ada invested $1000 at 5% compounded annually.

   a. Complete the table below to find the compound amount Ada will earn at the end of 5 years.

Year	Principal Amount	Earned Interest	Year End Total
1	$1000	$50	$1050
2	$1050	$52.50	
3			
4			
5			

   b. Use the compound interest formula to determine the compound amount Ada will earn in 5 years.

3. Find the compound amount and the earned interest.

   a. $100 compounded annually at 9% for 5 years.

   b. $100 compounded semi-annually at 9% for 5 years.

   c. $100 compounded quarterly at 9% for 5 years.

   d. $100 compounded monthly at 9% for 5 years.

   e. $100 compounded daily at 9% for 5 years.

4. When Penny was born her parents put $500 in a special fund paying 12% compounded quarterly.

   a. How much will the fund be worth when Penny turns 10 years old?

   b. When she turns 18 years old?

5. a. Agnes is 35. If she invests $1000 now, and she hopes to earn 10% compounded annually, how much will her investment be worth when she retires at the age of 65?

   b. Suppose Agnes invested the $1000 at 10% when she was 25. How much would it be worth when she is 65?

6. Imagine you had $10 000 and a choice between buying a bond paying 8% simple interest or buying a savings certificate paying 7.75% compounded monthly.

   a. How much will the bond earn (in interest) in one year?

   b. How much interest will the savings certificate earn after one year?

   c. Which is the better deal and by how much?

7. You have \$2500 to invest, compounded monthly, over a period of 10 years. Compare the compound amount when interest rates are as follows:

   a. 6%

   b. 12%

   c. 18%

   d. Notice that the interest rate of 18% is triple that of the 6% rate. How many times higher is the compound amount earned by the 18% interest rate than the amount earned by the 6% rate?

8. L. Shark says that he will lend you the \$5000 you need but he wants 50% compounded daily on the loan. (And, you had better pay him within 90 days.)

   How much will you owe him in:

   a. 30 days

   Hint: here $n = 365$ and $t = \frac{30}{365}$

   b. 60 days

   c. 90 days

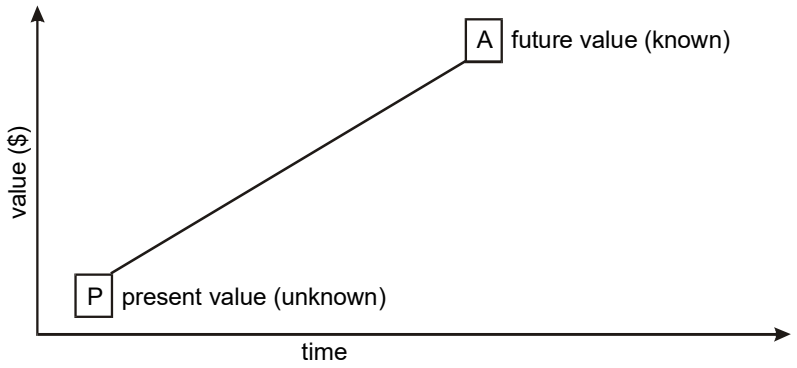
   d. Does it make sense to pay off a loan quickly?

Answers are on pages 65.

Unit 4: Variations on compound interest – present value

Imagine you wanted to know how much principal you needed to invest right **now** in order to have $5000 (the compound amount) six years from now at a certain compound interest rate.

The amount you need **now** to earn a desired amount in the future is called **the present value**.



The present value can be calculated by solving the compound interest formula for P.

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$\frac{A}{\left(1+\frac{r}{n}\right)^{nt}} = P \qquad \text{divide both sides by } \left(1+\frac{r}{n}\right)^{nt}$$

The **present value formula** is:

$$P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}}$$

where P = present value

A = desired future amount

r = interest rate

n = number of times interest is calculated in one year

t = time (in years)

Example 1

Pat and her friends are planning a reunion in five years. She estimates that the cost of the trip plus expenses will be approximately $2000. How much should she invest right now in order to have $2000 five years from now, if she thinks her money will earn 6% compounded quarterly?

Solution

$P = ?$
$A = \$2000$
$r = 6\% = 0.06$
$n = 4$
$t = 5$ years

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$$

$$P = \frac{2000}{\left(1 + \frac{0.06}{4}\right)^{4(5)}}$$ Replace the variables with their values

$$P = \frac{2000}{\left(1 + \frac{0.06}{4}\right)^{20}}$$ Multiply 4(5) = 20

$$P = \frac{2000}{(1 + 1.015)^{20}}$$ Divide $\frac{0.06}{4} = 0.015$

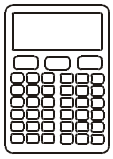
$$P = \frac{2000}{(1.015)^{20}}$$ Add 1 + 0.015 = 1.015

$$P = \frac{2000}{1.346\ 855}$$ Raise $(1.015)^{20} = 1.346\ 855$

$P = 1484.94$

The present value is **$1484.94**

In other words, if Pat invested $1484.94 now at 6% compounded quarterly, then in 5 years the compound amount would be $2000.



Calculator Tip

To calculate $P = \dfrac{2000}{\left(1 + \dfrac{0.06}{4}\right)^{20}}$ press,

2000 [÷] [(] 1 [+] 0.06 [÷] 4 [)] [y^x] 20 [=]

Did you get 1484.940 836? If not, check with your instructor.

Now complete Exercise 4 and check your answers.

Exercise 4

1. Find the present value in each of the following.

   a. \$100 due in 5 years at 6% compounded semi-annually.

   b. \$2500 due in 2 years at 9% compounded monthly.

   c. \$4000 due in 10 years at 10% compounded yearly.

   d. \$650 due in $3\frac{1}{2}$ years at 4% compounded quarterly.

   e. \$1000 due in 6 months at $8\frac{1}{2}$% compounded monthly.

2. In 6 years, Sylvia's son will be going to college. Sylvia estimates that her son will then need about \$8000 to get started in the first year of his education. How much should she invest now if she can earn 7% compounded monthly?

3. The Smiths inherited \$20 000. They would like to spend some now, but still have \$20 000 fifteen years from now when they retire. They think they could average 10% compounded yearly over this time.

   a. How much should they invest now?

   b. How much of the \$20 000 can they spend now?

4. A certain certificate will pay the owner $1000 in the year 2020. If money is worth 8% compounded semi-annually, how much must you pay for the certificate in

   a. 2015
   b. 2010
   c. 2005
   d. this year

Answers are on page 65.

Unit 5: Nominal and effective rates of interest

Bank A has a savings plan that offers 6.25% compounded monthly and Bank B offers 6.5% compounded semi-annually. Which offers the better rate of return? There is a formula for dealing with this problem. The formula converts a **nominal interest** rate into an **effective interest rate**. Here, both the 6.25% compounded monthly and the 6.5% compounded semi-annually are the nominal rates. The effective rate reflects the rate of interest **actually** earned in one year.

Again, the formula for effective rate is a variation on the compound interest formula, $A = P\left(1 + \frac{r}{n}\right)^{nt}$. See Appendix B for the derivation.

The **effective interest rate formula** is,

$f = \left(1 + \frac{r}{n}\right)^{n} - 1$ where f = effective interest rate

r = nominal interest rate (annual interest rate)

n = number of times in one year that interest is calculated

Example 1

What is the effective rate of 4% compounded monthly?

Solution

$f = ?$
$r = 4\% = 0.04$
$n = 12$

$$f = \left(1 + \frac{r}{n}\right)^{n} - 1$$

$$f = \left(1 + \frac{0.04}{12}\right)^{12} - 1$$ Replace the variables with their values

$$f = (1 + 0.003\,333\,3)^{12} - 1$$ Divide 0.04 by 12

$$f = (1.003\,333\,3)^{12} - 1$$ Add 1 + 0.003 333 3

$$f = 1.040\,742 - 1$$ Raise $(1.003\,333\,3)^{12}$

$f = 0.040\,742 = 4.07\%$

The rate of 4% compounded monthly is the same as, or effectively, **4.07%**.

Example 2

What is the effective rate of 4% compounded yearly?

Solution

$f = ?$
$r = 4\% = 0.04$
$n = 1$

$$f = \left(1 + \frac{r}{n}\right)^{n} - 1$$

$$f = \left(1 + \frac{0.04}{1}\right)^{1} - 1$$

$$f = (1 + 0.04) - 1$$

$$f = 1.04 - 1 = 0.04$$

The effective rate of 4% compounded annually is still 4%. Notice that the effective rate will equal the **yearly** compound rate, but will always be more than the semi-annual, quarterly, monthly, weekly or daily nominal rate.

Example 3

Bank Alberta offers 6.25% compounded monthly while Bank BC offers 6.5% compounded semi-annually. Which bank offers the better effective rate of return?

Solution

For Bank Alberta,

$f = ?$
$r = 6.25\% = 0.0625$
$n = 12$

$$f = \left(1 + \frac{0.0625}{12}\right)^{12} - 1$$

$$f = (1 + 0.005\,208\,3)^{12} - 1$$

$$f = (1.005\,208\,3)^{12} - 1$$

$$f = 1.064\,322 - 1$$

$$f = 0.064\,322 = 6.43\%$$

We can stop here. Notice that Bank Alberta's effective rate, 6.43%, is still less than Bank BC's nominal rate of 6.5%. Bank BC's effective rate will be even greater than 6.5%, so Bank BC offers the better effective rate of return. (Bank BC's effective rate is 6.61%.)

Now complete Exercise 5 and check your answers.

Exercise 5

1. Determine the effective rates for the following when they are compounded monthly.

   a. 8%

   b. 10%

   c. 12%

   d. 5%

2. Determine the effective rate when 10% is compounded

   a. yearly

   b. semi-annually

   c. quarterly

   d. monthly

   e. weekly

   f. daily

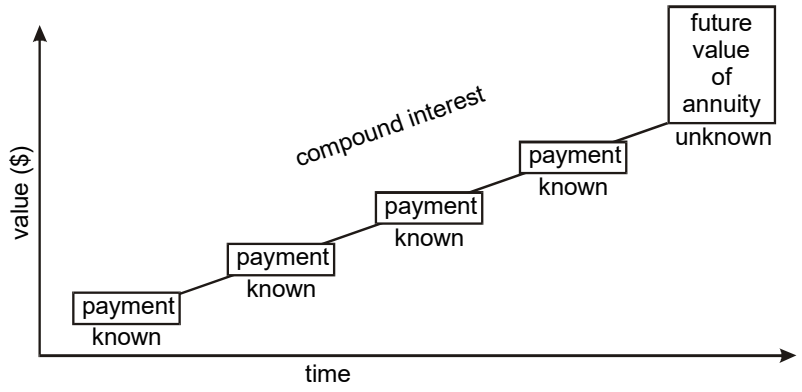
3. You have a choice between purchasing a savings bond paying 9% simple interest or putting your money in a savings account at $8\frac{3}{4}\%$ compounded monthly. What is the better rate of return and what is the difference between the effective percentage rates?

4. What simple interest rate would give you the same return as 6% compounded monthly?

Answers are on page 65.

Unit 6: Ordinary annuities

Imagine you plan to open a savings account and deposit $100 at the end of each month. A sequence of equal payments at equal intervals like this is called an ordinary annuity.* When compound interest is calculated on each payment, the value of the annuity can grow substantially.



An obvious question you might ask about an annuity is how much will the annuity be worth in 6 months, or 1 year, or 5 years or even 20 years?

To see how the annuity process works, study the table below. In this case, $100 is deposited at the end of each month into an account that pays 6% interest compounded monthly. The process is continued for 6 months.

Date	Total amount from previous month	Interest earned on total amount	Monthly deposit
Jan. 31	—	—	$100
Feb. 28	$100	$0.50	$100
March 31	$200.50	$1.0025	$100
April 30	$301.5025	$1.5075	$100
May 31	$403.01	$2.0151	$100
June 30	$505.0251	$2.5251	$100
July 1	$607.55		

Notice above that the interest is calculated on the previous month's total amount **before** the current month's deposit is made using the formula $I = Prt$. For example, the interest earned on May 31, is

* There are other types of annuities. For example, if payments are made at the beginning of each time interval, the annuity is called an annuity due. In this unit, we will only be concerned with ordinary simple annuities.

$$\mathrm{I} = \mathrm{Pr}\,t = 403.01(0.06)\left(\frac{1}{12}\right) = \$2.0151$$

There is a formula for calculating the annuity amount.

The **ordinary annuity formula** is,

$$A = \frac{nP\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{r}$$

where

A = amount of annuity
P = periodic payment amount
r = annual interest rate
n = number of times interest is calculated in one year
t = time (in years)

Example 1

Use the annuity formula to find the annuity amount in 6 months if \$100 is deposited monthly at 6% compounded monthly. Compare this answer to the answer obtained in the table.

Solution

A = ?
P = \$100
r = 6% = 0.06
n = 12
t = 6 months = 0.5 years

$$A = \frac{nP\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{r}$$

$$A = \frac{12(100)\left[\left(1+\frac{0.06}{12}\right)^{12(0.5)} - 1\right]}{0.06}$$ Replace variables

$$A = \frac{1200\left[(1+0.005)^{6} - 1\right]}{0.06}$$ Divide and multiply

$$A = \frac{1200\left[(1.005)^{6} - 1\right]}{0.06}$$ Add

$$A = \frac{1200(1.030\,377\,5 - 1)}{0.06}$$ Calculate the power

$$A = \frac{1200(0.030\,377\,5)}{0.06}$$ Subtract

$$A = \frac{36.45301}{0.06} = 607.55$$ Multiply and divide

The annuity is worth **$607.55**. This answer agrees perfectly with the table answer.

Example 2

How much would an annuity be worth in 2 years at 6% compounded monthly if the periodic payments are $50 per month?

Solution

A = ?
P = $50
r = 6% = 0.06
n = 12
t = 2

$$A = \frac{nP\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{r}$$

$$A = \frac{12(50)\left[\left(1+\frac{0.06}{12}\right)^{12\times 2} - 1\right]}{0.06}$$

$$A = \frac{600\left[(1+0.005)^{24} - 1\right]}{0.06}$$

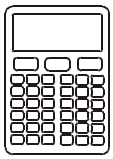
$$A = \frac{600\left[(1.005)^{24} - 1\right]}{0.06}$$

$$A = \frac{600(1.12716 - 1)}{0.06}$$

$$A = \frac{600(0.12716)}{0.06}$$

$$A = \frac{76.296}{0.06} = 1271.60$$

The annuity is worth **$1271.60** after 2 years.
The total interest earned was $1271.60 – 1200 = $71.60



Calculator Tip

To calculate $A = \dfrac{12(50)\left[\left(1+\dfrac{0.06}{12}\right)^{24}-1\right]}{0.06}$ press,

[(] 1 [+] 0.06 [÷] 12 [)] [x^y] 24 [=] [−] 1 [=] [×] 12 [=] [×] 50 [=] [÷] 0.06 [=]

Did you get 1271.59762? If not, check with your instructor.

Now complete Exercise 6 and check your answers.

Exercise 6

1. a. Complete the table below, where $2000 is deposited annually for 5 years at 5% compounded annually.

Date	Total amount from previous year	Interest earned on total amount	Annual deposit
Dec. 31/01	—	—	$2000
Dec. 31/02	$2000	$100	$2000
Dec. 31/03	$4100		
Dec. 31/04			
Dec. 31/05			
Jan. 1/06			

b. Use the annuity formula to calculate the above amount. Do your formula and table amounts agree?

2. Find the annuity amount when

a. a period payment of $1000 per year earns 8% compounded annually for 10 years

b. a payment of $100 per month earns 4% compounded monthly for 5 years

c. a payment of $200 quarterly earns 10% compounded quarterly for 7 years

3. Daniel wishes to save $100 a month. He hopes to earn 6% compounded monthly. What would his annuity be worth in

a. 1 year?

b. 2 years?

c. 5 years?

d. 10 years?

4. The Andersons plan to retire in 25 years and want to start saving for it now. They hope to be able to earn about 10% compounded annually. Determine the amount of their annuity if they make the following periodic payments.

a. $500 per year

b. $1000 per year

c. $2000 per year

d. $3500 per year

5. Imagine the Andersons had the choice of either investing $1200 a year at 10% compounded annually for 25 years or investing $100 per month at 10% compounded monthly for 25 years.

a. How much would the annual annuity be worth in 25 years?

b. How much would the monthly annuity be worth in 25 years?

c. Which investment (yearly or monthly annuity) earns the greater amount and by how much?

6. The Grinders plan to save for their child's education by depositing $40 a month into a special savings plan which pays about 8% compounded monthly.

   a. How much would the annuity be worth after 1 year?

   b. How much after 18 years?

7. Imagine you start saving for your retirement and contribute $1000 yearly and average 10% compounded annually. Of course, the amount of the annuity depends on the length of the annuity. Complete the table below.

Years	Annuity Amount
20	
25	
30	
35	
40	

Note: Not only are retirement savings plans excellent ways to save for the future, the yearly payments you make along the way are tax-deductible. (i.e. If you were in the 19% tax bracket, a $1000 R.S.P. would save you at least $190 a year in taxes.)

8. In question 7 above, what is the effect of saving for your retirement over a 40 year period as opposed to a 20 year period?

Answers are on page 65.

Unit 7: Annuity payments

Often when you are saving, you have some goal in mind. For example, suppose you need about $5000 for a down payment on a mobile home that you would like to buy 3 years from now. If money can earn 7% compounded monthly, what monthly payment is needed to accumulate $5000 in 3 years?

The above problem involves solving the annuity formula for P.

The **periodic payment formula** is:

$$P = \frac{A\left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{nt}-1}$$

where

P = periodic payment amount
A = annuity amount
r = annual interest rate
n = number of times interest is calculated in one year
t = time (in years)

Example 1

What monthly payment is necessary for an annuity to be worth $5000 in 3 years at 7% compounded monthly?

Solution

$P = ?$
$A = \$5000$
$r = 7\% = 0.07$
$t = 3$

$$P = \frac{A\left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{nt}-1}$$

$$P = \frac{5000\left(\frac{0.07}{12}\right)}{\left(1+\frac{0.07}{12}\right)^{12\times 3}-1}$$

Replace variables with their values

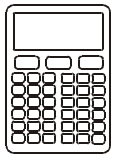
$$P = \frac{5000(0.005\,833\,3)}{(1.005\,833\,3)^{36}-1}$$

Divide 0.07 by 12

$$P = \frac{29.16667}{1.23292 - 1}$$ Multiply and calculate the power

$$P = \frac{29.16667}{0.23292} = 125.22$$

The periodic payment is **$125.22**



Calculator Tip

To calculate $P = \dfrac{5000\left(\dfrac{0.07}{12}\right)}{\left(1+\dfrac{0.07}{12}\right)^{36}-1}$ press,

5000 [×] 0.07 [÷] 12 [÷] [(] [(] 1 [+] 0.07 [÷] 12 [)] [x^y] 36 [−] 1 [)] [=]

Did you get 125.218 817 7? If not, check with your instructor.

Now complete Exercise 7 and check your answers.

Exercise 7

1. Find the periodic payment needed for an annuity of

   a. $1000 at 5% compounded monthly for 1 year

   b. $20 000 at 10% compounded yearly for 15 years

   c. $5000 at 8% compounded quarterly for 3 years

2. Mike wants to buy a $1500 stereo 9 months from now. How much will he have to deposit every month into a savings plan paying 6.5% compounded monthly?

3. Suppose you would like to save $1000 every year. What monthly payment would you have to make if money can earn 7% compounded monthly?

4. Imagine you would like to retire with a $100 000 annuity. And, suppose that over the years you hope that money will be worth an average of 10% compounded annually. Your yearly payments, of course, will depend upon the length of the annuity. Complete the table below.

Years	Yearly Payment
15	
20	
25	
30	
35	
40	

Do you see the difference between planning for retirement early as opposed to later in life?

5. The Wests need $9000 for their child's education 6 years from now. How much should they put aside every month if they hope to earn 8.5% compounded monthly?

6. Imagine you wanted to be a millionaire 30 years from now. How much would you have to put into an annuity every year if you think you could earn 12% compounded yearly?

Answers are on page 65.

Unit 8: Loans

Until now, we have looked at the various ways our investments earned money (simple interest, compound interest and annuities). Now it's time to consider how the financial institutions charge us for the use of their money.

When you take out a loan with a company or institution, you receive the principal amount and you agree to pay back in a fixed amount of time through equal payment amounts at a certain interest rate. For example, suppose you borrow \$1000 from a bank at 12% and agree to pay off the loan in 6 months with regular monthly payments. The bank would then calculate your monthly payments to be \$172.55. The way in which your debt would be disposed of is shown below.

Month	Amount Owing at beginning of Month	Interest Owing at end of Month $\left(\frac{r}{n}=\frac{0.12}{12}=0.01 \text{ or } 1\%\right)$	Monthly Payment	Amount of Loan Repaid at End of Month
1	1000.00	10.00	172.55	162.55
2	837.45	8.37	172.55	164.18
3	673.27	6.73	172.55	165.82
4	507.45	5.07	172.55	167.48
5	339.97	3.40	172.55	169.15
6	170.82	1.71	172.55	170.84
Totals		35.28	1035.30	1000.02

What is happening above is that at the end of the first month the bank charges you 12% ÷ 12 = 1% on the amount owing or \$10 interest for the use of the \$1000. The bank takes \$10 from the \$172.55 payment you gave them. The remaining \$162.55 is then subtracted from the original \$1000 loan. At the beginning of the second month, you now owe \$1000 - \$162.55 = \$837.45. At the end of the second month the bank will again take the interest (\$8.37) owing them and reduce your loan accordingly. This continues until the loan is reduced to \$0.00.

Naturally, mathematicians have devised a formula for determining the periodic payments necessary to pay down a loan.

The **periodic payment on a loan formula** is:

$$P=\frac{A\left(\frac{r}{n}\right)}{1-\left(1+\frac{r}{n}\right)^{-nt}}$$

where

P = periodic payment amount
A = amount of loan
r = annual interest rate
n = number of payments made in one year
t = time (in years)

Example 1

Audrey decides to borrow $3000 to buy a computer for her home based bookkeeping business.

Determine the monthly payments needed to pay down a loan of $3000 at 9% over 2 years.

Solution

$P = ?$
$A = \$3000$
$r = 9\% = 0.09$
$n = 12$
$t = 2$

$$P = \frac{A\left(\frac{r}{n}\right)}{1-\left(1+\frac{r}{n}\right)^{-nt}}$$

$$P = \frac{3000\left(\frac{0.09}{12}\right)}{1-\left(1+\frac{0.09}{12}\right)^{-12(2)}}$$

Replace variables with their values

$$P = \frac{3000(0.0075)}{1-(1.0075)^{-24}}$$

Divide 0.09 by 12

$$P = \frac{22.5}{1-0.83583}$$

Multiply and calculate the power

$$P = \frac{22.5}{0.16417} = 137.05$$

Subtract and divide

The monthly payments would be **$137.05**

In the above example, the borrower paid a total of ($137.05 × 12 × 2) = $3289.20 to the lender.

The borrower paid back the original $3000 plus an extra $289.20. This $289.20 is called the **finance charge** on the loan.

The **finance charge formula** is:

$F.C. = ntP - A$ where

$F.C.$ = finance charge
P = periodic payment amount
A = loan amount
n = number of payments made in a year
t = time (in years)

Example 2

Lee could buy a used truck for $8900 with a $1000 down payment and finance the rest at 6% over three years. What would the monthly payments on the truck be, and what is the finance charge?

Solution

A = \$8900 - \$1000
= \$7900
r = 6% = 0.06
n = 12
t = 3

$$P = \frac{A\left(\frac{r}{n}\right)}{1-\left(1+\frac{r}{n}\right)^{-nt}}$$

$$P = \frac{7900\left(\frac{0.06}{12}\right)}{1-\left(1+\frac{0.06}{12}\right)^{-12(3)}}$$

$$P = \frac{7900(0.005)}{1-(1.005)^{-36}}$$

$$P = \frac{39.5}{1-0.83564}$$

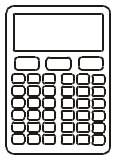
$$P = \frac{39.5}{0.16436} = 240.33$$

The monthly payment would be **\$240.33**

To find the finance charge,

$F.C. = ntP - A$
$F.C. = 12(3)(240.33) - 7900$
$F.C. = 8651.88 - 7900$
$F.C. = 751.88$

The finance charge is **$751.88**



Calculator Tip

To calculate $P = \dfrac{7900\left(\dfrac{0.06}{12}\right)}{1-\left(1+\dfrac{0.06}{12}\right)^{-36}}$ press,

7900 × 0.06 ÷ 12 ÷ (1 − (1 + 0.06 ÷ 12) x^y 36 +/−) =

Did you get 240.333 305 9? If not, check with your instructor.

Now complete Exercise 8 and check your answers.

Exercise 8

1. Determine the monthly payments needed to pay down a loan of $1000 at 14% in 1 year. Also determine the finance charge.

2. Complete the table below where a $5000 loan is to be paid off in 3 years in monthly payments.

Interest Rate	Monthly Payment	Finance Charge
6%		
9%		
13%		
19%		

Notice the effects of high interest rates.

3. Complete the table below. This time, suppose the loan is $2000 at 12% and the payments are monthly.

Term of Loan	Monthly Payment	Finance Charge
6 Months		
1 Year		
2 Years		
5 Years		

Notice here that the longer the term, the lower the payments **but** the higher the finance charge.

4. A living room set is on sale for $1495. You make a down payment of $600 but will have to finance the rest at 5% over 1 year. What would your monthly payments be? What would the finance charge be?

5. An electronics discount store has a stereo system on sale for $1199 for only $99 down, $99 per month for one year or you could get a loan from the bank for 12%.

   a. What would the monthly payments amount to if you financed through the bank for one year? Assume that you do make the $99 down payment.

   b. What is the electronics discount store's finance charge?

   c. What is the bank's finance charge?

   d. From whom would you receive the best deal?

Answers are on pages 65.

Unit 9: Mortgages

When you borrow money to buy a house or property, the loan is called a **mortgage**. The terms of a mortgage can be rather complicated and varied. Some facts about mortgages:

1. The term (length) of a mortgage can vary from a few years to 30 years.

2. During the term of the mortgage, the interest rate can change dramatically. For example, you can begin with a fixed rate of interest for the first 6 months, 1, 2, 3 or 5 years of the mortgage. At the end of this fixed time, a new interest rate is established.

3. Depending on the lending institution, you can periodically make lump sum payments to reduce the amount of the mortgage.

4. You can significantly reduce the term of the mortgage by changing monthly payments to weekly payments.

5. If you fail to make payments on the mortgage, the mortgage holder has the right to take possession of "your" property.

6. As a general rule, if you purchase a home:

   a. it should not cost more than $2\frac{1}{2}$ times your annual gross income,

   b. you should be able to make at least a 10% down payment on the cost of the house, and

   c. your monthly payments should not exceed $\frac{1}{3}$ of your net monthly income.

Since a mortgage is really just a long-term loan, the **mortgage formula** is identical to the loan formula.

$$P = \frac{A\left(\frac{r}{n}\right)}{1-\left(1+\frac{r}{n}\right)^{-nt}}$$

Where A = amount of the mortgage and n = number of payments per year
P = mortgage payment t = time (in years)
r = annual interest rate

The formula for determining the finance charge of a mortgage is also identical:

$$F.C. = ntP - A$$

Example 1

Determine the monthly payments needed to pay down a mortgage of $100 000 at 7.5% over 20 years. Also, determine the finance charge on the mortgage.

Note that the mortgage rate will change over the course of 20 years. Nevertheless, we can still get a sense of what the final finance charge might approximate.

Solution

P = ?
A = $100 000
r = 7.5% or 0.075
n = 12
t = 20

$$P = \frac{A\left(\frac{r}{n}\right)}{1-\left(1+\frac{r}{n}\right)^{-nt}}$$

$$P = \frac{100\,000\left(\frac{0.075}{12}\right)}{1-\left(1+\frac{0.075}{12}\right)^{-12\times 20}}$$

$$P = \frac{100\,000(0.00625)}{1-(1+0.00625)^{-240}}$$

$$P = \frac{625}{1-(1.00625)^{-240}}$$

$$P = \frac{625}{1 - 0.22417}$$

$$P = \frac{625}{0.77583} = 805.59$$

The monthly payment on the mortgage is **$805.59**

The finance charge is,

$F.C. = ntP - A = 12(20)(805.59) - 100\,000 = 93\,341.60$

The finance charge is an amazing **$93 341.60**

Now complete Exercise 9 and check your answers.

Exercise 9

1. As stated on page 46, as a general rule, if you purchase a home:

   a. it should not cost more than $2\frac{1}{2}$ times your annual gross income,

   b. you should be able to make at least a 10% down payment on the cost of house, and

   c. your monthly payments should not exceed $\frac{1}{3}$ of your net monthly income.

   Imagine you wanted to buy a condominium for $120 000. According to the "rules" suggested in a, b, and c above, determine the necessary:

   a. annual gross income

   b. 10% down payment

   c. monthly payments if you net income was $2000 per month

2. Determine the monthly payment needed to pay down a $80 000 mortgage over 20 years at 7%. Calculate the finance charge on this mortgage.

3. Complete the following table where the mortgage is $90 000 for 20 years.

Interest Rate	Monthly Payment	Finance Charge
8%		
10%		
12%		
14%		

Note how the finance charge increases as the interest rate increases.

4. Complete the table below where the mortgage is $100 000 and the interest rate is 7.5%.

Length of Mortgage	Monthly Payment	Finance Charge
15 years		
20 years		
25 years		
30 years		

Note how the finance charge increases as the length of the mortgage increases.

5. The Jones have $8500 for a down payment on a $63 900 cottage. What would their **weekly** payments be if the mortgage was 10.5% for 15 years?

6. The Waters are willing to sell their cottage for $29 500. The Pipes will buy the property for $6000 down if the Waters agree to hold the mortgage at 9% for 10 years.

a. If the Waters agree, what monthly payment should they charge the Pipes?

b. At the end of 10 years, what is the finance charge earned by the Waters?

c. The Waters decide to put the monthly payments from the mortgage into an **annuity** paying 9% compounded monthly. How much will the annuity be worth in 10 years?

Answers are on page 65

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Unit 10: Interest rates on loans

Suppose a computer system is selling for $3333 (taxes included). The store offers a payment plan of $99 down and $99 monthly payments for three years. You may wish to determine what interest rate the store is charging for this loan. The problem involves determining the *r* value in the loan formula. The interest rate can be found using the Annual Percentage Rate Table on Appendix C.

Example 1

What interest rate is charged on a loan of $3333 if a down payment of $99 is made on the loan and monthly payments of $99 are made on the loan for three years?

Solution

Step One	Determine the amount of the loan (or the amount being financed).	$A = 3333 - 99 = 3234$ $A = \$3234.00$
Step Two	Determine the finance charge, *F.C.* where $n = 12$, $t = 3$, and $P = \$99$	$F.C. = ntP - A$ $F.C. = 12(99) - 3234 = 330$ $F.C. = \$330.00$
Step Three	Determine the finance charge per $100.	$\frac{F.C.}{A} \times \$100 = \frac{330}{3234} \times 100 = 10.20$
Step Four	Go to Appendix C and read down the "Number of Payments" column until you come to 36 (since there is a total of 36 payments made on this loan). Read across the row until you come to 10.20 (or as close to 10.20 as you can). The closest value is 10.34. Reading up the column, you see that the Annual Percentage Rate is 6.5%.	

The store charges an interest rate of 6.5**%**.

Now complete Exercise 10 and check your answers.

Exercise 10

1. A used car is selling for $2500 or $500 down and $175 per month for 12 months.

   a. Find the amount financed.

   b. Find the finance charge.

   c. Determine the annual percentage rate charged for this loan.

2. Anthony borrowed $1400 and will be paying $65.58 a month for the next two years. What interest rate is he being charged?

3. A dining room set is selling for $1500. You could pay $500 down and $88 for 12 months or $100 down and $95 for 16 months. What percentage rate is being charged in each situation?

4. Tracey has accumulated $4800 in student loans. She decides to pay it off over the next five years. She is told that her monthly payments will be $96 per month. What interest rate is Tracey being charged?

5. After the down payment, John will owe $5600 on his truck at $197 monthly for 3 years, if the deal is through Fred's Friendly Motors. He could also get a personal loan at his bank for 12.5%.

   What interest rate is Fred's charging and what would the best deal be?

Answers are on page 65.

Review Questions

1. George had a \$100 bond earning $7\frac{3}{4}\%$ interest per annum. At the end of the year, he received a cheque for \$7.75. What kind of interest did he earn (simple or compound)?

2. Sal put \$100 into a savings account. At the end of each month, the interest was calculated and deposited into Sal's account. Sal did not make any withdrawals from his account until the sixth month. What kind of interest was his account earning (simple or compound)?

3. Calculate the interest earned on a \$350 investment at 9% (simple interest) for 3 months.

4. A \$1500 investment earned \$420 interest after 3 years. What simple interest rate produced these earnings?

5. How much would a \$2500 certificate be worth in 5 years if it earns 10% compounded quarterly?

6. Wendy's grandparents would like to give her $1000 when she turns 16, six years from now. How much should they invest now if they can earn 8.5% compounded monthly?

7. What is the effective rate of 9.5% compounded weekly?

8. Terry decides to start saving $50 a month. He can deposit it in an account yielding 4% compounded monthly. How much will his annuity be worth in 5 years?

9. Mary figures she will need about $7000 in 10 years to help pay for her children's education. A savings fund pays 9.5% compounded quarterly. How much should she deposit every 3 months into the fund to reach her goal?

10. Peter can buy a $9950 truck for $2000 down and monthly payments at 9.1% for 3 years.

    a. What would his monthly payments be?

    b. What is his finance charge?

11. Joan is paying $65.50 per month for 3 years on a $2000 loan. What interest rate is she being charged?

12. The Browns have a $38 000 second mortgage over 20 years at 10%.

    a. Calculate their monthly payments.

    b. Calculate the finance charge on this mortgage.

Answers are on page 65.

Appendix A

Simple Interest

$I = Prt$ — I = interest amount

$P = \dfrac{I}{rt}$ — P = principal amount

$r = \dfrac{I}{Pt}$ — r = annual interest rate

$t = \dfrac{I}{Pr}$ — t = years

Compound Interest

$A = P\left(1 + \dfrac{r}{n}\right)^{nt}$ — A = compound amount; n = number of times interest is calculated in one year

$I = A - P$ — P = principal (or present value); t = time (in years)

$P = \dfrac{A}{\left(1 + \dfrac{r}{n}\right)^{nt}}$ — r = annual interest rate; I = interest amount

Effective Interest Rate

$f = \left(1 + \dfrac{r}{n}\right)^{n} - 1$

f = effective rate
r = nominal rate (annual interest rate)
n = number of times interest is calculated in one year

Formulas

Ordinary Annuities

$$A = \frac{nP\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{r}$$

$$P = \frac{A\left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{nt} - 1}$$

A = amount of annuity

P = periodic payment

r = annual interest rate

n = number of times interest is calculated in one year

t = time (in years)

Loans and Mortgages

$$P = \frac{A\left(\frac{r}{n}\right)}{1-\left(1+\frac{r}{n}\right)^{-nt}}$$

$$F.C. = ntP - A$$

P = periodic payment

A = amount of loan or mortgage

r = annual interest rate

n = number of payments in one year

t = time (in years)

$F.C.$ = finance charge

Appendix B

When an investment earns compound interest, the actual interest earned is given by the formula,

$$\mathrm{I} = A - P \text{ or } \mathrm{I} = P\left(1 + \frac{r}{n}\right)^{nt} - P$$

If we let t = 1 year and P = \$1.00, the amount of interest earned in one year is,

$$\mathrm{I} = 1\left(1 + \frac{r}{n}\right)^{n(1)} - 1 = \left(1 + \frac{r}{n}\right)^{n} - 1$$

Since the effective interest rate, f, is the same as the simple interest rate, r, earned on \$1.00 after 1 year, using $r = \frac{\mathrm{I}}{Pt}$,

$$f = r = \frac{\mathrm{I}}{Pt} = \frac{\mathrm{I}}{1(1)} = \mathrm{I} = \left(1 + \frac{\mathrm{r}}{\mathrm{n}}\right)^{n} - 1$$

The effective rate is,

$$f = \left(1 + \frac{r}{n}\right)^{n} - 1$$

Appendix C

Number of Payments	ANNUAL PERCENTAGE RATE															
	6.00%	6.25%	6.50%	6.75%	7.00%	7.25%	7.50%	7.75%	8.00%	8.25%	8.50%	8.75%	9.00%	9.25%	9.50%	9.75%
1	0.50	0.52	0.54	0.56	0.58	0.60	0.62	0.65	0.67	0.69	0.71	0.73	0.75	0.77	0.79	0.81
2	0.75	0.78	0.81	0.84	0.88	0.91	0.94	0.97	1.00	1.03	1.06	1.10	1.13	1.16	1.19	1.22
3	1.00	1.04	1.09	1.13	1.17	1.21	1.21	1.29	1.34	1.38	1.42	1.46	1.50	1.55	1.59	1.63
4	1.25	1.31	1.36	1.41	1.40	1.51	1.57	1.62	1.67	1.72	1.78	1.83	1.88	1.93	1.99	2.04
5	1.50	1.57	1.63	1.69	1.76	1.82	1.88	1.95	2.01	2.07	2.13	2.20	2.26	2.32	2.39	2.45
6	1.76	1.83	1.90	1.98	2.05	2.13	2.20	2.27	2.35	2.42	2.49	2.57	2.64	2.72	2.79	2.86
7	2.01	2.09	2.18	2.26	2.35	2.43	2.52	2.60	0.68	2.77	2.85	2.94	3.02	3.11	3.19	3.28
8	2.26	2.36	2.45	2.55	2.64	2.74	2.83	2.93	3.02	3.12	3.21	3.31	3.40	3.50	3.60	3.69
9	2.52	2.62	2.73	2.83	2.94	3.05	3.15	3.26	3.36	3.47	3.57	3.68	3.79	3.89	4.00	4.11
10	2.77	2.89	3.00	3.12	3.24	3.35	3.47	3.59	3.70	3.82	3.94	4.05	4.17	4.29	4.41	4.52
11	3.02	3.15	3.28	3.41	3.53	3.66	3.79	3.92	4.04	4.17	4.30	4.43	4.56	4.68	4.81	4.94
12	3.28	3.42	3.56	3.69	3.83	3.97	4.11	4.25	4.39	4.52	4.66	4.80	4.94	5.08	5.22	5.36
13	3.53	3.68	3.83	3.98	4.13	4.28	4.43	4.58	4.73	4.88	5.03	5.18	5.33	5.48	5.63	5.78
14	3.79	3.95	4.11	4.27	4.43	4.59	4.75	4.91	5.07	5.23	5.39	5.55	5.72	5.88	6.04	6.20
15	4.05	4.22	4.39	4.56	4.73	4.90	5.07	5.24	5.42	5.59	5.76	5.93	6.10	6.28	6.45	6.62
16	4.30	4.48	4.67	4.85	5.03	5.21	5.40	5.58	5.76	5.94	6.13	6.31	6.49	6.68	6.86	7.05
17	4.56	4.75	4.95	5.14	5.33	5.52	5.72	5.91	6.11	6.30	6.49	6.69	6.88	7.08	7.27	7.47
18	4.82	5.02	5.22	5.43	5.63	5.84	6.04	6.25	6.45	6.66	6.86	7.07	7.28	7.48	7.69	7.90
19	5.07	5.29	5.50	5.72	5.94	6.15	6.37	6.58	6.80	7.00	7.23	7.45	7.67	7.89	8.10	8.32
20	5.33	5.56	5.78	6.01	6.24	6.46	6.69	6.92	7.15	7.38	7.60	7.83	8.06	8.29	8.52	8.75
21	5.59	5.83	6.07	6.30	6.54	6.78	7.02	7.26	7.50	7.74	7.97	8.21	8.46	8.70	8.94	9.18
22	5.85	6.10	6.35	6.60	6.84	7.09	7.34	7.59	7.84	8.10	8.35	8.60	8.85	9.10	9.36	9.61
23	6.11	6.37	6.63	6.89	7.15	7.41	7.67	7.93	8.19	8.46	8.72	8.98	9.25	9.51	9.77	10.04
24	6.37	6.64	6.91	7.18	7.45	7.73	8.00	8.27	8.55	8.82	9.09	9.37	9.64	9.92	10.19	10.47
25	6.63	6.91	7.19	7.48	7.76	8.04	8.33	8.61	8.90	9.18	9.47	9.75	10.04	10.33	10.62	10.90
26	6.89	7.18	7.48	7.77	8.07	8.36	8.66	8.95	9.25	9.55	9.84	10.14	10.44	10.74	11.04	11.34
27	7.15	7.46	7.76	8.07	8.37	8.68	8.99	9.29	9.60	9.91	10.22	10.53	10.84	11.15	11.46	11.77
28	7.41	7.73	8.05	8.36	8.68	9.00	9.32	9.64	9.96	10.28	10.60	10.92	11.24	11.56	11.89	12.21
29	7.67	8.00	8.33	8.66	8.99	9.32	9.65	9.98	10.31	10.64	10.97	11.31	11.64	11.98	12.31	12.65
30	7.94	8.28	8.61	8.96	9.30	9.64	9.98	10.32	10.66	11.01	11.35	11.70	12.04	12.39	12.74	13.09
31	8.20	8.55	8.90	9.25	9.60	9.96	10.31	10.67	11.02	11.38	11.73	12.09	12.45	12.81	13.17	13.53
32	8.46	8.82	9.19	9.55	9.91	10.28	10.64	11.01	11.38	11.74	12.11	12.48	12.85	13.22	13.59	13.97
33	8.73	9.10	9.47	9.85	10.22	10.60	10.98	11.36	11.73	12.11	12.49	12.88	13.26	13.64	14.02	14.41
34	8.99	9.37	9.76	10.15	10.53	10.92	11.31	11.70	12.09	12.48	12.88	13.27	13.66	14.06	14.45	14.85
35	9.25	9.65	10.05	10.45	10.85	11.25	11.65	12.05	12.45	12.85	13.26	13.66	14.07	14.48	14.89	15.29

Number of Payments	ANNUAL PERCENTAGE RATE continued															
	6.00%	6.25%	6.50%	6.75%	7.00%	7.25%	7.50%	7.75%	8.00%	8.25%	8.50%	8.75%	9.00%	9.25%	9.50%	9.75%
36	9.52	9.93	10.34	10.75	11.16	11.57	11.98	12.40	12.81	13.23	13.64	14.06	14.48	14.90	15.32	15.74
37	9.78	10.20	10.63	11.05	11.47	11.89	12.32	12.74	13.17	13.60	14.03	14.46	14.89	15.32	15.75	16.19
38	10.05	10.48	10.91	11.35	11.78	12.22	12.66	13.09	13.53	13.97	14.41	14.85	15.30	15.74	16.19	16.63
39	10.32	10.76	11.20	11.65	12.10	12.54	12.99	13.44	13.89	14.35	14.80	15.25	15.71	16.17	16.62	17.07
40	10.58	11.04	11.49	11.95	12.41	12.87	13.33	13.79	14.26	14.72	15.19	15.65	16.12	16.59	17.06	17.53
41	10.85	11.32	11.78	12.25	12.72	13.20	13.67	14.14	14.62	15.10	15.57	16.05	16.53	17.01	17.50	17.98
42	11.12	11.60	12.08	12.56	13.04	13.52	14.01	14.50	14.98	15.47	15.96	16.45	16.95	17.44	17.94	18.43
43	11.38	11.87	12.37	12.86	13.36	13.85	14.35	14.85	15.35	15.85	16.35	16.86	17.36	17.87	18.38	18.89
44	11.65	12.15	12.66	13.16	13.67	14.18	14.69	15.20	15.71	16.23	16.74	17.26	17.78	18.30	18.82	19.34
45	11.92	12.44	12.95	13.47	13.99	14.51	15.03	15.55	16.08	16.61	17.13	17.66	18.19	18.73	19.26	19.79
46	12.19	12.72	13.24	13.77	14.31	14.84	15.37	15.91	16.45	16.99	17.53	18.07	18.61	19.16	19.70	20.25
47	12.46	13.00	13.54	14.08	14.62	15.17	15.72	16.26	16.81	17.37	17.92	18.47	19.03	19.59	20.15	20.71
48	12.73	13.28	13.83	14.39	14.94	15.50	16.06	16.62	17.18	17.75	18.31	18.88	19.45	20.02	20.59	21.16
49	13.00	13.56	14.13	14.69	15.26	15.83	16.74	16.98	17.55	18.13	18.71	19.29	19.87	20.45	21.04	21.62
50	13.27	13.84	14.42	15.00	15.58	16.16	16.75	17.33	17.92	18.51	19.10	19.69	20.29	20.89	21.48	22.08
51	13.54	14.13	14.72	15.31	15.90	16.50	17.09	17.69	18.29	18.89	19.50	20.10	20.71	21.32	21.93	22.55
52	13.81	14.41	15.01	15.62	16.22	16.83	17.44	18.05	18.66	19.28	19.89	20.51	21.13	21.76	22.38	23.01
53	14.08	14.69	15.31	15.92	16.54	17.16	17.78	18.41	19.03	19.66	20.29	20.92	21.56	22.19	22.83	23.47
54	14.36	14.98	15.61	16.23	16.86	17.50	18.13	18.77	19.41	20.05	20.69	21.34	21.98	22.63	23.28	23.94
55	14.63	15.26	15.90	16.54	17.19	17.83	18.48	19.13	19.78	20.43	21.09	21.75	22.41	23.07	23.73	24.40
56	14.90	15.55	16.20	16.85	17.51	18.17	18.83	19.49	20.15	20.82	21.49	22.16	22.83	23.51	24.19	24.87
57	15.17	15.84	16.50	17.17	17.83	18.50	19.18	19.85	20.53	21.21	21.89	22.58	23.26	23.95	24.64	25.34
58	15.45	16.12	16.80	17.48	18.16	18.84	19.53	20.21	20.91	21.60	22.29	22.99	23.69	24.39	25.10	25.80
59	15.72	16.41	17.10	17.79	18.48	19.18	19.88	20.58	21.28	21.99	22.70	23.41	24.12	24.84	25.55	26.27
60	16.00	16.70	17.40	18.10	18.81	19.52	20.23	20.94	21.66	22.38	23.10	23.82	24.55	25.28	26.01	26.75

Number of Payments	ANNUAL PERCENTAGE RATE															
	10.00%	10.25%	10.50%	10.75%	11.00%	11.25%	11.50%	11.75%	12.00%	12.25%	12.50%	12.75%	13.00%	13.25%	13.50%	13.75%
1	0.83	0.85	0.87	0.90	0.92	0.94	0.96	0.98	1.00	1.02	1.04	1.06	1.08	1.10	1.12	1.15
2	1.25	1.28	1.31	1.35	1.38	1.41	1.44	1.47	1.50	1.53	1.57	1.60	1.63	1.66	1.69	1.72
3	1.67	1.71	1.76	1.80	1.84	1.88	1.92	1.96	2.01	2.05	2.09	2.13	2.17	2.22	2.26	2.30
4	2.09	2.14	2.20	2.25	2.30	2.35	2.41	2.46	2.51	2.57	2.62	2.67	2.72	2.78	2.83	2.88
5	2.51	2.58	2.64	2.70	2.77	2.83	2.89	2.96	3.02	3.08	3.15	3.21	3.27	3.34	3.40	3.46
6	2.94	3.01	3.08	3.16	3.23	3.31	3.38	3.45	3.53	3.60	3.68	3.75	3.83	3.90	3.97	4.05
7	3.36	3.45	3.53	3.62	3.70	3.78	3.87	3.95	4.04	4.12	4.21	4.29	4.38	4.47	4.55	4.64
8	3.79	3.88	3.98	4.07	4.17	4.26	4.36	4.46	4.55	4.65	4.74	4.84	4.94	5.03	5.13	5.22
9	4.21	4.32	4.43	4.53	4.64	4.75	4.85	4.96	5.07	5.17	5.28	5.39	5.49	5.60	5.71	5.82
10	4.64	4.76	4.88	4.99	5.11	5.23	5.35	5.46	5.58	5.70	5.82	5.94	6.05	6.17	6.29	6.41
11	5.07	5.20	5.33	5.45	5.58	5.71	5.84	5.97	6.10	6.23	6.36	6.49	6.62	6.75	6.88	7.01
12	5.50	5.64	5.78	5.92	6.06	6.20	6.34	6.48	6.62	6.76	6.90	7.04	7.18	7.32	7.46	7.60
13	5.93	6.08	6.23	6.38	6.53	6.68	6.84	6.99	7.14	7.29	7.44	7.59	7.75	7.90	8.05	8.20
14	6.36	6.52	6.69	6.85	7.01	7.17	7.34	7.50	7.66	7.82	7.99	8.15	8.31	8.48	8.64	8.81
15	6.80	6.97	7.14	7.32	7.49	7.66	7.84	8.01	8.19	8.36	8.53	8.71	8.88	9.06	9.23	9.41
16	7.23	7.41	7.60	7.78	7.97	8.15	8.34	8.53	8.71	8.90	9.08	9.27	9.46	9.64	9.83	10.02
17	7.67	7.86	8.06	8.25	8.45	8.65	8.84	9.04	9.24	9.44	9.63	9.83	10.03	10.23	10.43	10.63
18	8.10	8.31	8.52	8.73	8.93	9.14	9.35	9.56	9.77	9.98	10.19	10.40	10.61	10.82	11.03	11.24
19	8.54	8.76	8.98	9.20	9.42	9.64	9.86	10.08	10.30	10.52	10.74	10.96	11.18	11.41	11.63	11.85
20	8.98	9.21	9.44	9.67	9.90	10.13	10.37	10.60	10.83	11.06	11.30	11.53	11.76	12.00	12.23	12.46
21	9.42	9.66	9.90	10.15	10.39	10.63	10.88	11.12	11.36	11.61	11.85	12.10	12.34	12.59	12.84	13.08
22	9.86	10.12	10.37	10.62	10.88	11.13	11.39	11.64	11.90	12.16	12.41	12.67	12.93	13.19	13.44	13.70
23	10.30	10.57	10.84	11.10	11.37	11.63	11.90	12.17	12.44	12.71	12.97	13.24	13.51	13.78	14.05	14.32
24	10.75	11.02	11.30	11.58	11.86	12.14	12.42	12.70	12.98	13.26	13.54	13.82	14.10	14.38	14.66	14.95
25	11.19	11.48	11.77	12.06	12.35	12.64	12.93	13.22	13.52	13.81	14.10	14.40	14.69	14.98	15.28	15.57
26	11.64	11.94	12.24	12.54	12.85	13.15	13.45	13.75	14.06	14.36	14.67	14.97	15.28	15.59	15.89	16.20
27	12.09	12.40	12.71	13.03	13.34	13.66	13.97	14.29	14.60	14.92	15.24	15.56	15.87	16.19	16.51	16.83
28	12.53	12.86	13.18	13.51	13.84	14.16	14.49	14.82	15.15	15.48	15.81	16.14	16.47	16.80	17.13	17.46
29	12.98	13.32	13.66	14.00	14.33	14.67	15.01	15.35	15.70	16.04	16.38	16.72	17.07	17.41	17.75	18.10
30	13.43	13.78	14.13	14.48	14.83	15.19	15.54	15.89	16.24	16.60	16.95	17.31	17.66	18.02	18.38	18.74
31	13.89	14.25	14.61	14.97	15.33	15.70	16.06	16.43	16.79	17.16	17.53	17.90	18.27	18.63	19.00	19.38
32	14.34	14.71	15.09	15.46	15.84	16.21	16.59	16.97	17.35	17.73	18.11	18.49	18.87	19.25	19.63	20.02
33	14.79	15.18	15.57	15.95	16.34	16.73	17.12	17.51	17.90	18.29	18.69	19.08	19.47	19.87	20.26	20.66
34	15.25	15.65	16.05	16.44	16.85	17.25	17.65	18.05	18.46	18.86	19.27	19.67	20.08	20.49	20.90	21.31
35	15.70	16.11	16.53	16.94	17.35	17.77	18.18	18.60	19.01	19.43	19.85	20.27	20.69	21.11	21.53	21.95

Number of Payments	ANNUAL PERCENTAGE RATE continued															
	10.00%	10.25%	10.50%	10.75%	11.00%	11.25%	11.50%	11.75%	12.00%	12.25%	12.50%	12.75%	13.00%	13.25%	13.50%	13.75%
36	16.16	16.58	17.01	17.43	17.86	18.29	18.71	19.14	19.57	20.00	20.43	20.87	21.30	21.73	22.17	22.60
37	16.62	17.06	17.49	17.93	18.37	18.81	19.25	19.69	20.13	20.58	21.02	21.46	21.91	22.36	22.81	23.25
38	17.08	17.53	17.98	18.43	18.88	19.33	19.78	20.24	20.69	21.15	21.61	22.07	22.52	22.99	23.45	23.91
39	17.54	18.00	18.46	18.93	19.39	19.86	20.32	20.79	21.26	21.73	22.20	22.67	23.14	23.61	24.09	24.56
40	18.00	18.48	18.95	19.43	19.90	20.38	20.86	21.34	21.82	22.30	22.79	23.27	23.76	24.25	24.73	25.22
41	18.47	18.95	19.44	19.93	20.42	20.91	21.40	21.89	22.39	22.88	23.38	23.88	24.38	24.88	25.38	25.88
42	18.93	19.43	19.93	20.43	20.93	21.44	21.94	22.45	22.96	23.47	23.98	24.49	25.00	25.51	26.03	26.55
43	19.40	19.91	20.42	20.94	21.45	21.97	22.49	23.01	23.53	24.05	24.57	25.10	25.62	26.15	26.68	27.21
44	19.86	20.39	20.91	21.44	21.97	22.50	23.03	23.57	24.10	24.64	25.17	25.71	26.25	26.79	27.33	27.88
45	20.33	20.87	21.41	21.95	22.49	23.03	23.58	24.12	24.67	25.22	25.77	26.32	26.88	27.43	27.99	28.55
46	20.80	21.35	21.90	22.46	23.01	23.57	24.13	24.69	25.25	25.81	26.37	26.94	27.51	28.08	28.65	29.22
47	21.27	21.83	22.40	22.97	23.53	24.10	24.68	25.25	25.82	26.40	26.98	27.56	28.14	28.72	29.31	29.89
48	21.74	22.32	22.90	23.48	24.06	24.64	25.23	25.81	26.40	26.99	27.58	28.18	28.77	29.37	39.97	30.57
49	22.21	22.80	23.39	23.99	24.58	25.18	25.78	26.38	26.98	27.59	28.19	28.80	29.41	30.02	30.63	31.24
50	22.69	23.29	23.89	24.50	25.11	25.72	26.33	26.95	27.56	28.18	28.80	29.42	30.04	30.67	31.29	31.92
51	23.16	23.78	24.40	25.02	25.64	26.26	26.89	27.52	28.15	28.78	29.41	30.05	30.68	31.32	31.96	32.60
52	23.64	24.27	24.90	25.53	26.17	26.81	27.45	28.09	28.73	29.38	30.02	30.67	31.32	31.98	32.63	33.29
53	24.11	24.76	25.40	26.05	26.70	27.35	28.00	28.66	29.32	29.98	30.64	31.30	31.97	32.63	33.30	33.97
54	24.59	25.25	25.91	26.57	27.23	27.90	28.56	29.23	29.91	30.58	31.25	31.93	32.61	33.29	33.98	34.66
55	25.07	25.74	26.41	27.09	27.77	28.44	29.13	29.81	30.50	31.18	31.87	32.56	33.26	33.95	34.65	35.35
56	25.55	26.23	26.92	27.61	28.30	28.99	29.69	30.39	31.09	31.79	32.49	33.20	33.91	34.62	35.33	36.04
57	26.03	26.73	27.43	28.13	28.84	29.54	30.25	30.97	31.68	32.39	33.11	33.83	34.56	35.28	36.01	36.74
58	26.51	27.23	27.94	28.66	29.37	30.10	30.82	31.55	32.27	33.00	33.74	34.47	35.21	35.95	36.69	37.43
59	27.00	27.72	28.45	29.18	29.91	30.65	31.39	32.13	32.87	33.61	34.36	35.11	35.86	36.62	37.37	38.13
60	27.48	28.22	28.96	29.71	30.45	31.20	31.96	32.71	33.47	34.23	34.99	35.75	36.52	37.29	38.06	38.83

Number of Payments	ANNUAL PERCENTAGE RATE															
	14.00%	14.25%	14.50%	14.75%	15.00%	15.25%	15.50%	15.75%	16.00%	16.25%	16.50%	16.75%	17.00%	17.25%	17.50%	17.75%
1	1.17	1.19	1.21	1.23	1.25	1.27	1.29	1.31	1.33	1.35	1.37	1.40	1.42	1.44	1.46	1.48
2	1.75	1.78	1.82	1.85	1.88	1.91	1.94	1.97	2.00	2.04	2.07	2.10	2.13	2.16	2.19	2.22
3	2.34	2.38	2.43	2.47	2.51	2.55	2.59	2.64	2.68	2.72	2.76	2.80	2.85	2.89	2.93	2.97
4	2.93	2.99	3.04	3.09	3.14	3.20	3.25	3.30	3.36	3.41	3.46	3.51	3.57	3.62	3.67	3.73
5	3.53	3.59	3.65	3.72	3.78	3.84	3.91	3.97	4.04	4.10	4.16	4.23	4.29	4.35	4.42	4.48
6	4.12	4.20	4.27	4.35	4.42	4.49	4.57	4.64	4.72	4.79	4.87	4.94	5.02	5.09	5.17	5.24
7	4.72	4.81	4.89	4.98	5.06	5.15	5.23	5.32	5.40	5.49	5.58	5.66	5.75	5.83	5.92	6.00
8	5.32	5.42	5.51	5.61	5.71	5.80	5.90	6.00	6.09	6.19	6.29	6.38	6.48	6.58	6.67	6.77
9	5.92	6.03	6.14	6.25	6.35	6.46	6.57	6.68	6.78	6.89	7.00	7.11	7.22	7.32	7.43	7.54
10	6.53	6.65	6.77	6.88	7.00	7.12	7.24	7.36	7.48	7.60	7.72	7.84	7.96	8.08	8.19	8.31
11	7.14	7.27	7.40	7.53	7.66	7.79	7.92	8.05	8.18	8.31	8.44	8.57	8.70	8.83	8.96	9.09
12	7.74	7.89	8.03	8.17	8.31	8.45	8.59	8.74	8.88	9.02	9.16	9.30	9.45	9.59	9.73	9.87
13	8.36	8.51	8.66	8.81	8.97	9.12	9.27	9.43	9.58	9.73	9.89	10.04	10.20	10.35	10.50	10.66
14	8.97	9.13	9.30	9.46	9.63	9.79	9.96	10.12	10.29	10.45	10.62	10.78	10.95	11.11	11.28	11.45
15	9.59	9.76	9.94	10.11	10.29	10.47	10.64	10.82	11.00	11.17	11.35	11.53	11.71	11.88	12.06	12.24
16	10.20	10.39	10.58	10.77	10.95	11.14	11.33	11.52	11.71	11.90	12.09	12.28	12.46	12.65	12.84	13.03
17	10.82	11.02	11.22	11.42	11.62	11.82	12.02	12.22	12.42	12.62	12.83	13.03	13.23	13.43	13.63	13.83
18	11.45	11.66	11.87	12.08	12.29	12.50	12.72	12.93	13.14	13.35	13.57	13.78	13.99	14.21	14.42	14.64
19	12.07	12.30	12.52	12.74	12.97	13.19	13.41	13.64	13.86	14.09	14.31	14.54	14.76	14.99	15.22	15.44
20	12.70	12.93	13.47	13.41	13.64	13.88	14.11	14.35	14.59	14.82	15.06	15.30	15.54	15.77	16.01	16.25
21	13.33	13.58	13.82	14.07	14.32	14.57	14.82	15.06	15.31	15.56	15.81	16.06	16.31	16.56	16.81	17.07
22	13.96	14.22	14.48	14.74	15.00	15.26	15.52	15.78	16.04	16.30	16.57	16.83	17.09	17.36	17.62	17.88
23	14.59	14.87	15.14	15.41	15.68	15.96	16.23	16.50	16.78	17.05	17.32	17.60	17.88	18.15	18.43	18.70
24	15.23	15.51	15.80	16.08	16.37	16.65	16.94	17.22	17.51	17.80	18.09	18.37	18.66	18.95	19.24	19.53
25	15.87	16.17	16.46	16.76	17.06	17.35	17.65	17.95	18.25	18.55	18.85	19.15	19.45	19.75	20.05	20.36
26	16.51	16.82	17.13	17.44	17.75	18.06	18.37	18.68	18.99	19.30	19.62	19.93	20.24	20.56	20.87	21.19
27	17.15	17.47	17.80	18.12	18.44	18.76	19.09	19.41	19.74	20.06	20.39	20.71	21.04	21.37	21.69	22.02
28	17.80	18.13	18.47	18.80	19.14	19.47	19.81	20.15	20.48	20.82	21.16	21.50	21.84	22.18	22.52	22.86
29	18.45	18.79	19.14	19.49	19.83	20.18	20.53	20.88	21.23	21.58	21.94	22.29	22.64	22.99	23.35	23.70
30	19.10	19.45	19.81	20.17	20.54	20.90	21.26	21.62	21.99	22.35	22.72	23.08	23.45	23.81	24.18	24.55
31	19.75	20.12	20.47	20.87	21.24	21.61	21.99	22.37	22.74	23.12	23.50	23.88	24.26	24.64	25.02	25.40
32	20.40	20.79	21.17	21.56	21.95	22.33	22.72	23.11	23.50	23.89	24.28	24.68	25.07	25.46	25.86	26.25
33	21.06	21.46	21.85	22.25	22.65	23.06	23.46	23.86	24.26	24.67	25.07	25.48	25.88	26.29	26.70	27.11
34	21.72	22.13	22.54	22.95	23.37	23.78	24.19	24.64	25.03	25.44	25.86	26.28	26.70	27.12	27.54	27.97
35	22.38	22.80	23.23	23.65	24.08	24.51	24.94	25.36	25.79	26.23	26.66	27.09	27.52	27.96	28.39	28.83

Number of Payments	ANNUAL PERCENTAGE RATE continued															
	14.00%	14.25%	14.50%	14.75%	15.00%	15.25%	15.50%	15.75%	16.00%	16.25%	16.50%	16.75%	17.00%	17.25%	17.50%	17.75%
36	23.04	23.48	23.92	24.35	24.80	25.24	25.68	26.12	26.57	27.01	27.46	27.90	28.35	28.80	29.25	29.70
37	23.70	24.16	24.61	25.06	25.51	25.97	26.42	26.88	27.34	27.80	28.26	28.72	29.18	29.64	30.10	30.57
38	24.37	24.84	25.30	25.77	26.24	26.70	27.17	27.64	28.11	28.59	29.06	29.53	30.01	30.49	30.96	31.44
39	25.04	25.52	26.00	26.48	26.96	27.44	27.92	28.41	28.89	29.38	29.87	30.36	30.85	31.34	31.83	32.32
40	25.71	26.20	26.70	27.19	27.69	28.18	28.68	29.18	29.68	30.18	30.68	31.18	31.68	32.19	32.69	33.20
41	26.39	26.89	27.40	27.91	28.41	28.92	29.44	29.95	30.46	30.97	31.49	32.01	32.52	33.04	33.56	34.08
42	27.06	27.58	28.10	28.62	29.15	29.67	30.19	30.72	31.25	31.78	32.31	32.84	33.37	33.90	34.44	34.97
43	27.74	28.27	28.81	29.34	29.88	30.42	30.96	31.50	32.04	32.58	33.13	33.67	34.22	34.76	35.31	35.86
44	28.42	28.97	29.52	30.07	30.62	31.17	31.72	32.28	32.83	33.39	33.95	34.51	35.07	35.63	36.19	36.76
45	29.11	29.67	30.23	30.79	31.36	31.92	32.49	33.06	33.63	34.20	34.77	35.35	35.92	36.50	37.08	37.66
46	29.79	30.36	30.94	31.52	32.10	32.68	33.26	33.84	34.43	35.01	35.60	36.19	36.78	37.37	37.96	38.56
47	30.48	31.07	31.66	32.25	32.84	33.44	34.03	34.63	35.23	35.83	36.43	37.04	37.64	38.25	38.86	39.46
48	31.17	31.77	32.37	32.98	33.59	34.20	34.81	35.42	36.03	36.65	37.27	37.88	38.50	39.13	39.75	40.37
49	31.86	32.48	33.09	33.71	34.34	34.96	35.59	36.21	36.84	37.47	38.10	38.74	39.37	40.01	40.65	41.29
50	32.55	33.18	33.82	34.45	35.09	35.73	36.37	37.01	37.65	38.30	38.94	39.59	40.24	40.89	41.55	42.20
51	33.25	33.89	34.54	35.19	35.84	36.49	37.15	37.81	38.46	39.12	39.79	40.45	41.11	41.78	42.45	43.12
52	33.95	34.61	35.27	35.93	36.60	37.27	37.94	38.61	39.28	39.96	40.63	41.31	41.99	42.67	43.36	44.04
53	34.65	35.32	36.00	36.68	37.36	38.04	38.72	39.41	40.10	40.79	41.48	42.17	42.87	43.57	44.27	44.97
54	35.35	36.04	36.73	37.42	38.12	38.82	39.52	40.22	40.92	41.63	42.33	43.04	43.75	44.47	45.18	45.90
55	36.05	36.76	37.46	38.17	38.88	39.60	40.31	41.03	41.74	42.47	43.19	43.91	44.64	45.37	46.10	46.83
56	36.76	37.48	38.20	38.92	39.65	40.38	41.11	41.84	42.57	43.31	44.05	44.79	45.53	46.27	47.02	47.77
57	37.47	38.20	38.94	39.68	40.42	41.16	41.91	42.65	43.40	44.15	44.91	45.66	46.42	47.18	47.94	48.71
58	38.18	38.96	39.68	40.43	41.19	41.95	42.71	43.47	44.23	45.00	45.77	46.54	47.32	48.09	48.87	49.65
59	38.89	39.66	40.42	41.19	41.96	42.74	43.51	44.29	45.07	45.85	46.64	47.42	48.21	49.01	49.80	50.60
60	39.61	40.39	41.17	41.95	42.74	43.53	44.32	45.11	45.91	46.71	47.51	48.31	49.12	49.92	50.73	51.55

Answers

Exercise 1

1. a. 365 b. 52 c. 12
2. a. \$100 b. \$7.50 c. \$11.25 d. \$5.50
   e. \$3.50 f. \$2.00 g. \$44.52 h. \$30.29
3. interest \$56.25, altogether \$1500.00 + 56.25 = \$1556.25
4. I = 4.00(0.18)(1) = \$0.72. 1 share (now) = \$3.28
5. a. \$16 b. \$116 c. I = 116(0.12)(1) = \$13.92
   d. \$16.00 + \$13.92 = \$29.92
6. Mary owes Larry \$250 + \$4.32 = \$254.32
7. a. $I = 5000\ (0.0055)\ (\frac{6}{12}) = \$137.50$ b, $I = 5000\ (0.0055)5 = \$1375.00$

Exercise 2

1. a. \$2000.00 b. \$50.00 c. \$2561.40 d. \$48 484.85
2. a. 2.5% b. 6% c. 4.8% d. 4.4%
3. a. 0.5 years b. 0.292 days c. 0.53 years d. 0.3 years
4.

I	*P*	*r*	*t*
\$3.00	\$100.00	3%	1 year
\$50.00	\$2000.00	5%	6 months
\$3.42	\$631.38	6.5%	1 month
\$38.00	\$800.00	4.75%	1 year

5. 4.65%
6. 0.13 years or 47 days
7. 18.25%
8. a. 200 × \$18 = \$3600.00
   b. \$0.81 per share
   c. 200 × \$0.81 = \$162.00
   d. $r = \dfrac{162}{3600\left(\frac{9}{12}\right)} = 6\%$
   e. 200 × (22.50 – 18.00) = \$900.00
   f. Total earnings = \$162 + \$900 = \$1062
   g. $r = \dfrac{1062}{3600\left(\frac{9}{12}\right)} = 39.33\%$

Exercise 3

1. a. 1 b. 2 c. 4 d. 12 e. 52 f. 365

2. a.

Year	Principal Amount	Earned Interest	Year End Total
1	$1000	$50	$1050
2	$1050	$52.50	$1102.50
3	$1102.50	$55.13	$1157.63
4	$1157.63	$57.88	$1215.51
5	$1215.51	$60.78	$1276.29

b. $A = 1000\left(1 + \frac{0.05}{1}\right)^{1\times5} = \$1276.28$

3. a. $A = \$153.86$, $I = \$53.86$ b. $A = \$155.30$, $I = \$55.30$
   c. $A = \$156.05$, $I = \$56.05$ d. $A = \$156.57$, $I = \$56.57$
   e. $A = \$156.82$, $I = \$56.82$
4. a. $1631.02 b. $4200.01
5. a. $17 449.40 b. $45 259.26
6. a. $800 b. $808.13 c. The savings certificate by $8.13
7. a. A = $4,548.49 b. $8250.97 c. $14 923.31 d. 3.28 times
8. a. $5209.60 b. $5428.00 c. $5655.55 d. Yes!

Exercise 4

1. a. $74.41 b. $2089.58 c. $1542.17
   d. $565.48 e. $958.53
2. $5262.79
3. a. $4787.84 b. $15 212.16
4. a. $675.56 b. $456.39 c. $308.32
   d. check with your instructor

Exercise 5

1. a. 8.30% b. 10.47% c. 12.68% d. 5.12%
2. a. 10% b. 10.25% c. 10.38% d. 10.47%
   e. 10.51% f. 10.52%
3. The effective rate of $8\frac{3}{4}\%$ compounded monthly is 9.11% (the better deal) and the difference is 0.11%.
4. 6.17%

Exercise 6

1. a.

Date	Total amount from previous year	Interest earned on total amount	Annual deposit
Dec. 31/01	—	—	$2000
Dec. 31/02	$2000	$100	$2000
Dec. 31/03	$4100	$205	$2000
Dec. 31/04	$6305	$315.25	$2000
Dec. 31/05	$8620.25	$431.0125	$2000

Jan. 1/06	$11 051.26

b. $11 051.26 The answers should be the same.

2. a. $14 486.56 b. $6629.90 c. $7971.96
3. a. $1233.56 b. $2543.20 c. $6977.00 d. $16 387.93
4. a. $49 173.53 b. $98 347.06 c. $196 694.11 d. $344 214.70
5. a. $118 016.47 b. $132 683.33 c. Monthly by $14 666.87
6. a. $498.00 b. $19 203.45
7.

Years	Annuity Amount
20	$57 275.00
25	$98 347.06
30	$164 494.02
35	$271 024.36
40	$442 592.55

8. The annuity is worth (442 593 ÷ 57 275) almost 8 times more.

Exercise 7

1. a. $81.44 per month b. $629.48 per year c. $372.80 per quarter
2. $163.09
3. $80.69
4.

Years	Yearly Payment
15	$3147.38
20	$1745.96
25	$1016.81
30	$607.92
35	$368.97
40	$225.94

5. $96.26
6. $4143.65

Exercise 8

1. P = $89.79 and $F.C.$ = $77.48

2.

Interest Rate	Monthly Payment	Finance Charge
6%	$152.11	$475.96
9%	$159.00	$724.00
13%	$168.47	$1064.92
19%	$183.28	$1598.08

3.

Term of Loan	Monthly Payment	Finance Charge
6 Months	$345.10	$70.60
1 Year	$177.70	$132.40

2 Years	$94.15	$259.60
5 Years	$44.49	$669.40

4. P = \$76.62, $F.C.$ = \$24.44
5. a. \$97.73 b. \$88 c. \$72.76 d. the bank

Exercise 9

1. a. $\$120\,000 \div 2\frac{1}{2} = \$48\,000.00$

   b. 10% of \$120 000 = \$12 000.00

   c. $\frac{1}{3}$ of \$2000.00 = \$666.67
2. P = \$620.24, $F.C.$ = \$68 857.60
3.

Interest Rate	Monthly Payment	Finance Charge
8%	$752.80	$90 672.00
10%	$868.52	$118 444.80
12%	$990.98	$147 835.20
14%	$1119.17	$178 600.80

4.

Length of Mortgage	Monthly Payment	Finance Charge
15 years	$927.01	$66 861.80
20 years	$805.59	$93 341.60
25 years	$738.99	$121 697.00
30 years	$699.21	$151 715.60

5. \$141.13
6. a. \$297.69 b. \$12 222.73 c. \$57 607.27

Exercise 10

1. a. \$2000 b. \$100 c. 9.00%
2. 11.50%
3. 12 month loan is at 10.25%. 16 month loan is at 11.75%.
4. 7.5%
5. 16.00% The bank offers the best deal.

Review Questions

1. Simple
2. Compound
3. \$7.88
4. 9.33%
5. \$4096.54
6. \$612.95
7. 9.96%
8. \$3314.95
9. \$106.77
10. \$253.18 b. \$1164.48

11. 11%
12. a. $366.71 b. $50 010.40

Notes